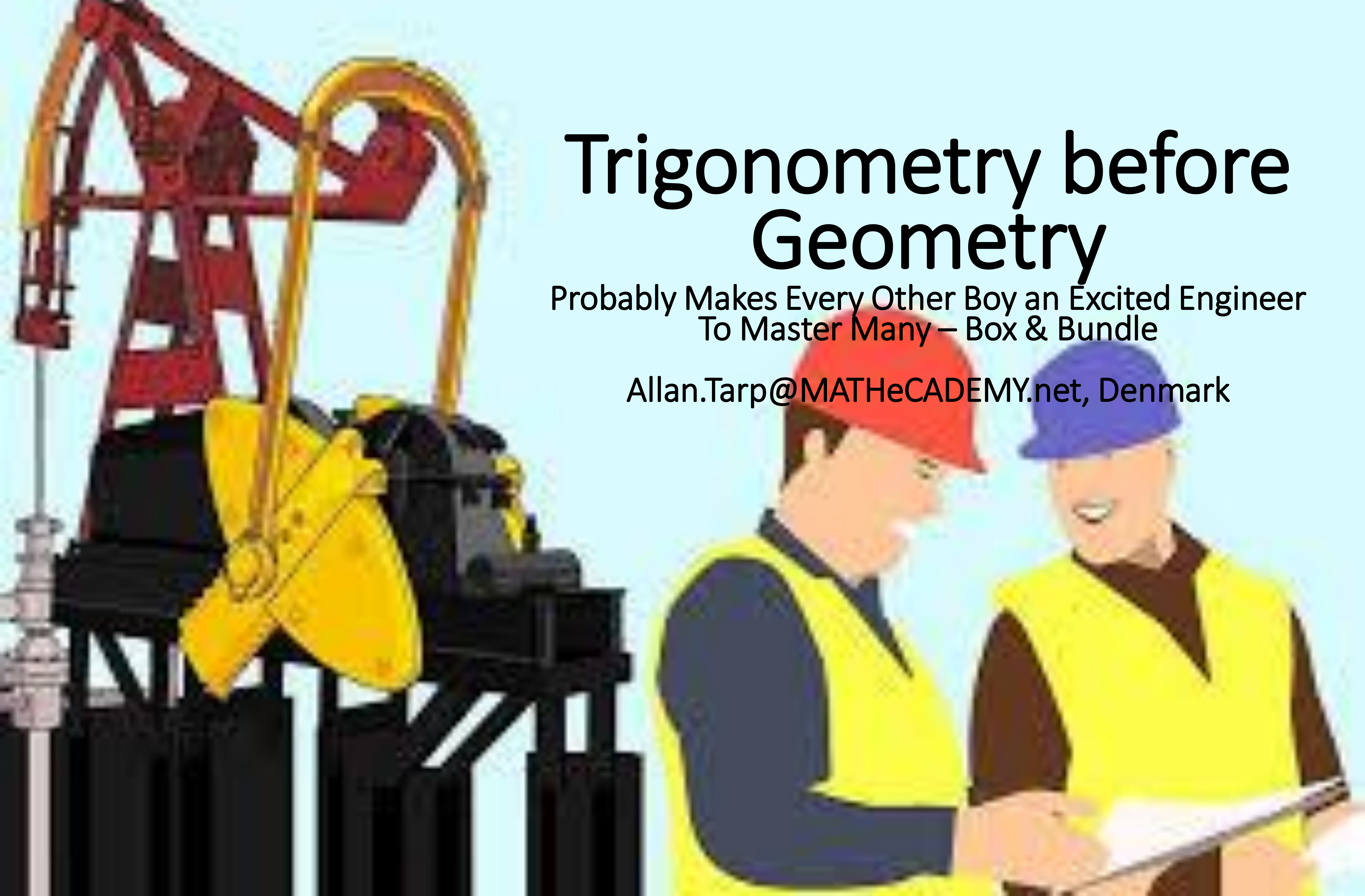

# Trigonometry before Geometry

Probably Makes Every Other Boy an Excited Engineer
To Master Many – Box & Bundle

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# In Mathematics Education, what is the goal?

## Mastery of Mathematics?

Or is this but a means to the real end-goal:

## Mastery of Many

so we ask: *Which Mastery of Many does a child have before school?*

*Can Mathematics be learned as a language through communication?*

Allowed to keep and develop their own flexible bundle-numbers,
within ½ a year they begin to master
digital numbers, negative numbers, decimals, fractions, functions,
solving equations, proportionality, calculus, trigonometry

# With a Plain & Latin English, is there also a Plain & Latin Math? Let's look at 'a Language House'

| | |
|---|---|
| The **WORD-language** assigns words in sentences with<br>The **NUMBER-language** assigns numbers instead with | • a subject<br>• a verb<br>• a predicate |

Both languages have a META-language, a grammar, describing the language, that is learned before the grammar. Why does mathematics teach language after and not before grammar?

| <u>The Twin Language Houses</u> | WORD-language | NUMBER-language |
|---|---|---|
| *META-language, grammar* | *'is' is a verb* | *'x' is an operation*<br>Mathematics |
| **INSIDE** Language | This is a chair<br>Mother tongue | T = 3x4 |
| **OUTSIDE** World | | |

# Is there a Plain Math communicating about Many? Let’s ask a 3year old: How Old Next Time?

The answer is 4, showing 4 fingers    I I I I

But, reacting strongly to 4 fingers held together 2 by 2:

“That is not four, that is two twos”    II  II

- <u>Observation 01</u>: Outside, children see what exists, and with units: bundles of **2s**, and 2 of them. So children use **BundleNumbers**
- <u>Observation 02</u>: Inside, children use full number-language sentences as in the word-language with a SUBJECT, a VERB, and a PREDICATE:

  ”That is two twos”, shortened to ”T = 2 **2s**”.

# Exploring the Plain Math growing from children's own Bundle-Numbers, we soon find: All is different

# Why Teach Children if they Already Know?

With education **curing** the diagnose **un-educated** we ask:

To **CURE**, be **SURE** the diagnose is not

- already cured
- self-referring: ***I teach math so you learn math***

Core Questions:

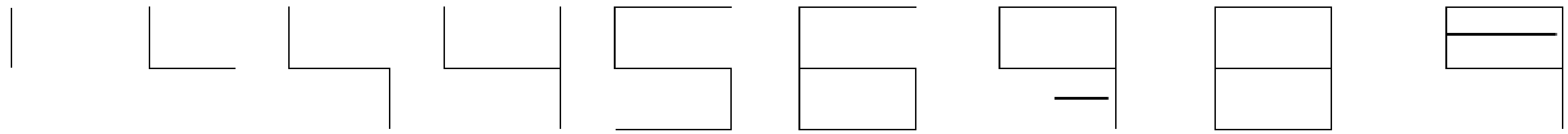
- What <u>Mastery of Many</u> does children develop when adapting to it?
- What is a <u>Child-Grounded-Curriculum</u> in <u>Plain Math</u>?

  Question-guided, please, to be used for home education in a virus situation.
- Will Plain Math make every other boy an <u>Exited Engineer</u>?

# Creating icons: IIII → IIII → 4 →

Children love making number-icons of cars, dolls, spoons, sticks.
Changing four ones to one fours creates a 4-icon with four sticks.
An icon contains as many sticks as it represents, if written less sloppy.
Once created, icons become **UNITS** when counting in bundles, as kids do.

| one | two | three | four | five | six | seven | eight | nine |
|---|---|---|---|---|---|---|---|---|
| I | II | III | IIII | IIIII | IIIIII | IIIIIII | IIIIIIII | IIIIIIIII |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

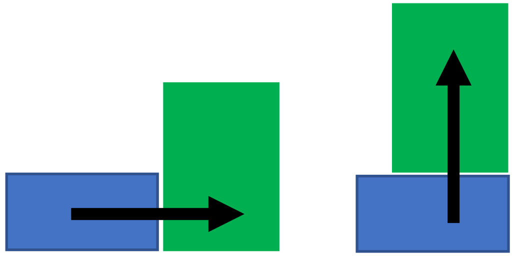


# Divide & Multiply & Subtract & Add may be '<u>de-modeled</u>' as Icons also

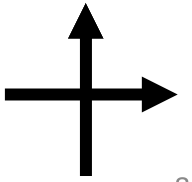
- From 9 **PUSH** away **4s** we write <u>9/4</u>
  <u>iconized</u> by a broom, called *division*.

- 2 times **LIFTING** the **4s** to a stack we write <u>2x4</u>
  <u>iconized</u> by a lift called *multiplication*.

- From 9 **PULL** away 2 **4s**' to find un-bundled we write <u>9 – 2x4</u>
  <u>iconized</u> by a rope, called *subtraction*.

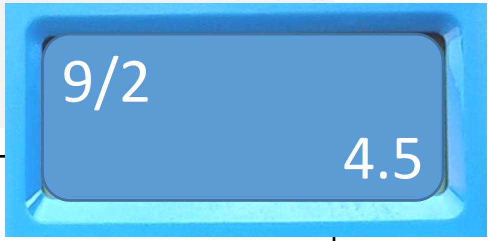


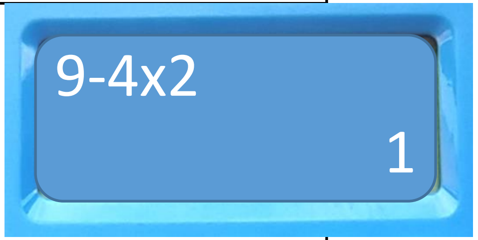
- **UNITING** next-to or on-top we write **A**+**C**
  <u>iconizing</u> the two directions, called *addition.*



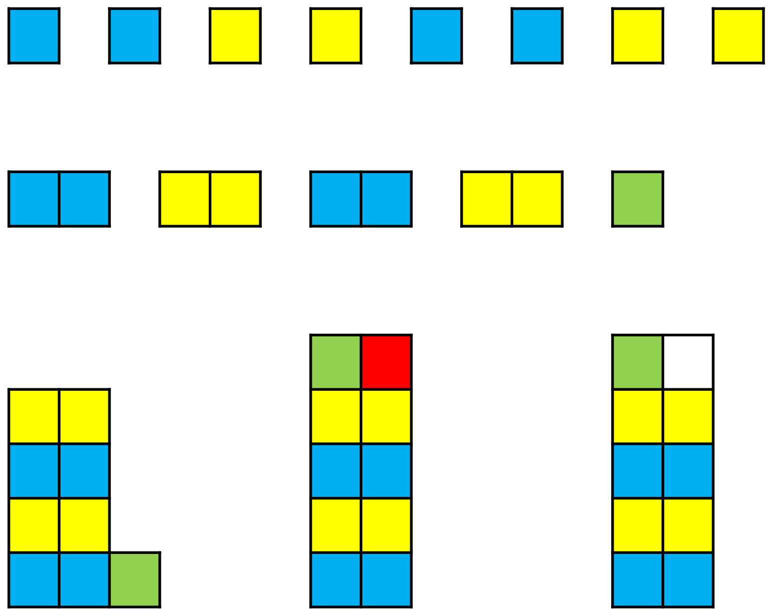
# BundleCounting a Total of 9 in **2s**

| | |
|---|---|
| 9/2 | 4.some |
| 9 – 4x2 | 1 |




## Outside Boxes



***9***
*bundled in **2s***
*with 1*
*unbundled*

*stacked as*
*4x2*
*with 1*
*unbundled*
*placed*
***next-to** or*
***on-top***

4B1 5B-1 4½B

**2s**

## Inside Bundles

From 9, 9/2 times push away 2.
From 9, pull away 4 **2s**, leaving 1.
Prediction by a calculator:

**T** = 9 = 4B1 **2s**

The unbundled can be placed
**next-to** the box
- iconized by a dot named a decimal point, 4.1 **2s**

**on-top** of the box
- counting what is missing in a full bundle, 5**B**-1 **2s**
- counted in bundles as 1/2, giving 4 1/2 **B 2s**

So, inside decimals & negatives & fractions tell how to outside place the undbundled singles.

