Proposals for Papers

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STEM PREVENTS A GOAL-DISPLACEMENT THAT MAKES MATHEMATICS A GOAL INSTEAD OF A MEANS

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Asking what is the purpose of mathematics education, US and UK mathematics educators say “to learn school mathematics”. Others say “to learn set-based mathematics as defined by university mathematics.” Focusing on competences leads to saying “to learn mathematical competences” or “to master mathematics”. Seldom, if ever, is heard that the goal is “to master many” or “to develop the number-language that children bring to school.”

Sociological imagination (Bauman, 1990) may prevent a goal displacement where a means becomes a goal instead. Historically, the Pythagoreans chose the word ‘mathematics’ meaning ‘knowledge’ in Greek as a common name for their knowledge about Many in space and time and by itself: astronomy, music, geometry and arithmetic. And today in North America, mathematics is still a common name for geometry and algebra, showing their outside goals in their original meanings, earth-measuring in Greek, and reuniting in Arabic. Integration and differentiation also name their tasks directly, to integrate small changes, and to differentiate a total change in small changes.

To avoid a goal displacement, mathematics must de-model (Tarp, 2019) its core ingredients: digits, operations, equations, fractions, functions etc. to allow primary school develop the flexible bundle-numbers children bring to school by teaching, not numbers to add, but numbering totals by counting, recounting and double-counting, where recounting 8 fingers in 2s as 8 = (8/2)*2 leads directly to the recount-formula $T = (T/B)*B$ with per-numbers that solve equations, that occur in most STEM-formulas typically predicting proportionality, and that become fractions when double-counting in the same unit.

Liberated from its goal displacement, mathematics education may have its own communicative turn as in the 1970s (Widdowson, 1978) such that from now on both the word- and the number-language are taught and learned through their use and not through their grammar, thus allowing all students to model outside quantities as to levels, change and distribution.

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GUIDING CHILDRENS' MASTERY OF MANY WITH BUNDLE NUMBERS

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Learning means adapting the inside brain to outside nature and culture. Vygotsky prioritizes culture and wiser-brain teaching, Piaget nature and peer-brain learning.

Adapting to Many, children answer the question 'How many?' with bundle-numbers as T = 2 3s containing two digits: 3 is a quantity-number in space, also called a cardinal-number taking on positive integer values; 2 is a counting-number in time taking on also decimal, fractional, and negative values as T = 7 = 2.1 3s = 2 1/3 3s = 3.-2 3s.

Quantity-numbers may add, and so may counting-numbers, but not in between. So, digits must be categorized before adding. Digits are not numbers but operators, needing a multiplier to become a number, T = 2 3s = 2*3, as seen when writing numbers fully as polynomials, as e.g. $T = 345 = 3*B^2 + 4*B + 5*1$

So, teaching digits as numbers is teaching wrong numbers. And bundle-numbers need not to be taught since children bring them to school, that should guide them to develop their number-language by learning that

- digits are icons with as many strokes as they represent.
- operations are icons also, rooted in the counting process: division is a broom pushing away bundles, multiplication is a lift stacking bundles, subtraction is a rope pulling away stacks to find unbundled, and addition is the two ways to unite stacks, on-top and next-to.
- recounting 8 in 2s gives a recount-formula: 8 = (8/2)*2, or $T = (T/B)*B$, used to solve the equation $u*2 = 8$ by recounting 8 in 2s to give the solution $u = 8/2$; thus solving most STEM-equations, typically predicting proportionality.

Later recounting between digit-units and tens leads to tables, and to equations when asking e.g. T = 4 6s = ? tens, and T = 42 = 4.2 tens = ? 7s.

So, childhood education should guide children develop the quantitative competence they bring to school using Kolb's experiential learning cycles.

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FROM STEAM TO STEEM TO INCLUDE ALSO ECONOMICS

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STEM integrates mathematics with its roots in science, technology and engineering, all using formulas from algebra and trigonometry to pre-dict the behavior of physical quantities. Statistics post-dicts unpredictable quantities by setting up probabilities for future behavior, using factual or fictitious numbers as median and fractals or average and deviation. Including economics so STEM becomes STEEM opens the door to statistics also. Art may be an appetizer, but not a main course since geometry and algebra should be always together and never apart to play a core role in STEEM.

