A Refugee Camp Curriculum

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The name ‘refugee camp curriculum’ is a metaphor for a situation where mathematics is taught from the beginning and with simple manipulatives. Thus, it is also a proposal for a curriculum for early childhood education, for adult education, for educating immigrants, and for learning mathematics outside institutionalized education.

It considers mathematics a number-language parallel to our word-language, both describing the outside world in full sentences, typically containing a subject and a verb and a predicate.

The task of the number-language is to describe the natural fact Many in space and time, first by counting and recounting and double-counting to transform outside examples of Many to inside sentences about the total; then by adding to unite (or split) inside totals in different ways depending on their units and on them being constant or changing.

This allows designing a curriculum for all students inspired by Tarp (2018) that focuses on proportionality, solving equations and calculus from the beginning, since proportionality occurs when recounting in a different unit, equations occur when recounting from tens to icons, and calculus occurs when adding block-numbers next-to and when adding per-numbers coming from double-counting in two units.

Talking about ‘refugee camp mathematics’ thus allows locating a setting where children do not have access to normal education, thus raising the question ‘What kind and how much mathematics can children learn outside normal education especially when residing outside normal housing conditions and without access to traditional learning materials?’.

This motivates another question ‘How much mathematics can be learned as ‘finger-math’ using the examples of Many coming from the body like fingers, arms, toes, and legs?’

So, the goal of ‘refugee camp mathematics’ is to learn core mathematics through ‘Finger-math’ disclosing how much math comes from counting the fingers.

Content

Focus 01. Digits as Icons with as Many Outside Sticks and Inside Strokes as They Present

Activity 01. With outside things (sticks, cars, dolls, animals), many ones are rearranged into one many-icon with as many things as it represents. Inside, we write the icon with as many strokes as it represents.

Observe that the actual digits from 1 to 9 are icons with as many strokes as they represent if written less sloppy. A discovery glass showing nothing is an icon for zero.

When counting by bundling in tens, ten become '1 Bundle, 0 unbundled' or 1B0 or just 10, thus needing no icon since after nine, a double-counting takes place of bundles and unbundled.

Focus 02. Counting Ten Fingers in Various Ways

Activity 01. Double-count ten fingers in bundles of 5s and in singles

● Outside, lift the finger to be counted; inside say "0 bundle 1, 0B2, 0B3, 0B4, 0B5 or 1B0. Then continue with saying "1B1, …, 1B5 or 2B".

● Outside, look at the fingers not yet counted; inside say "1 bundle less4, 1B-3, 1B-2, 1B-1, 1B or 1B0. Then continue with saying "2B-4, …, 2B or 2B0".

● Outside, show the fingers as ten ones.

● Outside, show ten fingers as 1 5s and 5 1s; inside say "The total is 1Bundle5 5s" and write 'T = 1B5 5s'.

● Outside, show ten fingers as 2 5s; inside say "The total is 2Bundle0 5s" and write 'T = 2B0 5s'.

Activity 02. Double-count ten fingers in bundles of tens and in singles

● Outside, lift the finger to be counted; inside say "0 bundle 1, 0B2, 0B3, …, 0B9, 0Bten, or 1B0".

● Outside, look at the fingers not yet counted; inside say "1 bundle less9, 1B-8, …, 1B-2, 1B-1, 1B or 1B0.

Activity 03. Counting ten fingers in bundles of 4s using 'flexible bundle-numbers'.

● Outside, show the fingers as ten ones, then as one tens.

● Outside, show ten fingers as 1 4s and 6 1s; inside say "The total is 1Bundle6 4s, an overload" and write 'T = 1B6 4s'

● Outside, show ten fingers as 2 4s and 2 1s; inside say "The total is 2Bundle2 4s, a standard form" and write 'T = 2B2 4s'.

● Outside, show ten fingers as 3 4s less 2; inside say “The total is 3Bundle, less2, 4s, an underload” and write ‘T = 3B-2 4s’.

Activity 04. Counting ten fingers in bundles of 3s using ‘flexible bundle-numbers’.

● Outside, show ten fingers as 1 3s and 7 1s; inside say “The total is 1Bundle7 3s, an overload” and write ‘T = 1B7 3s’.

● Outside, show ten fingers as 2 3s and 4 1s; inside say “The total is 2Bundle4 3s, an overload” and write ‘T = 2B4 3s’.