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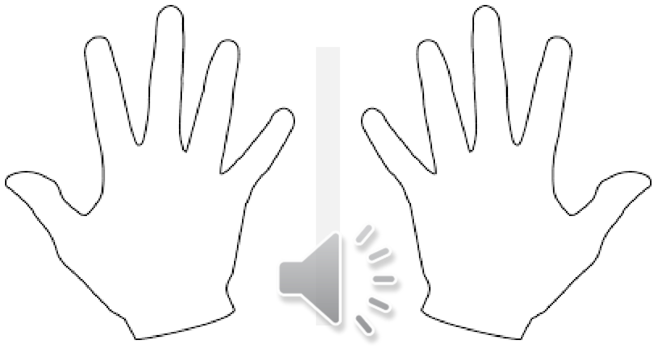
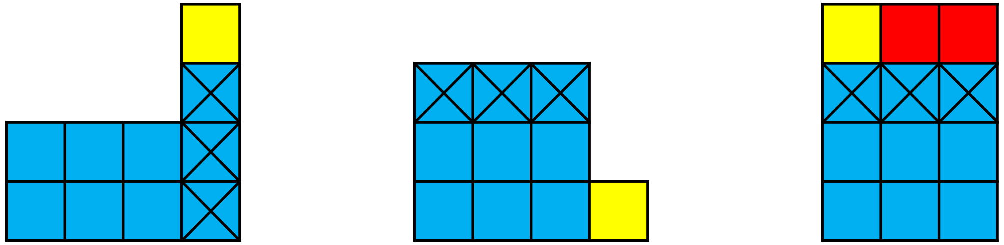
# BundleCounting Fingers in **3s** using Flexible Bundle-Numbers



| **Over-load, Normal, Under-load** | **Singles, Bundles, Bundle-Bundles** |
|---|---|
| Two hands bundle-counted in **3s**: | Counting-sequence bundle-counting in **3s**: |

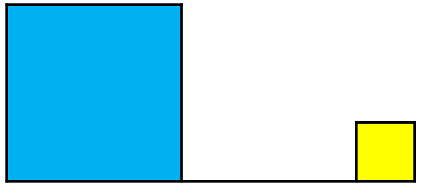


T = 2**B**4 over-load

T = 3**B**1 normal

T = 4**B**-2 under-load

***Flexible Bundle-Numbers***



T = 1**BB** 0**B** 1 = 101 **3s**

0**B**1, 0**B**2, ~~0**B**3~~ **no** 1**B**0,
1**B**1, 1**B**2, ~~1**B**3~~ **no** 2**B**0,
2**B**1, 2**B**2, ~~2**B**3~~ **no** 3**B**0,
3**B**1 or ten

But 3 **Bundles** is 1 **Bundle-of-Bundles.**

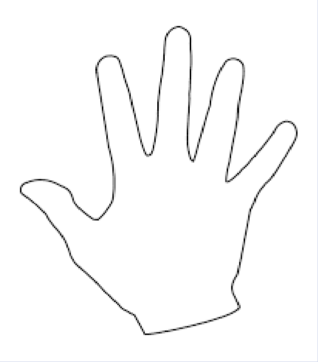
So T = 9 = 1**BB 3s** or

T = ten = 3**B**1 **3s** = 1**BB**1 **3s** or

T = ten = 1**BB**0**B**1 **3s** or

T = ten = 101 **3s**

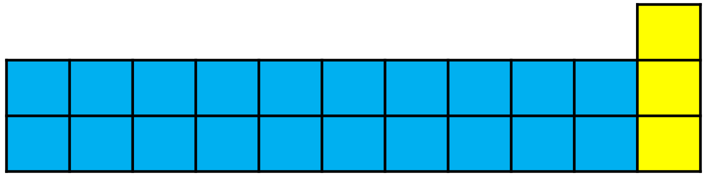
# BundleCounting fingers in 2s with flexible bundle-numbers: Bye-bye to place-values

| Outside | Inside |
|---|---|
| I I I I I<br>II I I I<br>II II I<br>II II II<br> | T = 5<br>T = 1**B**3 overload<br>T = 2**B**1 normal<br>T = 3**B**-1 (less 1) underload |
| Seven ten four<br>Three hundred seven ten four<br>Counting formula (polynomial) | T = 74 = 7**B**4 = 6**B**14 = 8**B**-6<br>T = 374 = 3**BB** 7**B** 4 = 2**BB** 15**B** 24<br>T = 3***B**^2 + 7***B** + 4 |

# Bundle-Counting in tens using Flexible Bundle-Numbers

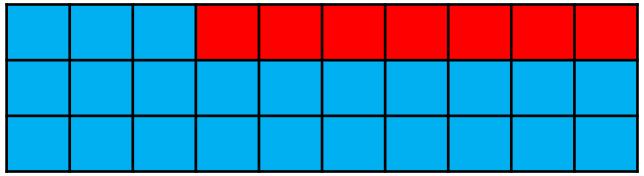
Counting in **tens**, an outside Total of 2 **tens** & 3 can be described inside as **T = 23,** if leaving out the unit and the decimal point.

- or as:



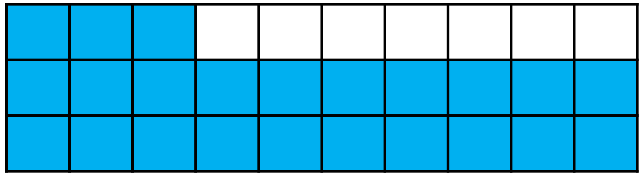
T = 2.3 **tens**

T = 2B3 **tens**



T = 3.-7 **tens**

T = 3B-7 **tens**



T = 2  3/10 **tens**

T = 2 3/10 B **tens**

# Flexible BundleNumbers Ease Operations

## Counting in tens, T = 78 = 7B8 = 6B18 = 8B-2

| Overload | Underload | Overload | Overload |
|---|---|---|---|
| 6 5<br>+ 2 7 | 6 5<br>- 2 7 | 7 x 48 | 336 /7 |
| 6B5<br>+ 2B7 | 6B5<br>- 2B7 | 7<br>x 4B8 | 33B6 /7 |
| 8B12 | 4B-2 | 28B56 | 28B56 /7 |
| 9B2 | 3B8 | 33B6 | 4B8 |
| **92** | **38** | **336** | **48** |

*No need to carry!*

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# Bundling-counting table

| 1BB0B0 | 1BB0B1 | 1BB0B2 | 1BB0B3 | 1BB0B4 | 1BB0B5 | 1BB0B6 | 1BB0B7 | 1BB0B8 | 1BB0B9 | ~~1BB0B10~~ |
|---|---|---|---|---|---|---|---|---|---|---|
| ~~10B0~~ | ~~10B1~~ | ~~10B2~~ | ~~10B3~~ | ~~10B4~~ | ~~10B5~~ | ~~10B6~~ | ~~10B7~~ | ~~10B8~~ | ~~10B9~~ | ~~10B10~~ |
| 9B0 | 9B1 | 9B2 | 9B3 | 9B4 | 9B5 | 9B6 | 9B7 | 9B8 | 9B9 | ~~9B10~~ |
| 8B0 | 8B1 | 8B2 | 8B3 | 8B4 | 8B5 | 8B6 | 8B7 | 8B8 | 8B9 | ~~8B10~~ |
| 7B0 | 7B1 | 7B2 | 7B3 | 7B4 | 7B5 | 7B6 | 7B7 | 7B8 | 7B9 | ~~7B10~~ |
| 6B0 | 6B1 | 6B2 | 6B3 | 6B4 | 6B5 | 6B6 | 6B7 | 6B8 | 6B9 | ~~6B10~~ |
| 5B0 | 5B1 | 5B2 | 5B3 | 5B4 | 5B5 | 5B6 | 5B7 | 5B8 | 5B9 | ~~5B10~~ |
| 4B0 | 4B1 | 4B2 | 4B3 | 4B4 | 4B5 | 4B6 | 4B7 | 4B8 | 4B9 | ~~4B10~~ |
| 3B0 | 3B1 | 3B2 | 3B3 | 3B4 | 3B5 | 3B6 | 3B7 | 3B8 | 3B9 | ~~3B10~~ |
| 2B0 | 2B1 | 2B2 | 2B3 | 2B4 | 2B5 | 2B6 | 2B7 | 2B8 | 2B9 | ~~2B10~~ |
| 1B0 | 1B1 | 1B2 | 1B3 | 1B4 | 1B5 | 1B6 | 1B7 | 1B8 | 1B9 | ~~1B10~~ |
| 0B0 | 0B1 | 0B2 | 0B3 | 0B4 | 0B5 | 0B6 | 0B7 | 0B8 | 0B9 | ~~0B10~~ |

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# Bundle-Numbers can Shift Units and create a RecountFormula

$8 = (8/2)x2$
$T = (T/B)xB$

Bundle-counting 8 in 2s: 8 = ? **2s**

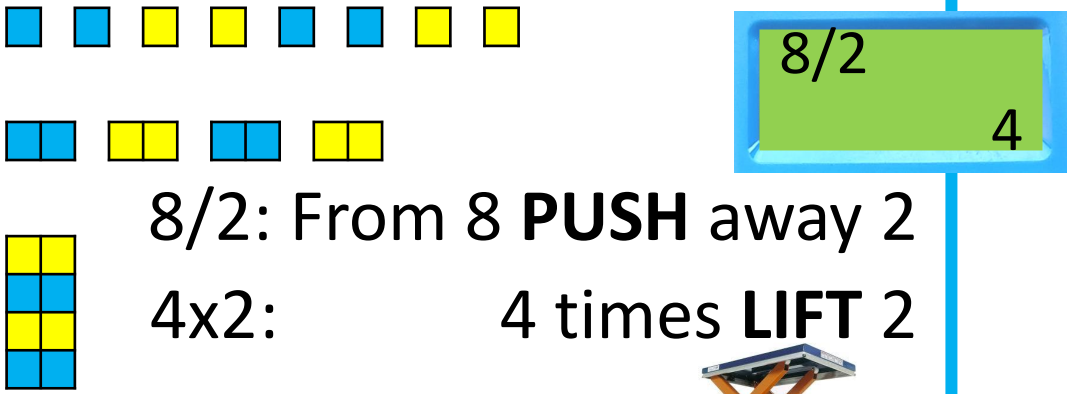
Fingers: 8 = 4 **2s**

Blocks bundle-counted: 8 = 8/2 **2s**

Bundles stacked: 4 **2s** = 4x2

Bundle-counting: 8 = (8/2)x2

Recount-Formula: **T = (T/B)xB**





8/2: From 8 **PUSH** away 2

4x2: 4 times **LIFT** 2

*u* x 2 = 8 = (8/2)x2
***u* = 8/2**

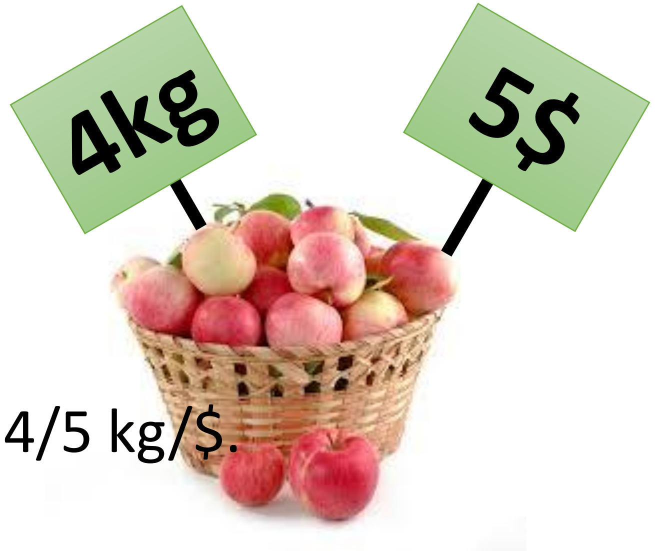
*OPPOSITE side & sign*

Equations ⟵ ⟶ Proportionality

| | |
|---|---|
| Shifting unit | y = k * x |
| Linearity | Δy = (**Δy/Δx**) * Δx = **m** * Δx |
| Local linearity | dy = (**dy/dx**) * dx = **y'** * dx |
| Trigonometry | a = (**a/b**) * b = **tanA** * b |
| Trade | \$ = (**\$/kg**) * kg = **price** * kg |
| STEM | meter = (**meter/sec**) * sec = **speed** * sec |

Observation ReCounting in BundleNumbers contains Core Mathematics & STEM

# ReCounting in two Units creates **PerNumbers** & Proportionality




ReCounting in kg & $, we get a PerNumber 4kg **per** 5$ = 4kg/5$ = 4/5 kg/$.

With like units, per-numbers become fractions: 4$/5$ = 4/5, and 4$/100$ = 4/100 = 4%.

With 4kg linked to 5$, we simply recount in the per-number.