Macroeconomics describes households and factories exchanging salary for goods on a market in a cycle having sinks and sources: savings and investments controlled by banks and stock markets; tax and public spending on investment, salary and transferals controlled by governments; and import and export controlled by foreign markets experiencing inflation and devaluation. Proportionality and linear formulas may be used as first and second order models for this economic cycle, using regression to set up formulas and spreadsheet for simulations using different parameters.

Microeconomics describes equilibriums in the individual cycles. On a market, shops buy and sell goods with a budget for fixed and variable cost, and with a profit depending on the volume sold and the unit-prices, all leading to linear equations. In the case of two goods, optimizing leads to linear programming. Competition with another shop leads to linear Game Theory. Market supply and demand determines the equilibrium price. Market surveys leads to statistics, as does insurance. In the households, spending comes from balancing income and transferals with saving and tax. In a bank, income come from simple and compound interest, from installment plans as well as risk taking. At a stock market, courses fluctuate. Governments must consider quadratic Laffer-curves describing a negative return of a tax-raise. To avoid units, factories use variations of Cobb-Douglas power elasticity production functions for modeling.

In English sentences may be analyzed on a word level as to the frequency of subjects, verbs, direct and indirect objects, predicates, and unspecified words.

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CONCRETE STEM SUBJECTS ALLOW MATHEMATICS LEARNING BY MODELING AND PEER-BRAIN TEACHING

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Traditionally, mathematics is considered one of the core subjects in education because of the many 'applications of mathematics'. This phrasing leads directly to the view that "of course mathematics must be learned first before it can be applied by others". Consequently, mathematics teaches the operation order addition, subtraction, multiplication and division with cardinal numbers, later expanded to integers, rational and real numbers, again followed by expressions including also unspecified numbers.

Talking instead about outside roots leads to the opposite view that "of course, mathematics must be learned through its outside roots, also constituting its basic applications". This 'de-modeling' view resonates with the fact that historically, the Pythagoreans chose the name mathematics, meaning knowledge in Greeks, as a common label for their four areas of knowledge about Many in time and pace, in time, in space and by itself: astronomy, music, geometry and arithmetic. Later the Arabs added algebra with polynomial numbers created by systematic bundling. Here the outside roots are evident through the original meanings of geometry and algebra: earth-measuring and reuniting.

So, mathematics grew and may still grow from counting, recounting and double-counting bundles, and from applying science, describing forces pumping motion in and out of matter when having the same or opposite directions.

Working in groups with science applications allows students to learn through peer-brain teaching instead of through wiser-brain teaching. As to matter, tasks could be to find its mass, its center, its density, and the heat transfer under collision between visible macro-matter and invisible micro-matter, applied in steam power, or when placing ice-cubes in water.

As to motion, tasks could be to describe traveling with constant or changing speed horizontally or on an incline, vertical motion, projectile orbits; and circular motion, swings or see-saws on a market place. As well as how to use electrons to store or transport motion and information.

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TO SUPPORT STEM, TRIGONOMETRY AND COORDINATE GEOMETRY SHOULD PRECEDE PLANE GEOMETRY

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Halved by its diagonal c, a rectangle becomes a right triangle ABC with base b and height h. Using the recount-formula $T = (T/B)*B$ coming from recounting 8 in 2s as 8 = (8/2)*2, mutual recounting gives trigonometry:

$h = (h/c)*c = \sin A*c$, $b = (b/c)*c = \cos A*c$, $h = (h/b)*b = \tan A*b$.

Splitting the diagonal in $c1$ and $c2$ by the triangle-height produces two triangles where $\cos A = c1/b = b/c$, making $b\text{^}2 = c*c1$, and $\cos B = c2/h = h/c$, making $h\text{^}2 = c*c2$, thus giving the Pythagoras rule $h\text{^}2 + b\text{^}2 = c\text{^}2$.

Finding √70 means squeezing 7 tens until becoming a square (8+t)^2 situated between 8^2 and 9^2. And having four parts as shown by two playing cards placed like an L: 8^2, and 8*t twice, and t^2. Neglecting t^2, we get the equation 8*t = (70-8^2)/2 = 3 = (3/8)*8, solved by recounting 8 in 3s, giving t = 3/8 = 0.375, so 8.375^2 = 70,14 = 70 approximately.

In a coordinate system, a circle with center in the origin and radius r gets the equation $x\text{^}2 + y\text{^}2 = r\text{^}2$, else $(\Delta x)\text{^}2 + (\Delta y)\text{^}2 = r\text{^}2$.