● Outside, show ten fingers as 3 3s and 1 1s; inside say “The total is 3Bundle1 3s, a standard form” and write ‘T = 3B1 3s’.

● Outside, show ten fingers as 4 3s less 2; inside say “The total is 4Bundle, less2, 3s, an underload” and write ‘T = 4B-2 3s’.

Activity 05. Counting ten fingers in bundles of 3s, now also using bundles of bundles.

● Outside, show ten fingers as 3 3s (a bundle of bundles) and 1 1s;

Inside say “The total is 1BundleBundle1 3s” and write ‘T = 1BB1 3s’.

Now, inside say “The total is 1BundleBundle 0 Bundle 1 3s” and write ‘T = 1BB 0B 1 3s’.

Now, inside say “The total is 1BundleBundle 1 Bundle, less2, 3s” and write ‘T = 1BB 1B -2 3s’.

Focus 03. Counting Ten Sticks in Various Ways

The same as Focus 02, but now with sticks instead of fingers.

Focus 04. Counting Ten Cubes in Various Ways

The same as Focus 02, but now with cubes, e.g., centi-cubes or Lego Bricks, instead of fingers.

When possible, transform multiple bundles into 1 block, e.g., 2 4s = 1 2x4 block; inside say “The total is 1 2x4 block” and write ‘T = 2B0 4s.’

Focus 05. Counting a Dozen Finger-parts in Various Ways

Except for the thumps, our fingers all have three parts. So, four fingers have three parts four times, i.e., a total of T = 4 3s = 1 dozen finger-parts.

Focus 05 is the same as focus 02, but now with a dozen finger-parts instead of ten fingers.

Focus 06. Counting a Dozen Sticks in Various Ways

Focus 06 is the same as focus 03, but now with a dozen sticks instead of ten.

Focus 07. Counting a Dozen Cubes in Various Ways

Focus 07 is the same as focus 04, but now with a dozen cubes instead of ten.

Focus 08. Counting Numbers with Underloads and Overloads.

Activity 01. Totals counted in tens may also be recounted in under- or overloads.

● Inside, rewrite T = 23 as T = 2B3 tens, then as 1B13 tens, then as 3B-7tens.

● Try other two-digit numbers as well.

● Inside, rewrite T = 234 as T = 2BB3B4 tens, then as T = 2BB 2B14, then as T = 2BB 4B-6. Now rewrite T = 234 as T = 23B4, then as 22B14, then as 24B-6. Now rewrite T = 234 as T = 3BB-7B4, then as 3BB-6B-6.

● Try other three-digit numbers as well.

Focus 09. Operations as Icons Showing Pushing, Lifting and Pulling

Activity 01. Transform the three outside counting operations (push, lift and pull) into three inside operation-icons: division, multiplication and subtraction.

● Outside, place five sticks as 5 1s.

● Outside, push away 2s with a hand or a sheet; inside say "The total 5 is counted in 2s by pushing away 2s with a broom iconized as an uphill stroke" and write 'T = 5 = 5/2 2s'.

● Outside, rearrange the 2 2s into 1 2x2 block by lifting up the bundles into a stack; inside say "The bundles are stacked into a 2x2 block by lifting up bundles iconized as a lift" and write 'T = 2 2s = 2x2'.

● Outside, pull away the 2x2 block to locate unbundled 1s; inside say "The 2x2 block is pulled away, iconized as a rope" and write 'T = 5 – 2x2 = 1'.

Five counted in 2s:

I I I I I   (push away 2s)

II II I   (lift to stack)

II
II I   (pull to find unbundles ones)

II
II   I

Focus 10. The Inside Recount-Formula T = (T/B)xB Predicts Outside Bundle-counting Results

Activity 01. Use a calculator to predict a bundle-counting result by a recount-formula T = (T/B)xB, saying "from T, T/B times, B is pushed away", thus using a full number-language sentence with a subject, a verb and a predicate.

● Outside, place five cubes as 5 1s.

● Outside, push away 2s with a ‘broom’; inside say “Asked ‘5/2’, a calculator answers ‘2.some’, meaning that 2 times we can push ways bundles of 2s.