*(Or we recount the units directly. Or we equate the per-numbers. Or we use the before 1900 'Rule of 3' (regula de tri) alternating the units, and, from behind, first multiply, then divide.)*

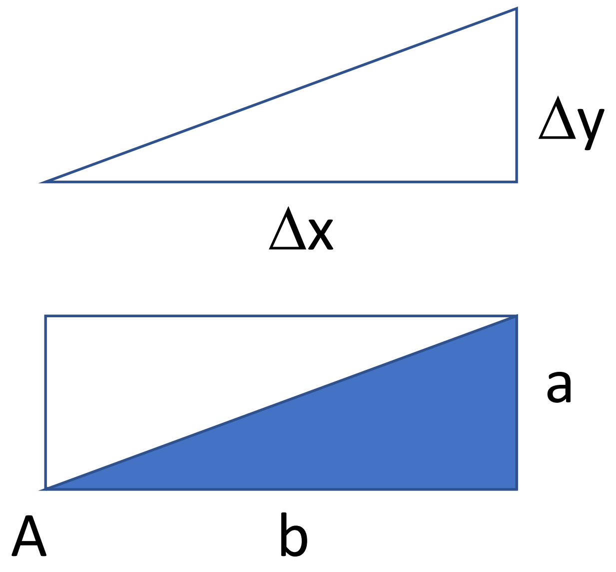
**Questions**:

| **12kg = ?$** | **20$ = ?kg** |
|---|---|
| 12kg = (12/4) x 4kg<br>= (12/4) x 5$ = 15$ | 20$ = (20/5) x 5$<br>= (20/5) x 4kg = 16kg |
| $ = ($/kg) x kg = 5/4 x 12 = 15 | kg = (kg/$) x $ = 4/5 x 20 = 16 |
| *u*/12 = 5/4, so *u* = 5/4 x 12 = 15 | *u*/20 = 4/5, so *u* = 4/5 x 20 = 16 |
| If 4kg is 5$, then 12kg is ?$; answer: 12x5/4 = 15 | If 5$ is 4kg, then 20$ is ?kg; answer: 20x4/5 = 16 |

# The ReCount formula is core mathematics & science

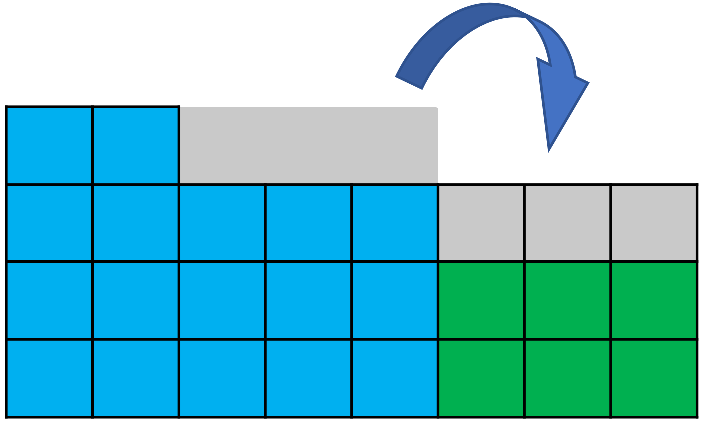
**T = (T/B)*B** is used to change unit, and is all over:

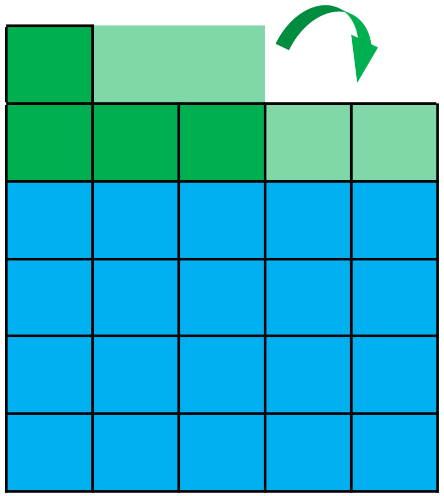
| | |
|---|---|
| Proportionality | $y = \mathbf{c} * x$ |
| Linearity | $\Delta y = (\mathbf{\Delta y / \Delta x}) * \Delta x = \mathbf{m} * \Delta x$ |
| Local linearity | $dy = (\mathbf{dy/dx}) * dx = \mathbf{y'} * dx$ |
| Trigonometry | $a = (\mathbf{a/b}) * b = \mathbf{tanA} * b$ |
| Trade | \$ = (**\$/kg**) * kg = **price** * kg |
| Science | meter = (**meter/second**) * second<br>= **speed** * second |



# Once Counted & Recounted, Totals may Add

| **BUT: NextTo** | **or OnTop** |
|---|---|
| 4 5s + 2 3s = 3B2 8s | 4 5s + 2 3s = 5B1 5s |
| The areas are integrated<br>*Adding areas = Integration* | The units are changed to be the same<br>*Change unit = ReCounting = Proportionality* |





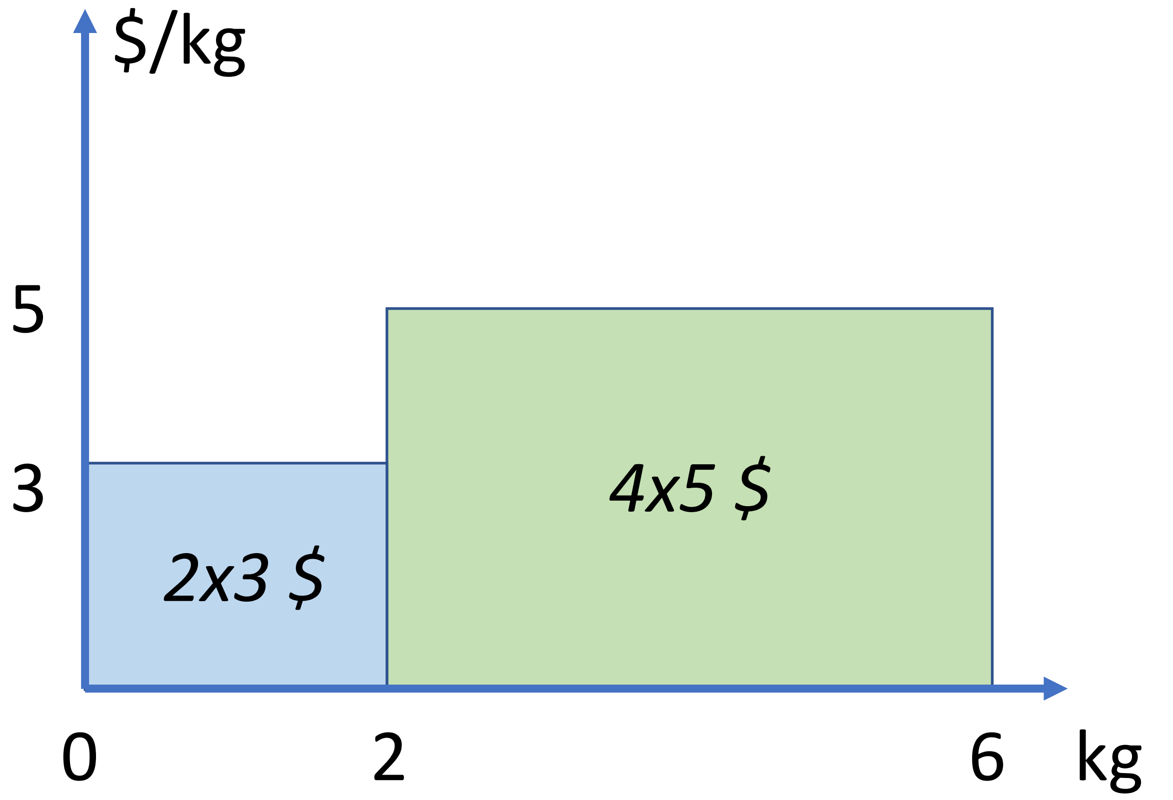
# Adding PerNumbers as Areas (Integral Calculus)

“2kg at **3$/kg** + 4kg at **5$/kg** = 6kg at **?** **$/kg**?”

2 kg at **3 $/kg**
\+ 4 kg at **5 $/kg**

(2+4) kg at **? $/kg**

- Unit-numbers add on-top.
- Per-numbers must be multiplied to unit-numbers, thus adding as **areas** under the per-number graph.
- Here, multiplication before addition
- So, per-numbers and fractions are not numbers, but operators needing numbers to be numbers.



# Subtracting PerNumbers (Differentiation)

"2kg at 3$/kg + 4kg at **what** = 6kg at 5$/kg?"

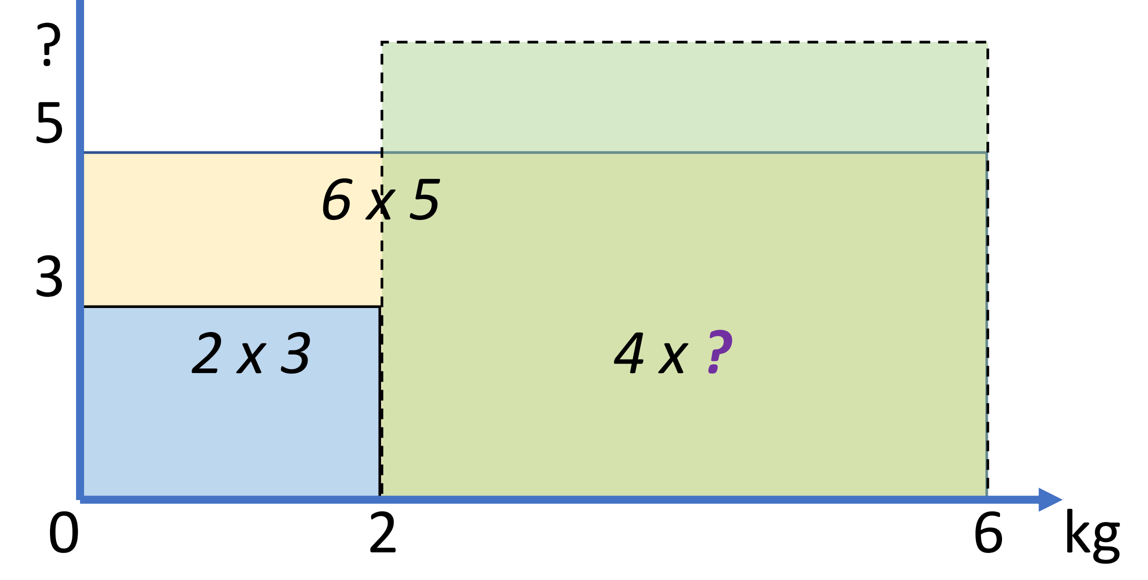
|   | | | |
|---|---|---|---|
| | 2 kg | at | 3 $/kg |
| + | 4 kg | at | ? $/kg |
| | 6 kg | at | 5 $/kg |

Reversed per-number addition is called differential calculus. Outside, we remove the initial 2x3 box, and recount the rest in 4s. So, we subtract areas to be reshaped by the recount-formula. So, here subtraction (giving a change, Δ) comes before division.

$2x3 + 4x? = 6x5$

$? = (6x5 – 2x3)/(6-2)$

$? = Δ\$/Δkg$



# Four Ways to Unite and Split a Total

A number-formula T = 345 = 3**BB**4**B**5 = 3***B**^2 + 4***B** + 5 (a polynomial) shows the 4 ways to unite: +, *, ^, next-to block-addition (integration). Addition and multiplication unite changing and constant unit-numbers. Integration and power unite changing and constant per-numbers. We might call this beautiful simplicity the 'Algebra Square' since in Arabic, algebra means to reunite. ● The 4 uniting operations each has a reverse splitting operation: Addition has subtraction (–), and multiplication has division (/). Power has factor-finding (root, √) and factor-counting (logarithm, log). Integration has per-number finding (differentiation dT/dn = T'). Reversing operations is solving equations, done by moving to **opposite side** with **opposite sign**.

| **Operations unite** / *split into* | **changing** | **constant** |
|---|---|---|
| **Unit-numbers** *m, s, \$, kg* | **T = a + n** *T – a = n* | **T = a * n** *T/n = a* |
| **Per-numbers** *m/s, \$/kg, m/(100m) = %* | **T = ∫ a dn** *dT/dn = a* | **T = a^n** $\log_a T = n$, $\sqrt[n]{T} = a$ |

# Uniting Reversed = Solving Equations

| OppoSite Side with OppoSite Sign | | | |
|---|---|---|---|
| u + 2 = 8 | u*2 = 8 | u^8 = 2 | 2^u = 8 |
| u = 8 – 2 | u = 8/2 | u = 8√2 | u = log2(8) |
| plus ↕ minus | multiply ↕ divide | exponent ↕ root | base ↕ logarithm |

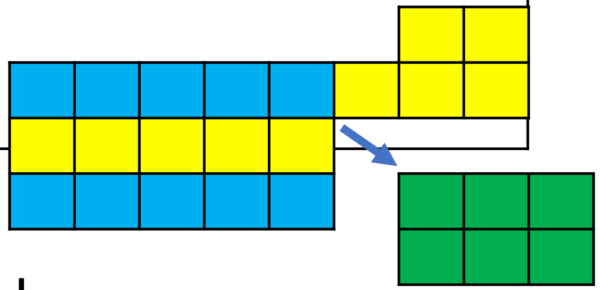
**Add NextTo, Integration**

2 **3s** + ? **5s** = 3.2 **8s**

? = (3.2 **8s** – 2 **3s**)/5

*Solved by differentiation: (T-T1)/5 = ΔT/5*

Integration
↕ Differentiation



**Hymn to Equations**

Equations are the best we know,
they are solved by isolation.
But first, the bracket must be placed
around multiplication.