In a horizontal right triangle, moving along the diagonal will change x and y with Δx and Δy. Recounting Δy in Δx gives

$\Delta y = (\Delta y/\Delta x)*\Delta x = m*\Delta x = \tan A*\Delta x$ that allows drawing lines from tables.

Intersection points between lines are predicted by a linear equation solved by technology or by moving to opposite side with opposite sign.

Intersection points between lines and circles or parabolas are predicted by a quadratic equation $x\text{^}2 + b*x + c = 0$, solved by two L-placed playing cards showing that

$(x+t)\text{^}2 = x\text{^}2 + 2*x*t + c + (t\text{^}2 - c)$

where the first three terms disappear with $t = b/2$.

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A FRESH-START YEAR10 (PRE)CALCULUS CURRICULUM

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Often precalculus suffers from lacking student knowledge. Three options exist: make mathematics non-mandatory, choose an application-based curriculum; or, to rebuild student self-confidence, design a fresh-start curriculum that also includes the core of calculus by presenting integral calculus first.

Writing a number out fully as a polynomial, e.g. T = 345 = 3*B^2 + 4*B + 5 shows the four ways to unite numbers, resonating with the Arabic meaning of the word algebra, to reunite: addition and multiplication unite changing and constant unit-numbers into totals; and next-to-block-addition (integration) and power unite changing and constant per-numbers, all having reverse operations that split totals into parts.

Addition, multiplication, and power are defined as counting-on, repeated addition and repeated multiplication. As reverse operations, x = 7-3 is defined as the number that added to 3 gives 7, thus solving the equation x+3 = 7 by moving to opposite side with opposite sign. Likewise, x = 7/3 solves x*3 = 7, the factor-finder (root) x = 3√7 solves x^3 = 7, and the factor-counter (logarithm) x = log3(7) solves 3^x = 7, again moving to opposite side with opposite sign.

Hidden brackets allow reducing a double calculation to a single:

2+3*x = 14 becomes 2+(3*x) = 14, solved by x = (14-2)/3.

Next transposing letter-equations as $T = a+b*c$^d really boost self-pride.

Future behavior of 2set unit-number tables is predicted by linear, exponential, or power models assuming constant change-number, change-percent, or elasticity.

1-4set per-number speed tables are modeled with lines, parabolas and double-parabolas, allowing technology to calculate the distance covered, thus introducing integral calculus, that also occurs when adding per-numbers in mixture-problems, and when adding percent in cross tables generated by statistical questionnaires.

Trigonometry comes from mutual double-counting sides in a rectangle halved by its diagonal, and is used to model distances to far away points, bridges, roads on hillsides, motion down an incline, and jumps from a swing.

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TO SUPPORT STEM, CALCULUS MUST TEACH ADDING BUNDLE-NUMBERS, PER-NUMBERS AND FRACTIONS ALSO

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Created to add locally constant per-numbers by their areas, integral calculus normally is the last subject in high school, and only taught to a minority of students. But, since most STEM-formulas express proportionality by means of per-numbers, the question is if integral calculus may be taught earlier. Difference research (Tarp, 2018) searching for hidden differences finds that the answer is yes.

Integral calculus occurs in grade one when performing next-to addition of bundle-numbers as e.g. T = 2 3s + 4 5s = ? 8s, leading on to differential calculus as the reverse question: 2 3s + ? 5s = 3 8s, solved by first removing 2 3s from 3 8s and then counting the rest in 5s, thus letting subtraction precede division, where integral calculus does the opposite by letting multiplication creating areas precede addition.

In middle school adding per-numbers by areas occurs in mixture problems: 2kg at 3$/kg + 4kg at 5$/kg = 6kg at ? $/kg, again with differential calculus coming from the reverse question: 2kg at 3$/kg + 4kg at ? $/kg = 6kg at 5 $/kg. Here the per-number graph is piecewise constant c, i.e. there exists a delta-interval so that for all positive epsilon, the distance between y and c is less than epsilon. With like units, per-numbers become fractions thus also added by their areas, and never without units.

In high school adding per-numbers occurs when the meters traveled with varying m/s speed P is found as the area under the per-number graph now being locally constant, formalized by interchanging epsilon and delta. Here the area A under the per-number graph P, is found by slicing the area thinly so that its change may be written as dA = P*dx in order to use that when differences add, all middle terms disappear leaving just the endpoint difference, thus motivating developing differential calculus to find A’ = dA/dx = P.