● Outside, stack the 2s into one 2x2 stack by lifting; inside say “We lift the 2 bundles into one 2x2 stack, and we write T = 2 2s = 2x2

● Outside, we locate the unbundled by, from 5 pulling away the 2x2 block; inside we say “Asked ‘5-2x2’, a calculator answers ‘1’. We write T = 2B1 2s and say “The recount-formula predicts that 5 recounts in 2s as T = 2B1 2s, which is tested by recounting five sticks manually outside.”

Activity 02. The same as activity 01, but now with 4 3s counted in 5s, 4s and 3s.

Focus 11. Discovering Decimals, Fractions and Negative Numbers.

Activity 01. When bundle-counting a total, the unbundled can be placed next-to or on-top.

● Outside, chose seven cubes to be counted in 3s.

● Outside, push away 3s to be lifted into a 2x3 stack to be pulled away to locate one unbundled single. Inside use the recount-formula to predict the result, and say “seven ones recounts as 2B1 3s” and write T = 2B1 3s.

● Outside, place the single next-to the stack. Inside say “Placed next-to the stack the single becomes a decimal-fraction ‘.1’ so now seven recounts as 2.1 3s” and write T = 2.1 3s.

● Outside, place the single on-top of the stack. Inside say “Placed on-top of the stack the single becomes a fraction-part 1 of 3, so now seven recounts as 2 1/3 3s” and write T = 2 1/3 3s. Now, inside say “Placed on-top of the stack the single becomes a full bundle less 2, so now seven recounts as 3.-2 3s” and write T = 3.-2 3s. Finally, inside say “With 3 3s as 1 bundle-bundle of 3s, seven recounts as 1BB-2 3s.”

Activity 02. The same as activity 01, but now with first 2 then 3 etc. until a dozen counted in 3s.

Activity 03. The same as activity 01, but now with first 2 then 3 etc. until a dozen counted in 4s.

Activity 04. The same as activity 01, but now with first 2 then 3 etc. until a dozen counted in 5s.

Focus 12. Recount in a New Unit to Change Units, Predicted by the Recount-Formula

Activity 01. When bundle-counting, all numbers have units that may be changed into a new unit by recounting predicted by the recount-formula.

● Outside, chose 3 4s to be recounted in 5s.

● Outside, rearrange the block in 5s to find the answer T = 3 4s = 2B2 5s. Inside use the recount-formula to predict the result, and say "three fours recounts as 2B2 5s" and write T = 3 4s = 2B2 5s = 3B-3 5s = 2 2/5 5s.

Repeat with other examples as e.g., 4 5s recounted in 6s.

Focus 13. Recount from Tens to Icons

Activity 01. A total counted in tens may be recounted in icons, traditionally called division.

● Outside, chose 29 or 2B9 tens to be recounted in 8s.

● Outside, rearrange the block in 8s to find the answer T = 29 = 3B5 8s and notice that a block that decreases its base must increase its height to keep the total the same. Inside use the recount-formula to predict the result, and say "With the recount-formula, a calculator predicts that 2 bundle 9 tens recounts as 3B5 8s" and write

T = 29 = 2B9 tens = 3B 5 8s = 4B-3 8s = 3 5/8 8s.

Repeat with other examples as e.g., 27 recounted in 6s.

* Now, inside reformulate the outside question 'T = 29 = ? 8s' as an equation using the letter u for the unknown number, u*8 = 24, to be solved by recounting 24 in 8s:

T = u*8 = 24 = (24/8)*8,

so that the unknown number is u = 24/8, attained by moving 8 to the opposite side with the opposite sign.

Use an outside ten-by-ten abacus to see that when a block decreases its base from ten to 8, it must increase its height from 2.4 to 3.

Repeat with other examples as e.g. 17 = ? 3s.

Focus 14. Recount from Icons to Tens

Activity 01. Oops, without a ten-button, a calculator cannot use the recount-formula to predict the answer if asking 'T = 3 7s = ? tens'. However, it is programmed to give the answer directly by using multiplication alone:

T = 3 7s = 3*7 = 21 = 2.1 tens,

only it leaves out the unit and misplaces the decimal point.

Use an outside ten-by-ten abacus to see that when a block increases its base from 7 to ten, it must decrease its height from 3 to 2.1.

Activity 02. Use 'less-numbers', geometrically on an abacus, or algebraically with brackets:

T = 3*7 = 3 * (ten less 3) = 3 * ten less 3*3 = 3ten less 9 = 3ten less (ten less1) = 2ten less less 1 = 2ten & 1 = 21.