We change the sign, and take away,
and only x itself will stay.
We just keep on moving, we never give up.
So feed us equations, we don't want to stop!

# ReCounting Sides in a Box gives Trigonometry

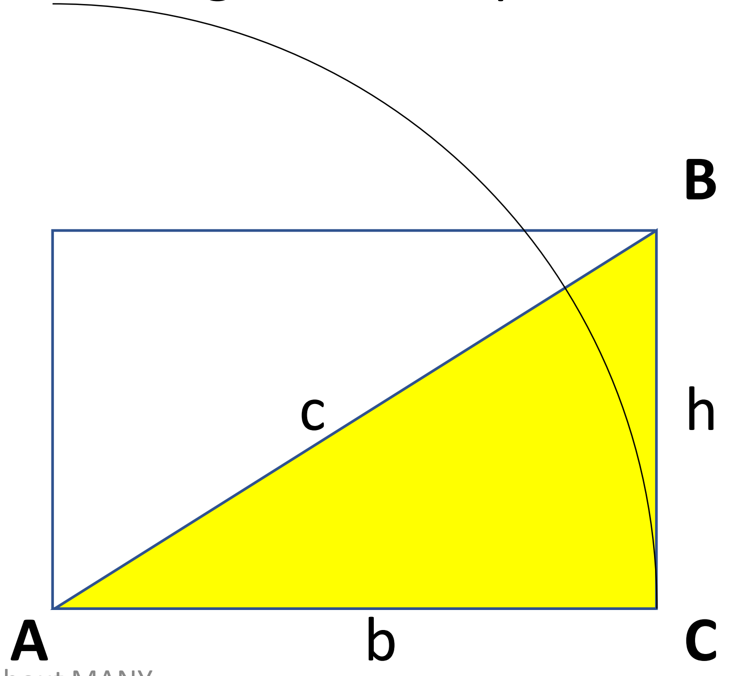
Geometry means to measure earth in Greek. The earth can be divided in triangles; that can be divided in right triangles; that can be seen as a box halved by its diagonal thus having three sides: the base *b*, the height *h* and the diagonal *c* connected with the angles by per-number formulas re-counting the sides pairwise.

$h = (h/c) \times c = \sin \boldsymbol{A} \times c;$

$b = (b/c) \times c = \cos \boldsymbol{A} \times c$

$h = (h/b) \times b = \tan \boldsymbol{A} \times b;$

$\tan \boldsymbol{A} = h/b = \Delta y/\Delta x =$ diagonal gradient



# A CIRCLE has Many Half-Boxes

To find the length of a circle, we see, that a CIRCLE contains Many small Half-Boxes.

The half circle's 180 degrees is split in 100 parts.

In the half-box, we re-count the height *h* in the radius *r*, and insert tan *A*.

The length of the half-circle then is close to 3.14 times *r*, or $\pi$ times *r*.

This gives a formula for pi

$$\pi = n * \tan(180/n)$$

for *n* large

$h = (h/r) * r$

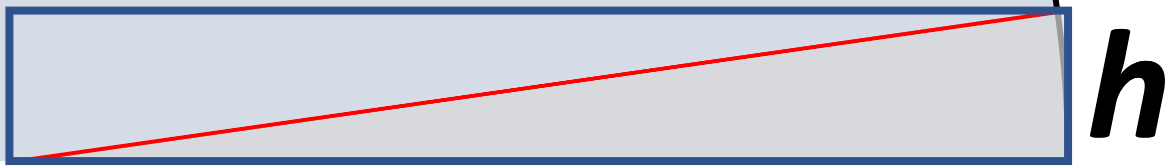
$= \tan A * r$

$= \tan(180/100) * r$

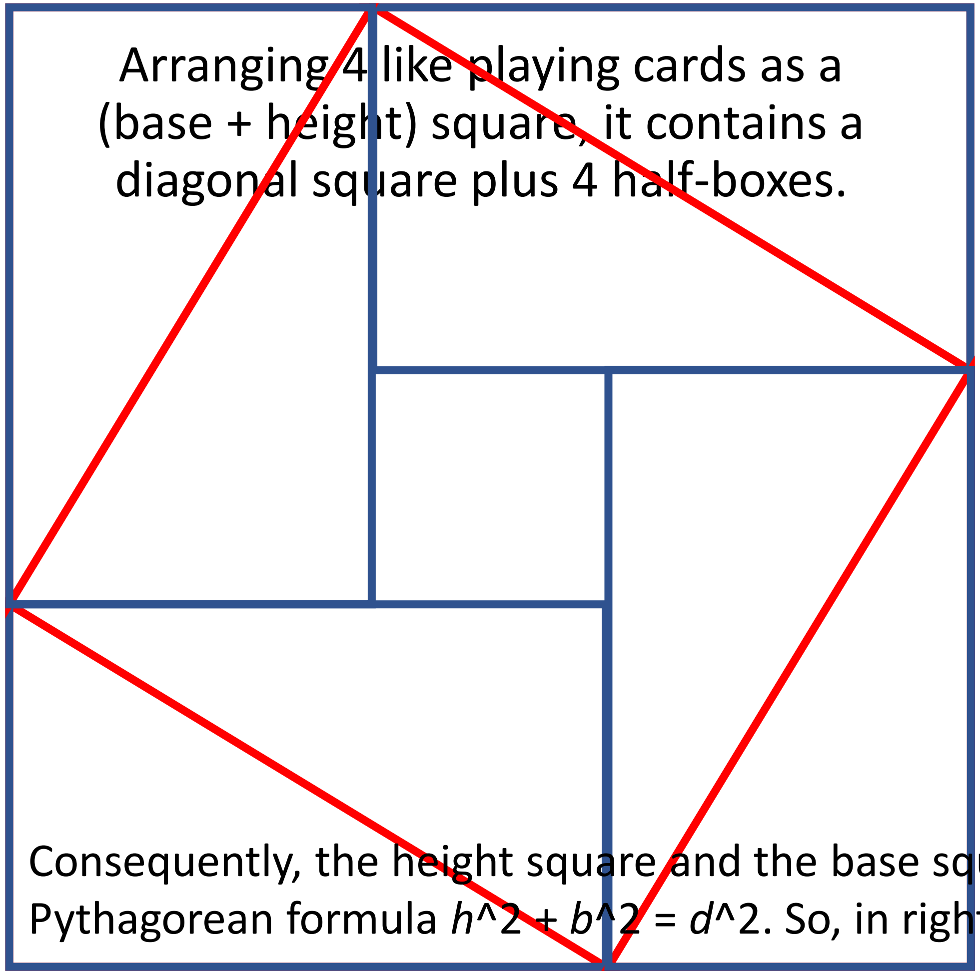
$C = 100 * h$

$= 100 * \tan(180/100) * r$

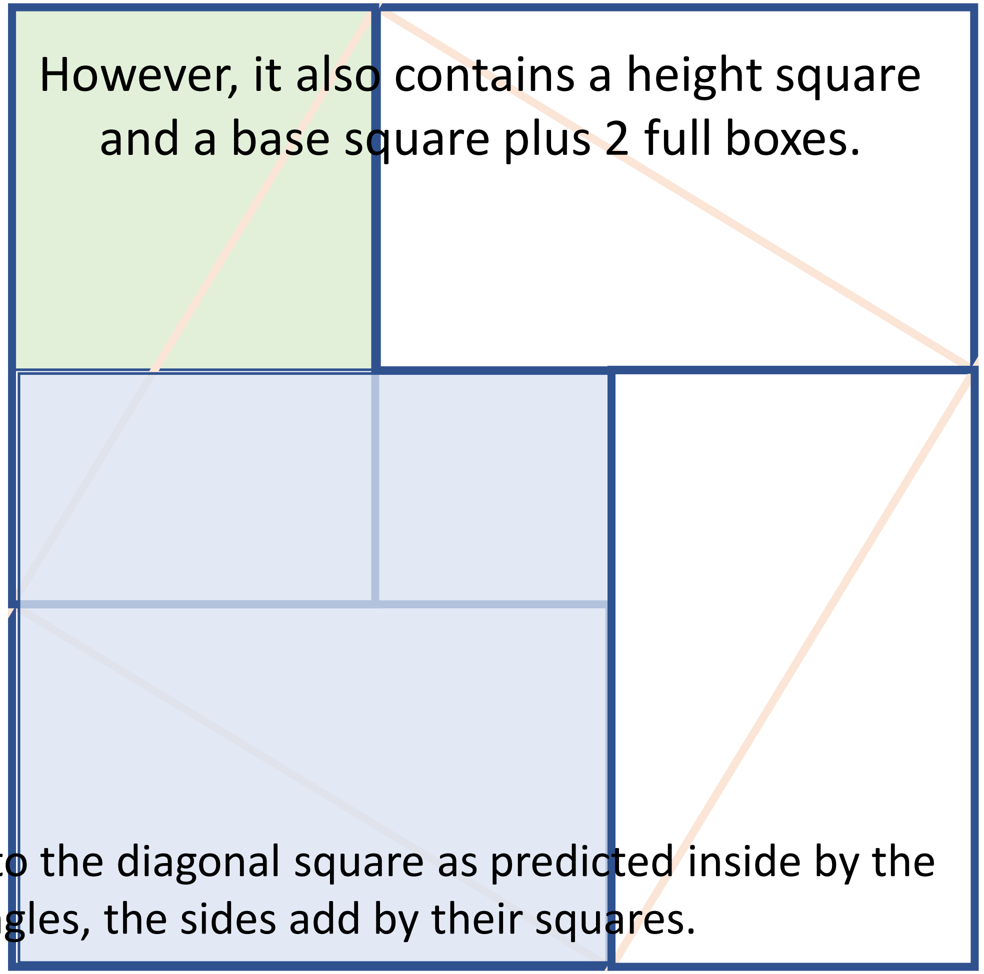
$= 3.1426 * r = \pi * r$



# Sides in a Half-Box add by squares
# $h^2 + b^2 = d^2$



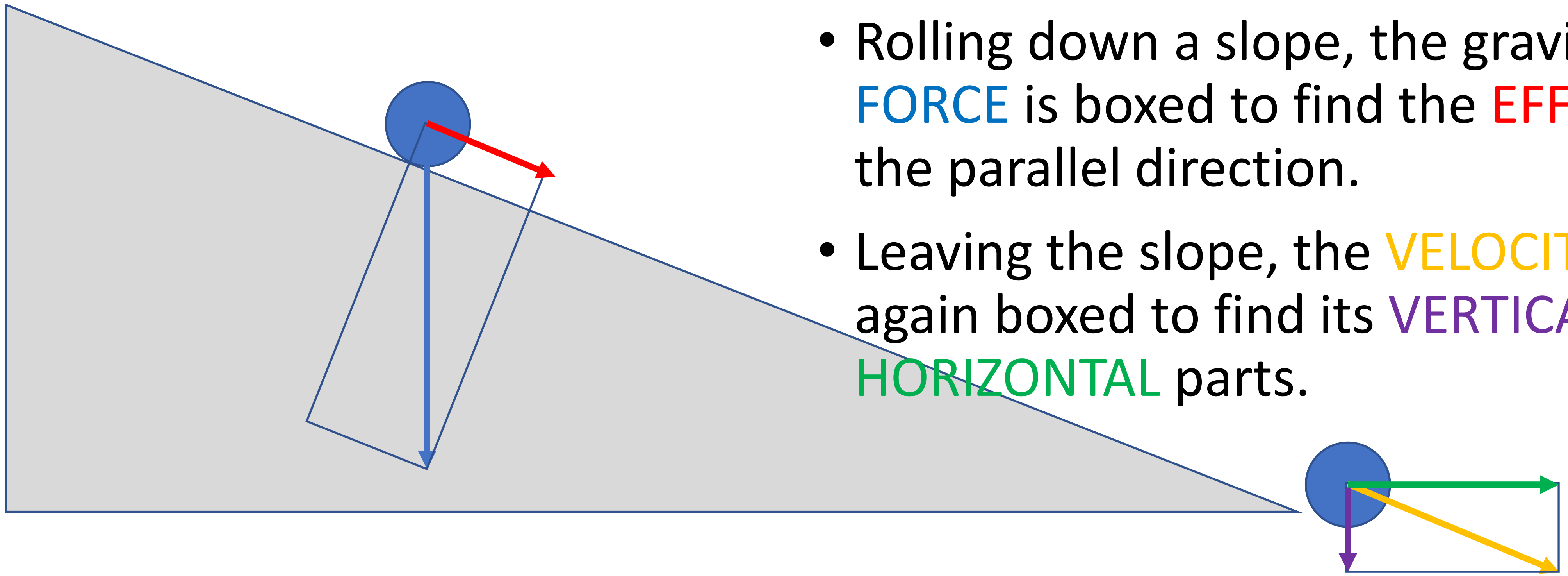
Arranging 4 like playing cards as a (base + height) square, it contains a diagonal square plus 4 half-boxes.