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CONFLICTING GRAND THEORIES CREATE 2X3X2 DIFFERENT MATHEMATICS EDUCATIONS

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As part of institutionalized education, mathematics falls under the focus of the three grand theories, philosophy, sociology and psychology, discussing different kinds of mathematics, of education and of learning; and recommending appropriate means to institutional goals. However, is the goal to master mathematics first, as a means to later master many; or to master many directly if mastering mathematics proves difficult?

As to learning, psychology sees coping coming from brains adapting to outside nature and culture, but which is more important? Vygotsky points to culture, mediated by a more knowledgeable wiser-brain, a teacher. Piaget points to nature, automatically creating inside schemata that accommodate if meeting outside resistance from nature or from peer-brain communication.

As to mathematics, philosophy has three conflicting views: Pre-modern mathematics is inspired by the Pythagoreans seeing mathematics as knowledge about Many in space and time and by itself as expressed in astronomy, music, geometry and arithmetic; and as part of the three basic Rs: reading, writing and 'rithmetic called reckoning in Germanic countries. Modern mathematics needs no outside examples for its concepts. Alternatively, postmodern scepticism sees mathematics as a number-language abstracting inside concepts from outside examples, and parallel to the word-language.

As to institutions, sociology recommends imagination to prevent a goal displacement making a means a goal instead. As to education, two conflicting views exits. One sees the student as a servant of the state forcing its population to choose between different multiyear tracks from upper secondary school, and forcing students back to start if changing track. One sees the state as a servant of the student by helping students to uncover and develop their personal talent in self-chosen half-year blocks after puberty.

So, two different learning forms, three different mathematics forms, and two different education forms create 2x3x2 different ways of conducting mathematics education.

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THE POWER OF PER-NUMBERS

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Uniting unit-numbers as 4$ and 5$, or per-numbers as 6$/kg and 7$/kg or 6% and 7%, we observe that addition and multiplication unite changing and constant unit-numbers into a total, and integration and power unite changing and constant per-numbers. Reversely, subtraction and division split a total into changing and constant unit-numbers, and integration and power split a total into changing and constant per-numbers.

Recounting 8 in 2s as 8 = (8/2)*2 creates a recount-formula $T = (T/B)*B$, saying 'From T, T/B times, T may be pushed away'; and used to change units when asking e.g. 2 6s = ? 3s, giving the prediction T = (2*6/3)*3 = 4*3 = 4 3s.

Recounting 8 in 2s also provides the solution $u = 8/2$ to the equations as $u*2 = 8 = (8/2)*2$; thus solving most STEM-equations, since the recount-formula occurs all over. In proportionality, $y = c*x$; in coordinate geometry as line gradients, $\Delta y = \Delta y/\Delta x = c*\Delta x$; in calculus as the derivative, $dy = (dy/dx)*dx = y'*dx$. In science as meter = (meter/second)*second = speed*second, etc. In economics as price formulas: $ = ($/kg)*kg = price*kg, $ = ($/day)*day = price*day, etc.

With physical units, recounting gives per-numbers bridging the units. Thus 4$ per 5kg or 4/5 $/kg gives T = 15kg = (15/5)*5kg = (15/5)*4$ = 3$; and T = 16$ = (16/4)*4$ = (16/4)*5kg = 20kg. With like units, per-numbers become fractions.

Trigonometry occurs as per-numbers when mutually recounting sides in a rectangle halved by its diagonal, $a = (a/c)*c = \sin A*c$, etc.

Modeling mixtures as 2kg at 3$/kg + etc, unit-numbers add directly, but per-numbers P add by the area A under the per-number graph, found by slicing it thinly so that the change may be written as $dA = P*dx$ in order to use that when differences add, all middle terms disappear leaving just the endpoint difference, thus motivating developing differential calculus to find the per-number $A' = dA/dx = P$.

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THE SIMPLICITY OF MATHEMATICS DESIGNING A STEM-BASED CORE CURRICULUM FOR REFUGEE CAMPS

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Numbers as 2345 evade the place value notation if seen as numbering unbundled, bundles, bundle-bundles, bundle-bundle-bundles. Here exponents occur as the number of bundling-repetitions, e.g. when counting ten fingers as T = ten = 3*B*1 3s = 1*BB*1 3s.