Consequently ‘less less 1’ means adding 1.

Focus 15. Double-Counting in Two Physical Units

Activity 01. We observe that double-counting in two physical units creates ‘per-numbers’ as e.g. 2$ per 3kg, or 2$/3kg.

To bridge units, we recount in the per-number. Asking ‘6$ = ?kg’ we recount 6 in 2s:

T = 6$ = (6/2)*2$ = (6/2)*3kg = 9kg; and

T = 9kg = (9/3)*3kg = (9/3)*2$ = 6$.

Repeat with other examples as e.g., 4$ per 5days.

Focus 16. Double-Counting in the Same Unit Creates Fractions

Activity 01. Double-counting in the same unit creates fractions and percent as 4$/5$ = 4/5, or 40$/100$ = 40/100 = 4%.

Finding 40% of 20$ means finding 40$ per 100$ so we re-count 20 in 100s:

T = 20$ = (20/100)*100$ giving (20/100)*40$ = 8$.

Finding 3$ per 4$ in percent, we recount 100 in 4s, that many times we get 3$:

T = 100$ = (100/4)*4$ giving (100/4)*3$ = 75$ per 100$, so ¾ = 75%.

We observe that per-numbers and fractions are not numbers, but operators needing a number to become a number.

Repeat with other examples as e.g. 2$/5$.

Focus 17. Mutually Double-Counting the Sides in a Block Halved by its Diagonal

Activity 01. Recount sides in a block halved by its diagonal?

Here, in a block with base b, height a, and diagonal c, recounting creates the per-numbers:

a = (a/c)*c = sinA*c;

b = (b/c)*c = cosA*c;

a = (a/b)*b = tanA*b.

Use these formulas to predict the sides in a half-block with base 6 and angle 30 degrees. Use these formulas to predict the angles and side in a half-block with base 6 and height 4.

Focus 18. Adding Next-to

Activity 01. With T1 = 2 3s and T2 = 3 5s, what is T1+T2 when added next-to as 8s?"

Here the learning opportunity is that next-to addition geometrically means adding by areas, so multiplication precedes addition. Algebraically, the recount-formula predicts the result. Since 3*5 is an area, adding next-to in 8s means adding areas, called integral calculus.

Asking a calculator, the two answers, '2.some' and '5', predict the result as 2B5 8s.

Focus 19. Reversed Adding Next-to

Activity 01. With T1 = 2 3s and T2 adding next-to as T = 4 7s, what is T2?"

Here the learning opportunity is that when finding the answer by removing the initial block and recounting the rest in 3s, subtraction precedes division, which is natural as reversed integration, also called differential calculus.

Asking '3 5s and how many 3s total 2B6 8s?', using sticks will give the answer 2B1 3s. Adding or integrating two stacks next-to each other means multiplying before adding.

Reversing integration then means subtracting before dividing, as shown in the gradient formula

$y' = \Delta y/t = (y2 – y1)/t$.

Focus 20. Adding On-top

Activity 01. With T1 = 2 3s and T2 = 3 5s, what is T1+T2 when added on-top as 3s; and as 5s?"

Here the learning opportunity is that on-top addition means changing units by using the recount-formula. Thus, on-top addition may apply proportionality; an overload is removed by recounting in the same unit.

Adding on-top in 5s, '3 5s + 2 3s = ? 5s?', re-counting must make the units the same. Asking a calculator, the two answers, '4.some' and '1', predict the result as 4B1 5s.

Focus 21. Reversed Adding On-top

Activity 01. With T1 = 2 3s and T2 as some 5s adding to T = 4 5s, what is T2?"

Here the learning opportunity is that when finding the answer by removing the initial block and recounting the rest in 5s, subtraction precedes division, again called differential calculus. An underload is removed by recounting.

Reversed addition is called backward calculation or solving equations.

Focus 22. Adding Tens

Activity 01. With T1 = 23 and T2 = 48, what is T1+T2 id added as tens?" Recounting removes an overload:

T1+T2 = 23 + 48 = 2B3 + 4B8 = 6B11 = 7B1 = 71.

Focus 23. Subtracting Tens

Activity 01. "If T1 = 23 and T2 add to T = 71, what is T2?"