However, it also contains a height square and a base square plus 2 full boxes.

Consequently, the height square and the base square add up to the diagonal square as predicted inside by the Pythagorean formula $h^2 + b^2 = d^2$. So, in right angled triangles, the sides add by their squares.

# STEM: Science Technology Engineering Mathematics
# Motion



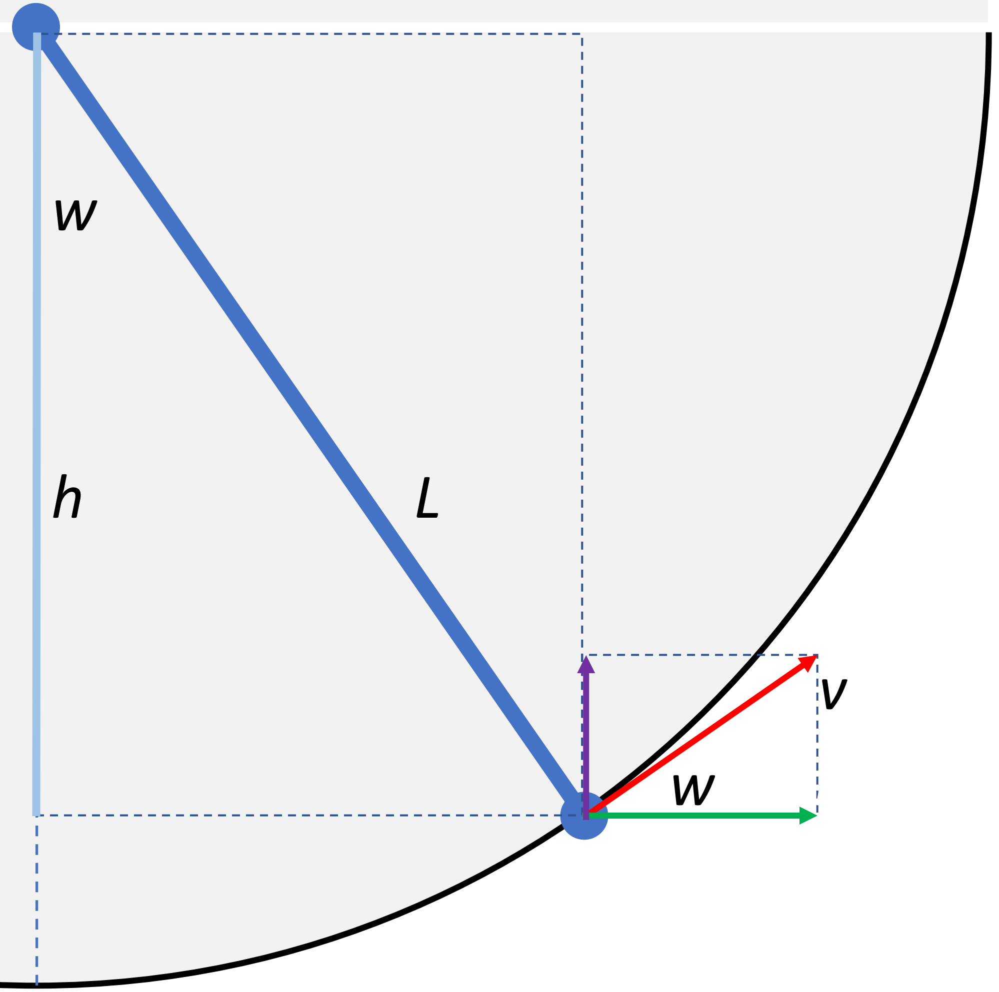
- Rolling down a slope, the gravity FORCE is boxed to find the EFFECT, in the parallel direction.
- Leaving the slope, the VELOCITY is again boxed to find its VERTICAL and HORIZONTAL parts.

# STEM: Science Technology Engineering Mathematics
## Jumping from a swing

● To find the length when jumping from a swing, first the rope ***L*** is boxed to find the height ***h***, giving the speed ***v*** = √(20*$L$*cos $w$).

● Then the speed is boxed to find its horizontal part, ***v*cos w***, and its vertical part, ***v*sin w***.

● Then formulas for the horizontal motion without, and the vertical motion with acceleration allow finding the length.



# STEM: Science Technology Engineering Mathematics
# Optics

From ***P*** to ***Q***

The shortest way is a direct line.

*But, with different speed, 3m/s and 2m/s,*

The quickest way is a broken line.

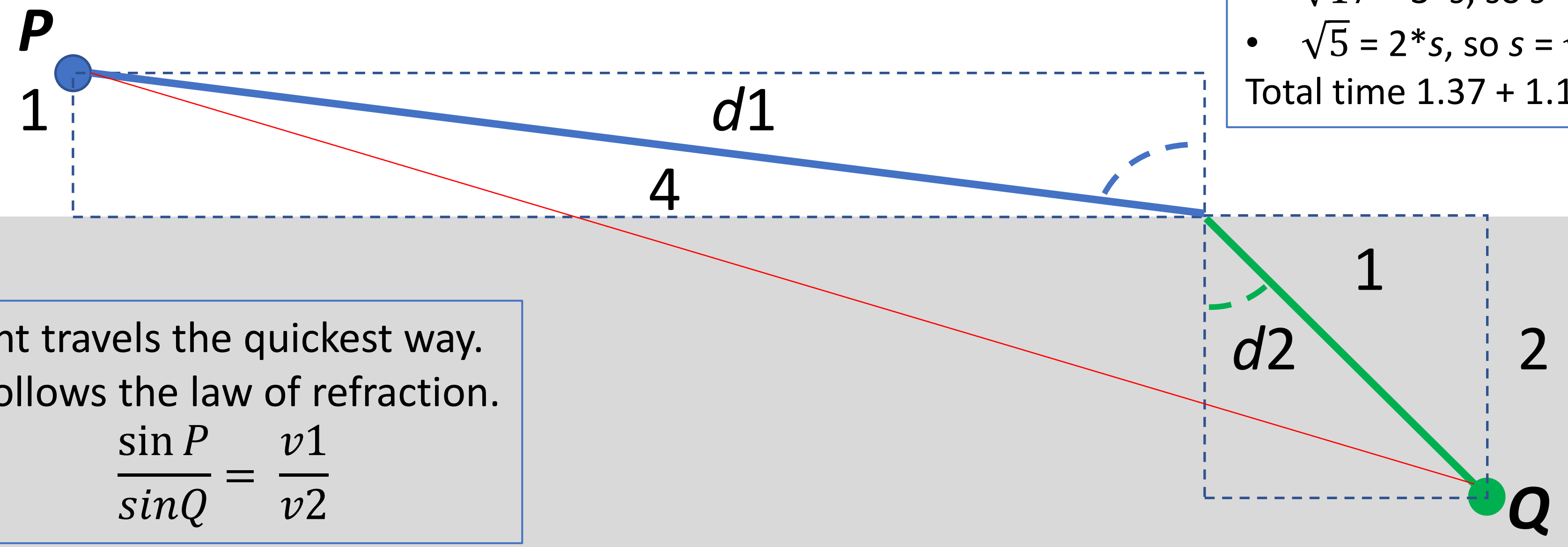
First we box the distance to find its length

- $d1 = \sqrt{1^2 + 4^2} = \sqrt{17}$
- $d2 = \sqrt{2^2 + 1^2} = \sqrt{5}$

Double-counting in meter and second gives the time: meter = (meter/second)*second

- $\sqrt{17} = 3*s$, so $s = \sqrt{17}\,/3 = 1.37$
- $\sqrt{5} = 2*s$, so $s = \sqrt{5}\,/2 = 1.12$

Total time 1.37 + 1.12 = 2.49

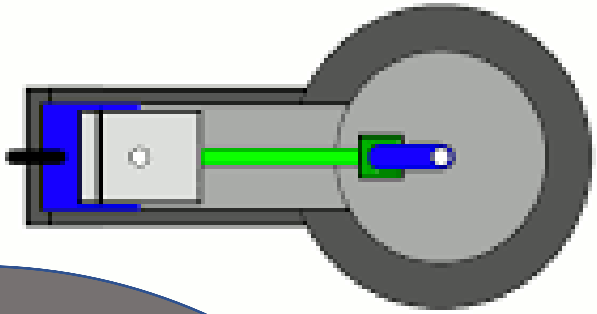



$v1$ = 3m/s
$v2$ = 2m/s

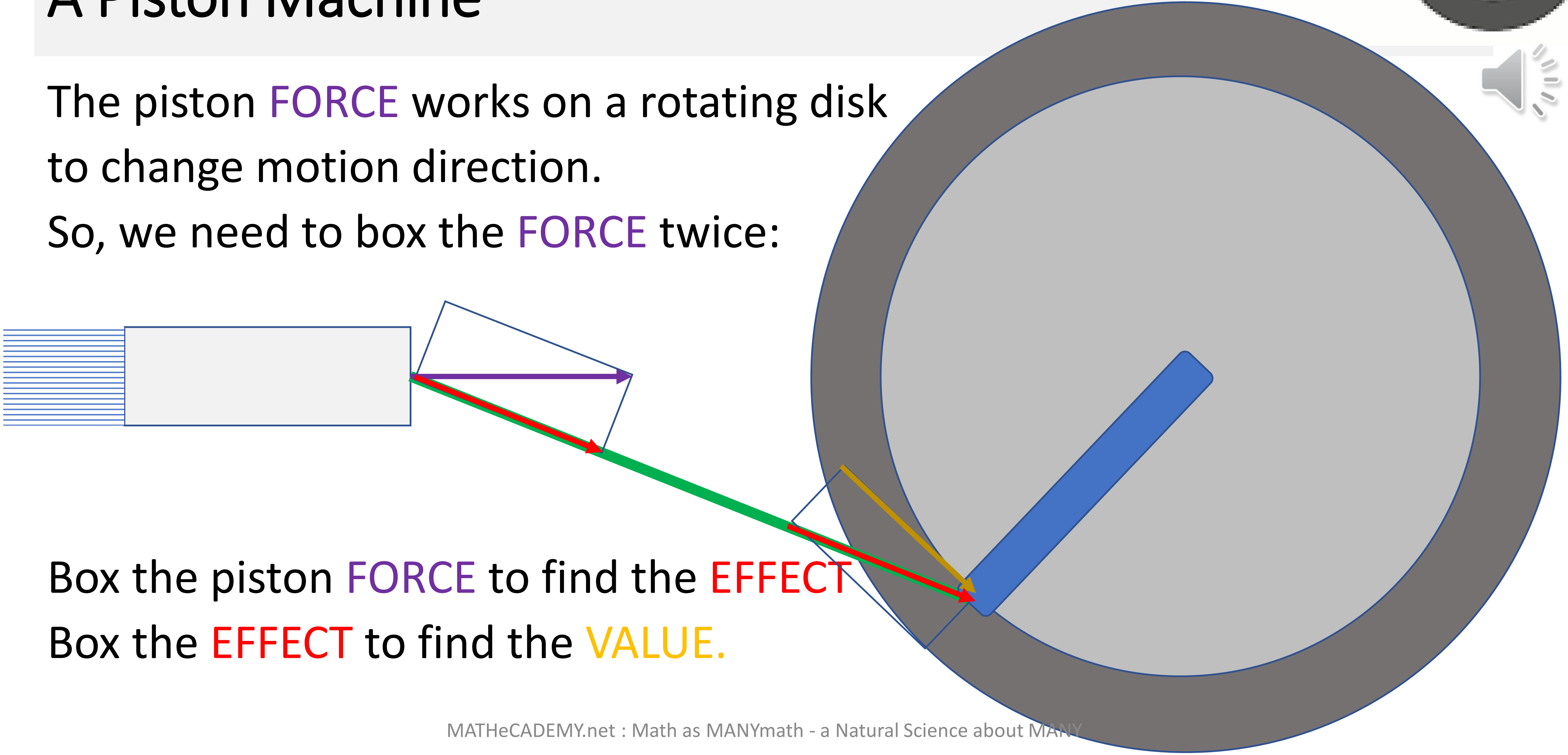
Light travels the quickest way.
It follows the law of refraction.

$$\frac{\sin P}{\sin Q} = \frac{v1}{v2}$$

# STEM: Science Technology Engineering Mathe
# A Piston Machine



The piston FORCE works on a rotating disk
to change motion direction.
So, we need to box the FORCE twice:



Box the piston FORCE to find the EFFECT
Box the EFFECT to find the VALUE.

# STEM: Science Technology Engineering Mathematics
# How many turns on a steep 1 km by 1 km hillside?

**On a 30-degree hillside, a 10-degree road is constructed.**

**How many turns will there be?**

In *BCD*, *B* is 30 degrees, and $BD = u * \cos 30$.

So Pythagoras gives

$u\text{^}2 = CD\text{^}2 + BD\text{^}2 = CD\text{^}2 + u\text{^}2 * (\cos 30)\text{^}2$, or

$CD\text{^}2 = u\text{^}2*(1 - (\cos 30)\text{^}2) = u\text{^}2 * (\sin 30)\text{^}2$.