Stacking bundles in blocks, the unbundled singles may be placed as a stack next-to leading to decimals, e.g. T = 7 = 2.1 3s; or on-top of the stack counted as bundles leading to fractions, T = 7 = 2 1/3 3s, or to negative numbers counting what is needed for another bundle, T = 7 = 3.-2 3s.

Bundles and negative numbers may be included in the counting sequence: 0*B*1, 0*B*2, …, 0*B*7, 1*B*-2; 1*B*-1, 1*B*0, 1*B*1, …, 9*B*7, 1*BB*-2, 1*BB*-1, 1*BB*.

Counting makes operations icons: division is a broom pushing away bundles, multiplication is a lift stacking bundles, subtraction is a rope pulling away stacks to find unbundled, and addition is the two ways to unite stacks, on-top and next-to.

Recounting 8 in 2s gives a recount-formula, 8 = (8/2)*2 or *T = (T/B)*B*, that changes unit from 1s to 2s (proportionality), that gives the equation u*2 = 8 the solution u = 8/2 (moving to opposite side with opposite sign), and that shows that per-numbers as 8/2 must be multiplied to areas before being added (integral calculus).

Once counted, stacks may add on-top after recounting changes the units to the same, or next-to by adding areas as in integral calculus. And reverse addition leads to differential calculus by pulling away the initial stack before pushing away bundles.

Recounting between digits and tens leads to tables and equations when asking e.g. T = 4 6s = ? tens, and T = 42 = ? 7s. Recounting in different units gives per-numbers bridging the units, becoming fractions with like units, and adding by areas. Mutually recounting sides in a block halved by its diagonal gives trigonometry.

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CALCULATION MODELS, FACT OR FICTION

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As qualitative literature, also quantitative literature has the genres fact and fiction when modeling real world situations.

Fact is 'since-then' calculations using numbers and formulas to quantify and to predict predictable quantities as e.g. 'since the base is 4 and the height is 5, then the area of the rectangle is T = 4*5 = 20'. Fact models can be trusted once the numbers and the formulas and the calculation have been checked. Special care must be shown with units to avoid adding meters and inches as in the case of the failure of the 1999 Mars-orbiter.

Fiction is 'if-then' calculations using numbers and formulas to quantify and to predict unpredictable quantities as e.g. 'if the unit-price is 4 and we buy 5, then the total cost is T = 4*5 = 20'. Fiction models build upon assumptions that must be complemented with scenarios based upon alternative assumptions before a choice is made.

This paper looks at three infection models, the standard logistic model and two alternatives, one formulated as a differential equation, one as a difference equation.

The models all assume that the population change is proportional to the population itself thus giving a doubling factor that is assumed to decrease with the number of infected as the first model assumes, or with time as the two others assume.

However, where the standard model cannot be validated since infection data are difficult or impossible to achieve, reliable data from the number of hospital beds points to the other fiction models. Here, the scientific principle of simplicity, known as Occam's razor, points to the difference equation, easy to set up in a spreadsheet.

It may be simple but it provides important information: its high degree of elasticity recommends a gradual reduction of the two central factors, group size and meeting time, instead of a complete shutdown.

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RISING AGAIN THE DROPOUTS

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Children come to school with a number-language developed through communicating about Many using two-dimensional bundle-numbers with units as 2 3s. However, the school ignores this and forces upon them one-dimensional line-numbers without units making many drop out.

A math dropout feels the full burden of the classroom as a 'pris-pital' (Foucault, 1995), i.e., a mixture of a prison forcing you to return again and again, and a hospital curing you for a self-referring diagnose 'as math-ignorant you must learn math.' Just entering makes your stomach turn because of negative associations. The teacher's look shows the negative expectations you also feel inside. You go to your place in the back where you are neglected or bullied. You are down with no chance to rise again. Then a miracle happens. A virus comes and forces all out of the classroom to home education, where all of a sudden you get a second chance to again do what you once mastered so fully, to communicate about Many (Widdowson, 1978). Your school even offers rise-again curricula (Tarp, 2021) saying that the goal of mathematics education is to master, not inside mathematics, but its outside root Many.

In MC01, micro-curriculum 1, at your folded right hand you point to 4, to 5, and to 6 when including the arm. You then rearrange a total of four sticks as one 4 icon, reported inside on paper by writing T = I I I I = 4. Likewise with 5 and 6. Thus observing that digits are icons with their number of sticks if written less sloppy.