Here, recounting removes an underload:

T2 = 71 – 23 = 7B1 – 2B3 = 5B-2 = 4B8 = 48; or

T2 = 956 – 487 = 9BB5B6 – 4BB8B7 = 5BB-3B-1 = 4BB7B-1 = 4BB6B9 = 469.

Since T = 19 = 2.-1 tens, T2 = 19 -(-1) = 2.-1 tens take away -1 = 2 tens = 20 = 19+1, so -(-1) = +1.

Focus 24. Multiplying Tens

Activity 01. "What is 7 43s recounted in tens?"

Here the learning opportunity is that also multiplication may create overloads:

T = 7*43 = 7*4B3 = 28B21 = 30B1 = 301; or

27*43 = 2B7*4B3 =8BB+6B+28B+21 =8BB34B21 = 8BB36B1 = 11BB6B1 = 1161, solved geometrically in a 2x2 block.

Focus 25. Dividing Tens

Activity 01. "What is 348 recounted in 6s?"

Here the learning opportunity is that recounting a total with overload often eases division:

T = 348 /6 = 34B8 /6 = 30B48 /6 = 5B8 = 58; and

T = 349 /6 = 34B9 /6 = 30B49 /6 = (30B48 +1) /6 = 58 + 1/6.

Focus 26. Adding Per-Numbers

Activity 01. "2kg of 3$/kg + 4kg of 5$/kg = 6kg of what?"

Here we see that the unit-numbers 2 and 4 add directly whereas the per-numbers 3 and 5 add by areas since they must first transform to unit-numbers by multiplication, creating the areas.

Here, the per-numbers are piecewise constant. Later, asking 2 seconds of 4m/s increasing constantly to 5m/s leads to finding the area in a 'locally constant' (continuous) situation defining local constancy by epsilon and delta.

Activity 02. Two groups of voters have a different positive attitude to a proposal. How to find the total positive attitude?

● Asking "20 voters with 30% positive + 60 voters with 10% positive = 80 voters with ? positive."

Here we see that the unit-numbers 20 and 40 add directly whereas the per-numbers 30% and 10% add by areas since they must first transform to unit-numbers by multiplication, creating the areas.

Focus 27. Subtracting Per-Numbers

Activity 01. "2kg of 3$/kg + 4kg of what = 6kg of 5$/kg?"

Here the learning opportunity is that unit-numbers 6 and 2 subtract directly whereas the per-numbers 5 and 3 subtract by areas since they must first transform into unit-number by multiplication, creating the areas.

Later, in a 'locally constant' situation, subtracting per-numbers is called differential calculus.

Focus 28. Adding Differences

Activity 01. Adding many numbers is time-consuming, but not if the numbers are changes, then the sum is simply calculated as the change from the start to the end-number.

● Write down ten numbers vertically. The first number must be 3 and the last 5, the rest can be any numbers between 1 and 9. In the next column write down the individual changes 'end-start'. In the third column add up the individual changes along the way. Try to explain why the result must be 5-3 regardless of the in-between numbers.

● Draw a square with side n. Let n have a small positive change t. Show that the square will change with two next blocks when disregarding the small txt square. This shows that the change in an n*n square is 2*n*t, so if we want to add arears under a y = 2*n curve we must add very many small areas y*t = 2*n*t. However, since each may be written as a change in a square, we just have to find the change of the square from the start-point to the end-point. That is how integral calculus works.

Focus 29. Finding Common Units

Activity 01. "Only add with like units, so how add T = 4ab^2 + 6abc?".

Here units come from factorizing:

T = 2*2*a*b*b + 2*3*a*b*c = 2*b*(2*a*b).

Focus 30. Finding Square Roots

Activity 01. A 7x7 square can be recounted in tens as 4.9 tens. The inverse question is how to transform a 6x7 block into a square, or in other words, to find the square root of 4.2 tens.

A quick way to approach a relevant number is to first find two consecutive numbers, p and p+1, that squared are too low and too high.

Then an approximate value for the square root may be calculated by using that

if p^2 < N < (p+1)^2, then √N ≈ $\frac{\text{N+ p\^{}2}}{p*2}$.

Reference

Tarp, A. (2018). Mastering many by counting, recounting and double-counting before adding on-top and next-to. *Journal of Mathematics Education, March 2018, 11*(1), 103-117.