In *ACD*, *A* is 10 degrees, and $AD = AC * \cos 10$.

So Pythagoras gives

$AC\text{^}2 = CD\text{^}2 + AD\text{^}2 = CD\text{^}2 + AC\text{^}2 * (\cos 10)\text{^}2$, or

$CD\text{^}2 = AC\text{^}2*(1 - (\cos 10)\text{^}2) = AC\text{^}2 * (\sin 10)\text{^}2$.

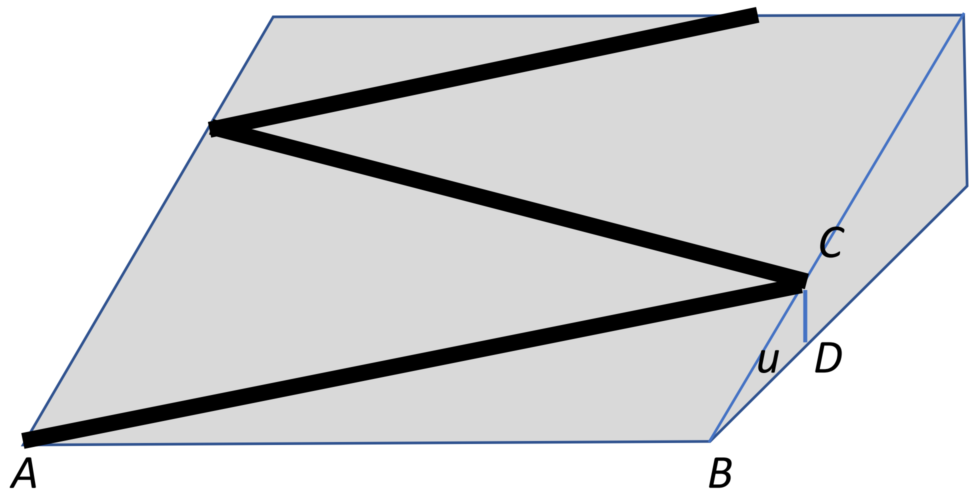
In *ACB*, $AB = 1$ and $BC = u$,

So Pythagoras gives

$AC\text{^}2 = 1\text{^}2 + u\text{^}2$, or $AC = \sqrt{}(1 + u\text{^}2)$, so

$u\text{^}2 * (\sin 30)\text{^}2 = AC\text{^}2 * (\sin 10)\text{^}2$, or

$u = AC * r$, where $r = \sin 10/\sin 30$




Consequently

$u = \sqrt{}(1+u\text{^}2) * r$, or

$u\text{^}2 = (1+u\text{^}2) * r\text{^}2$, or

$u\text{^}2 * (1-r\text{^}2) = r\text{^}2$, or

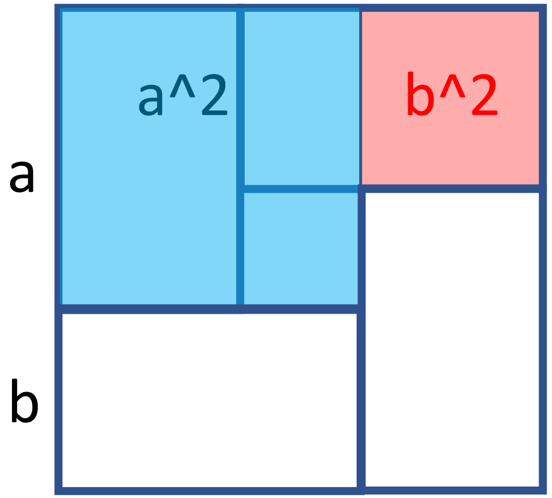
$u\text{^}2 = r\text{^}2/(1 - r\text{^}2) = 0.137$, so

$BC = u = \sqrt{}0.137 = 0.37$.

Thus, there will be 2 turns:

370 meter and 740 meter up the hillside.

# STEM: Science Technology Engineering Mathematics
# Algebra - with two playing cards

<table>
<tr><th>Quadratic Sum</th><th>Quadratic Difference</th><th>Sum & Difference</th></tr>
<tr>
<td>
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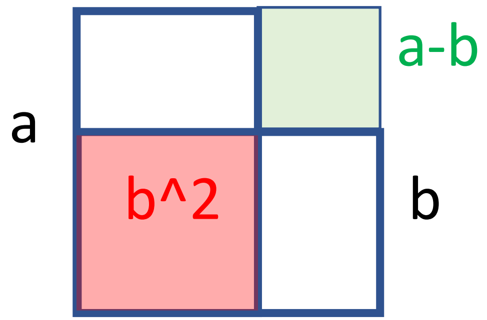
<td>
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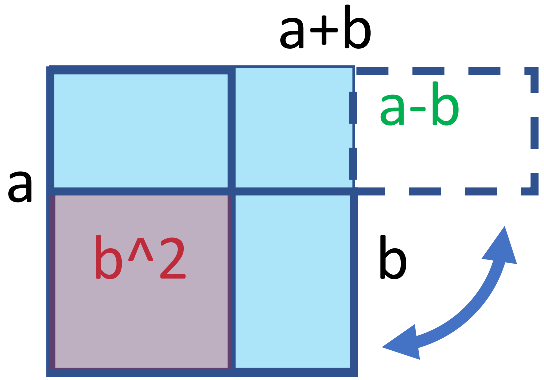
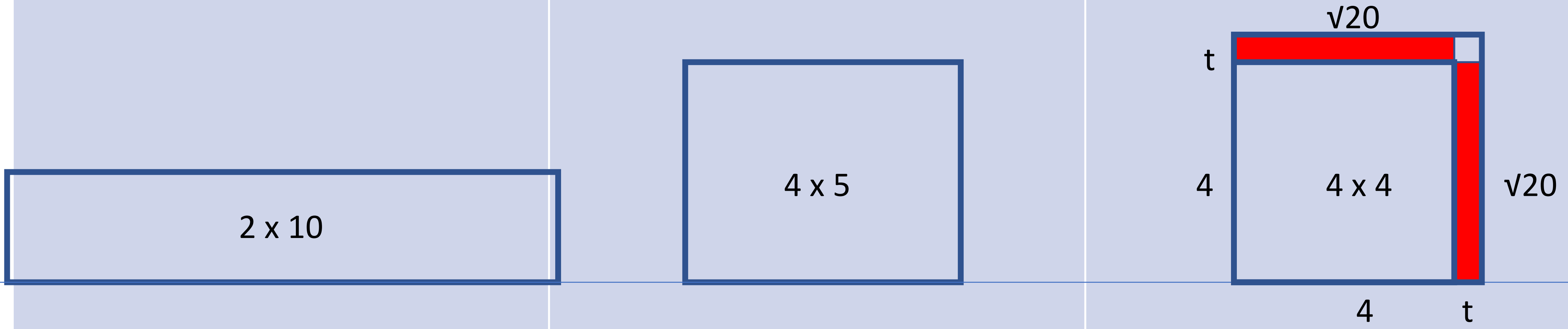
<td>
</td>
</tr>
<tr>
<td>$(a + b)^2$<br>$= a^2 + b^2 + 2 \text{ cards}$<br>$= a^2 + b^2 + 2*a*b$</td>
<td>$(a - b)^2$ *removed twice*<br>$= a^2 - 2 \text{ cards} + b^2$<br>$= a^2 + b^2 - 2*a*b$</td>
<td>$a^2 - b^2$<br>$= (a + b)*(a - b)$</td>
</tr>
</table>

# STEM: Science Technology Engineering Mathematics
## Algebra – Totals BOX'ed, TEN'ed or SQUARE'd

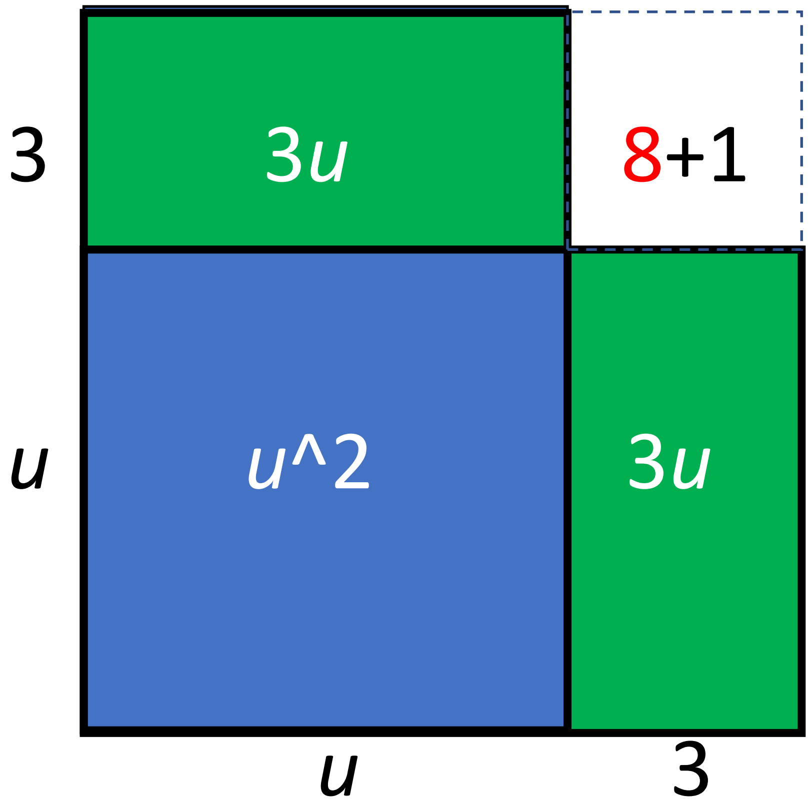
| TEN'ed | BOX'ed | SQUARE'd |
| --- | --- | --- |




20 – 4 * 4 = 4
shared by the
two 4 x t boxes,
gives t = 0.5
So √20 ≈ 4.5
Calculator: √20 = 4.472

# STEM: Science Technology Engineering Mathematics
# Algebra – squaring quadratic equations

Solving a quadratic equation with 2 playing cards




**Solve $u^2 + 6u + 8 = 0$**

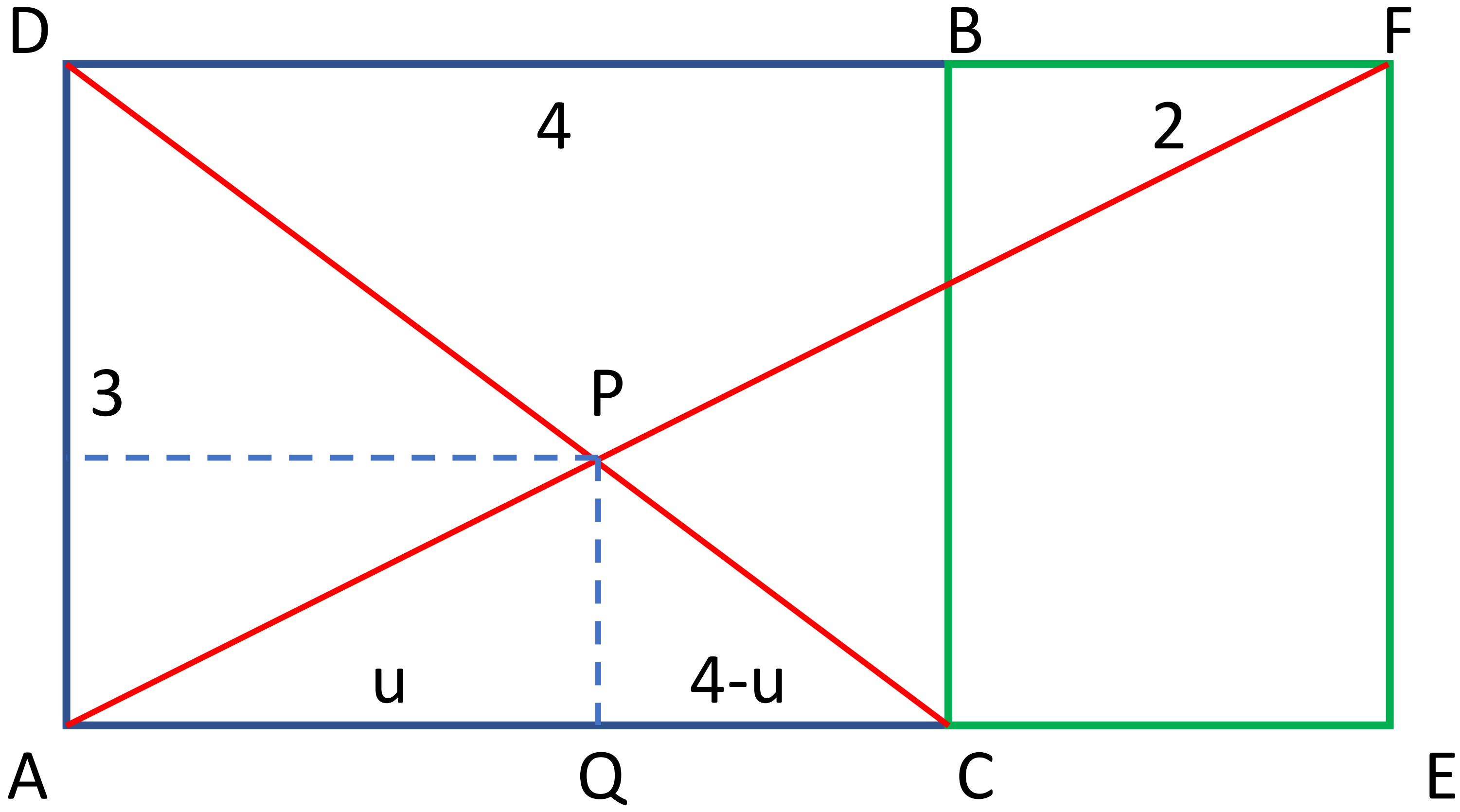
$(u+3)^2 = u^2 + 6u + 8 + 1$

$(u+3)^2 = 0 + 1$

$u = -3 \pm 1$

**$u = -4$ & $u = -2$**

# STEM: Science Technology Engineering Mathematics
# Geometry boxed - Intersection Line-Line




$A$ is part of $AQP$ and $AEF$, so

tan $A = \frac{QP}{u} = \frac{3}{6}$, so

$$\boldsymbol{QP = \frac{3}{6} * u}$$

$C$ is part of $CQP$ and $CAD$, so

tan $C = \frac{QP}{4-u} = \frac{3}{4}$ , so

$$\boldsymbol{QP = \frac{3}{4} * (4\text{-}u)}, \text{ so}$$

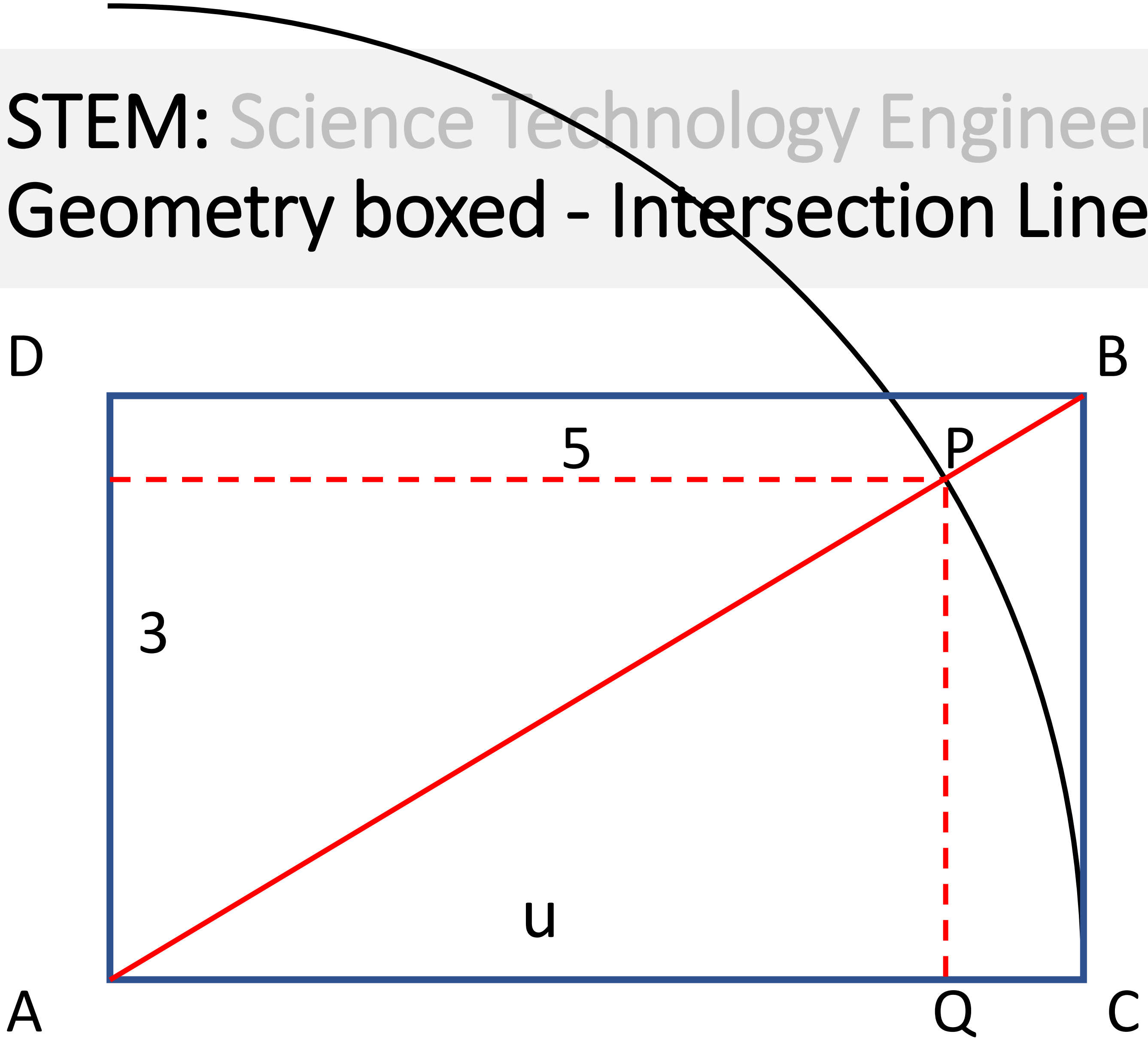
$\frac{3}{6} * u = \frac{3}{4} * (4\text{-}u)$, solved by

$u = 2.4$, so

$QP = \frac{3}{6} * u = \frac{3}{6} * 2.4 = 1.2$, so

$$\mathbf{AP} = \begin{pmatrix} \mathbf{2.4} \\ \mathbf{1.2} \end{pmatrix}$$

# STEM: Science Technology Engineering Mathematics
## Geometry boxed - Intersection Line-Circle




*P* is on *AB*, so

tan *A* = $\frac{QP}{u}$ = $\frac{3}{5}$ = 0.6, so

*QP* = 0.6 * *u*

*P* is on the circle, so

$u^2 + QP^2 = 5^2$, so

$u^2 + (0.6 * u)^2 = 25$

1.36 * $u^2$ = 25, so

*u* = $\sqrt{\frac{25}{1.36}}$ = 4.29, so

*QP* = 0.6 * 4.29 = 2.57, so

***AP*** = $\begin{pmatrix} \mathbf{4.29} \\ \mathbf{2.57} \end{pmatrix}$

Finding the intersection point P with the diagonal CD leads to the equation

+ 2.64 *u* – 11.8 = 0, solved by *u* = 2.36, giving *v* = 4.42, so ***AP*** = $\begin{pmatrix} \mathbf{-2.36} \\ \mathbf{4.42} \end{pmatrix}$

# STEM: Science Technology Engineering Mathematics
# Algebra & Geometry – the Coordinate System I

*A*(2,1) and *C*(7,3) and *B*(5,7) form a triangle boxed in a 6x5 rectangle.

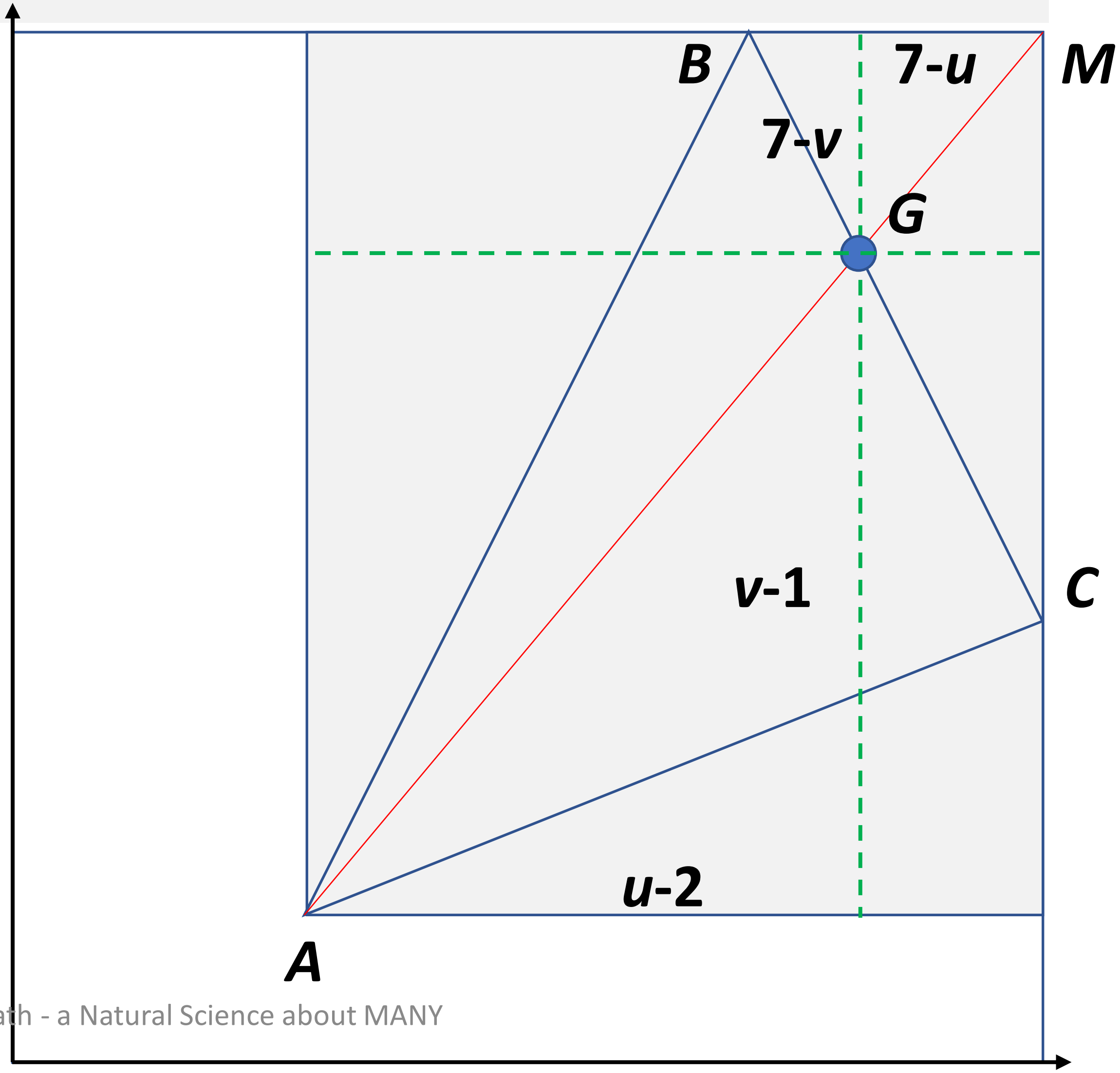
The line from *A* to *M*(7,7) intersect *BC* in the point *G*(*u*,*v*) found by equating tan *A* found in a lower and an upper right triangle

$$\tan A = \frac{v-1}{u-2} = \frac{7-v}{7-u} = \frac{6}{5}\text{, so}$$

$6(u – 2) = 5(v – 1)$ and

$6(7 – u) = 5(7 – v)$

The 2 equations with 2 unknowns are solved by $u = 5.75$ and $v = 5.5$



# STEM: Science Technology Engineering Mathematics
# Algebra & Geometry – the Coordinate System I

We may also find a formula for the points on the line *AG* by replacing *u* and *v* with *x* and *y*

$6(x – 2) = 5(y – 1)$, giving $y = 1.2(x – 2) + 1$.

Likewise with the formula for the line *BC*:

$\tan B = \frac{7-y}{x-5} = \frac{4}{2}$, so

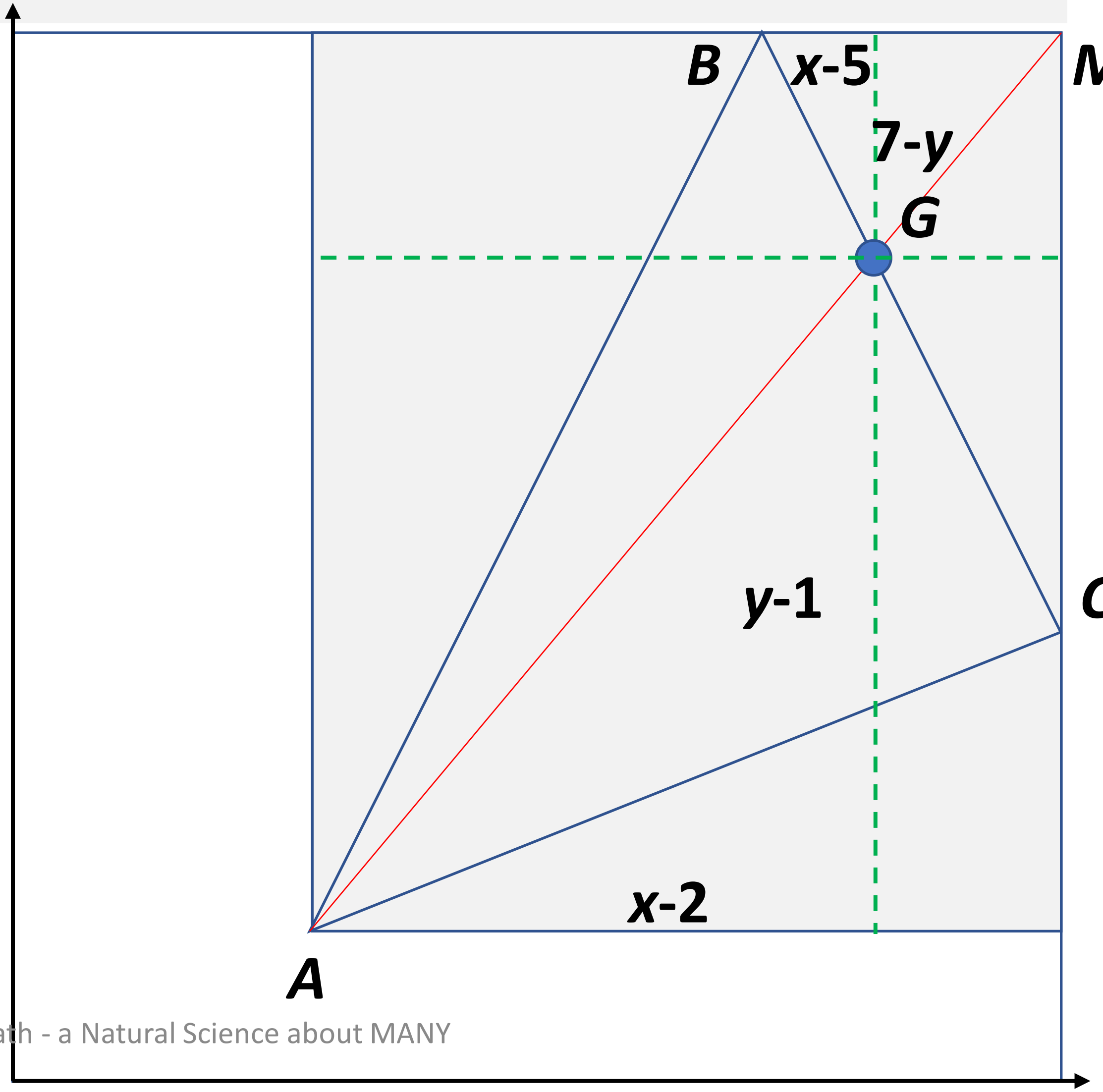
$y = -2(x – 5) + 7$

Since the intersection point is situated on both lines, we equate their *y*'s

$y = 1.2(x– 2) + 1 = -2(x– 5) + 7$,

solved by $x = 5.75$ and $y = 5.5$

In a similar way, intersection points between a line and a circle are quickly predicted by their formulas.



# STEM: Science Technology Engineering Mathematics
# Adding or Subtracting Unspecified Numbers

## Only add like units, so how to add $T = 4ab^2 + 6abc$?

Factors are units, so to find a common unit, we use a *factor-filter*

$T = 4ab^2 + 6abc = T_1 + T_2$

$= 2 * 2 * a * b * b \quad + \quad 2 * 3 * a * b * c$

$= 2 * b * (2*a*b) \quad + \quad 3 * c * (2*a*b)$

$= (2b+3c) * \mathbf{2ab}$

$= 2b+3c\ \mathbf{2ab}\text{s}$

| | | | | | |
|---|---|---|---|---|---|
| $T_1$ | 2 | 2 | $a$ | $b$ | $b$ |
| $T_2$ | 2 | 3 | $a$ | $b$ | $c$ |
| **Com. unit** | **2** | | $\boldsymbol{a}$ | $\boldsymbol{b}$ | |
| $T_1$ left | | 2 | | | $b$ |
| $T_2$ left | | 3 | | | $c$ |

# Discussion: What is the Difference?

| | | **Traditional Math** | **Box & Bundle Math** |
|---|---|---|---|
| Digit | 4 | Symbol | Icon with four strokes |
| Number | 456 | One number | Three numberings, 4**BB** 5**B** 6 |
| Division | 8/2 | 8 split in 2 | 8 counted in **2s** |
| Multiplication | 6*7 | 42 | 6 **7s** or 4**B**2 **tens** |
| Addition | 2+3 | 2+3 = 5 | 2 **4s** + 3 **5s** = 2**B**5 **9s** **&** 2 **4s** + 3 **5s** = 4**B**3 **5s** |
| Re-counting | 8 = 4 **2s** | Does not exist | **T = (T/B)*B** is core proportionality, changing units |
| Per-number | 2/3 | Does not exist | Created when recounting in a new physical unit |
| Fraction | 2/3 | Number<br>1/2 + 2/3 **IS** 7/6 | Per-number, i.e., operator, needing numbers to become numbers: 1/2 of 2 + 2/3 of 3 **IS** 3/5 of 5 |
| Function | T = 3*$u$ | Subset in a set-product | A number-language sentence predicting a total |
| Trigonometry | sin, cos, tan | After geometry | Recounting sides in a half-box. Before geometry |
| Calculus | $\int x\,dx$<br>dy/dx | First differential<br>Then integral | Adding piecewise or locally constant per-numbers<br>Before subtracting them |

# Is Solving Equations Different? Solve 2*u = 6

Many-math: To opposite side, with opposite sign

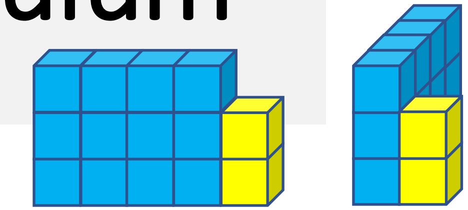
| | |
|---|---|
| 2 * u = 6 = (6/2) * 2 | Solved by re-counting 6 in 2s |
| u = 6/2 = 3 | Test: 2 * 3 = 6 OK |

Meta-math: Balance & neutralize, using abstract algebra group theory

| | |
|---|---|
| 2 * u = 6 | Multiplication has 1 as neutral element, and 2 has 1/2 as its inverse element |
| (2*u)*(1/2) = 6*(1/2) | Multiplying 2's inverse element to both number-names |
| (u * 2) * (1/2) = 3 | Applying the commutative law to u*2, 3 is the short number-name for 6*1/2 |
| u * (2 * (1/2)) = 3 | Applying the associative law |
| u * 1 = 3 | Applying the definition of an inverse element |
| u = 3 | Applying the definition of a neutral element. *With arrows, a test is not needed.* |

MATHeCADEMY.net

# The child’s own Box&Bundle math curriculum



1) Digits are (sloppy) icons, with as many sticks as they represent.

2) Totals are counted by bundling, giving **OUTSIDE** geometrical boxes, and (when turned to hide the units behind) **INSIDE** algebraic bundle-numbers.

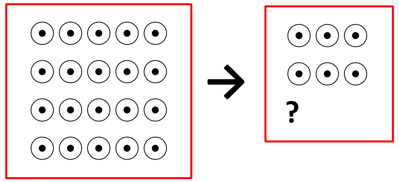
3) Operations are **INSIDE** icons, showing the 3 **OUTSIDE** counting steps: PUSHING & LIFTING bundles & PULLING stacks to find the unbundled ones.

4) The operation order is division first, then multiplication, then subtraction. Addition next-to & on-top comes later after totals are counted & re-counted.

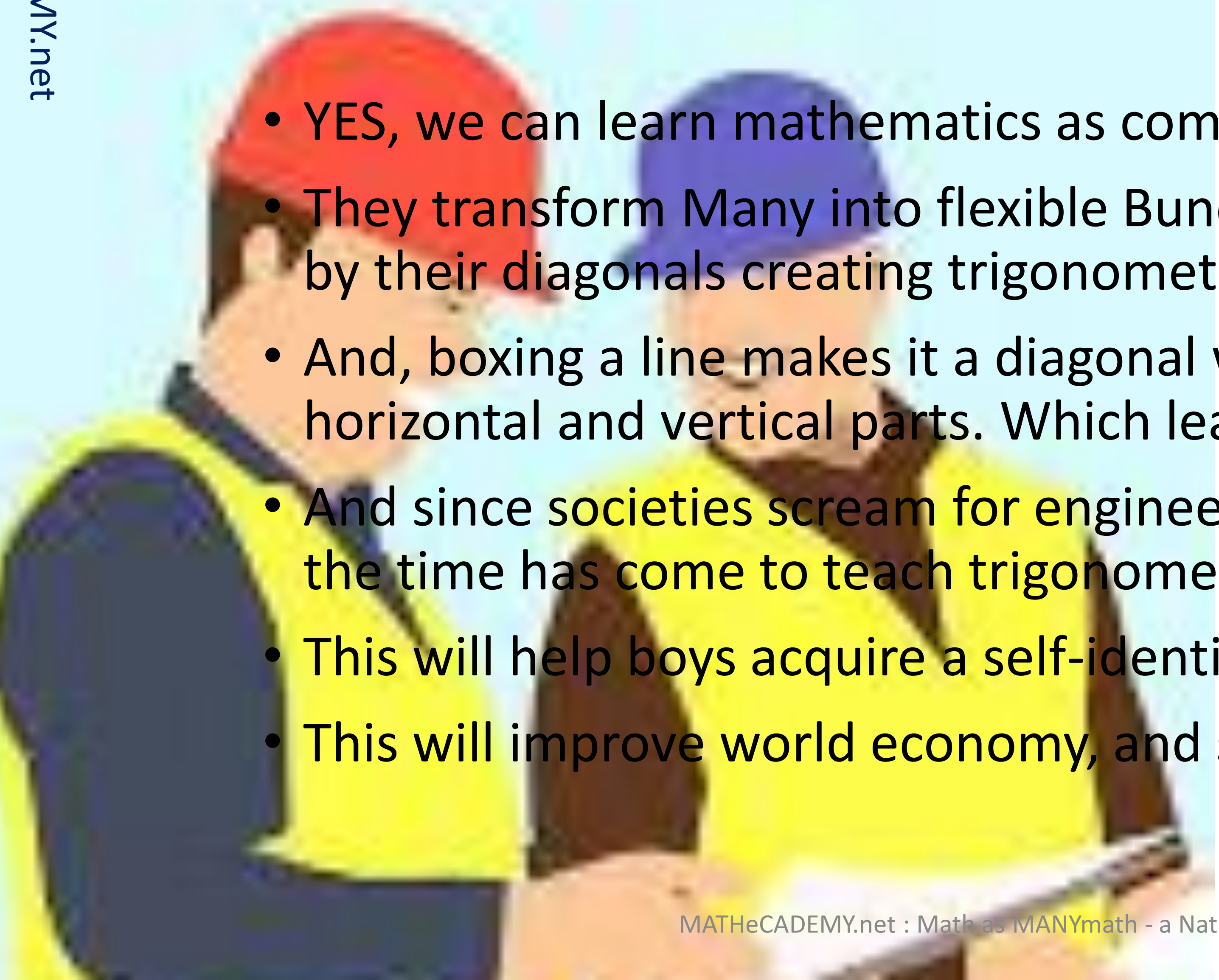
5) Counting & re-counting is big fun, when predicted by a calculator with the recount formula: **T = (T/B)xB** *(from T, T/B times, B can be pushed away)*

*Question: T = 4 **5s** = ? **3s*** ● *Answer: T = 4 **5s** = 6B2 **3s*** ● *Prediction:*

| | |
|---|---|
| 4x5/3 | 6.some |
| 4x5 – 6x3 | 2 |



# Conclusion



- YES, we can learn mathematics as communication about boxes.
- They transform Many into flexible Bundle-numbers. And they split in two by their diagonals creating trigonometry formulas when recounting sides.
- And, boxing a line makes it a diagonal with trigonometry predicting its horizontal and vertical parts. Which leads to core STEM examples.
- And since societies scream for engineers able to master STEM problems, the time has come to teach trigonometry before or instead of geometry.
- This will help boys acquire a self-identity as ‘Masters of Many’.
- This will improve world economy, and solve poverty and climate issues.

# Theoretical Background

Tarp, A. (2018). Mastering Many by counting and recounting before adding on-top and next-to. *Journal of Math Education, March 2018, 11*(1), 103-117.

Tarp, A. (2020). De-modeling numbers, operations and equations: from inside-inside to outside-inside understanding. *Ho Chi Minh City University of Education Journal of Science 17*(3), 453-466.