In MC02, inside you write T = 56 = 5ten6 = 5Bundle 6 = 5B 6 = 5xB + 6 = 5xB + 6x1. Thus observing that 56 is not one number, but two numberings of bundles, and of unbundled singles, to be shown outside in different ways, and as two boxes of bundles and singles.

In MC03, inside you write T = 456 = 4hundred 5ten 6 = 4tenten 5ten 6 = 4BB 5B 6 = 4xB^2 + 5xB + 6x1. Thus observing that 456 is three numberings of bundles-of-bundles, of bundles, and of unbundled singles, to be shown outside in different ways, and as three boxes.

In MC04, inside you draw the 'algebra-square' (Tarp, 2018) showing the 4 ways to unite as shown in the formula above. Here addition and multiplication unite changing and constant unit numbers, and box-addition (integration) and power unite changing and constant per-numbers. Now you see the simplicity of math.

In MC05, outside you re-count eight cubes in 2s by 4 times pushing away 2s with a card, inside iconized by an uphill stroke called division, and predicted by a calculator as '8/2 = 4'. Then you stack the bundles, inside iconized by a lift called multiplication, and predicted by a calculator as '4x2 = 8'. Then inside you report by writing, first 8 = (8/2)x2, then T = (T/B)xB with unspecified numbers, called a 'recount-formula' solving equations: ux2 = 8 is solved by recounting 8 in 2s, ux2 = 8 = (8/2)x2, so u = 8/2. And also changing units.

In MC06, outside you re-count cubes in dollars, given that there is 3$ per 5cubes called a per-number 3/5 $/cube. You then answer the two questions by recounting in the per-number:
Q01: "12$ = ? cubes." T = 12$ = (12/3) x 3$ = (12/3) x 5 cubes = 20 cubes
Q02: "? $ = 15 cubes." T = 15 cubes = (15/5) x 5 cubes = (15/5) x 3$ = 9$

In MC07, outside you use the diagonal to halve a box with base b, and height h, which inside allows you to set up trigonometry formula when mutually recounting the sides: h = (h/b) x b = tan$A$ x b, etc.

In MC08, outside you observe that two totals 2 3s and 4 5s may add both on-top after recounting provides like units, and next-to by areas as 8s, also called integration; becoming differentiation when asking the reverse question '2 3s and how many 5s total 3 8s?', thus subtracting the 2 3s before dividing by 5.

In MC09, outside in the bill '2kg at 3$/kg and 4kg at 5$/kg', the unit-numbers add directly, but the per-numbers must be multiplied to unit-numbers before adding. So, per-numbers add by areas, called integral calculus, that again may be reversed to differential calculus finding per-numbers.

Now, you are far ahead in mathematics, and will return to the classroom as a star teaching how to master Many.

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MATHEMATICS IS BORN AS STEEM

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In ancient Greece, the Pythagoreans chose the word mathematics as a label for their four knowledge areas: music, astronomy, geometry, and arithmetic, describing the physical fact Many in time, in time and space, in space, and by itself. With music and astronomy as independent areas, today mathematics is often used as a common label for the two remaining areas, geometry and algebra, both born as natural sciences as shown by their Greek and Arabic meanings: earth-measuring, and reuniting.

The core of the science of reuniting is seen in the Algebra-square (Tarp, 2018). Writing out fully a total of 456 as a sum of boxes, T = 4xB^2 + 5xB + 6x1, shows the four ways to unite: addition and multiplication unite changing and constant plus-numbers, and box-addition, or integration, and power unite changing and constant factor-numbers. All have reverse operations to predict the result of splitting a total into parts: subtraction, division, differentiation, and root and logarithm.

The basic number-science transforms outside examples of Many into inside numbers. Recounting a total of 8 in 2s means 4 times pushing away bundles of 2s with a card iconized by an uphill stroke called division, 8/2 = 4. Next, the bundles are stacked 4 times iconized by a lift called multiplication, 4x2 = 8. Outside recounting thus is predicted inside by 8 = (8/2)x2, or T = (T/B)xB with unspecified numbers, called a recount-, proportionality- or linearity formula that is used to solve equations:

The equation ux2 = 8 is solved by recounting 8 in 2s, ux2 = 8 = (8/2)x2, so u = 8/2.

And that also is used to change units as when recounting cubes in dollars, given that there is 3$ per 5cubes called a per-number 3/5 $/cube, used for recounting: 12$ = (12/3) x 3$ = (12/3) x 5 cubes = 20 cubes. And, when recounting from $ to fractions occurring as per-numbers with like units, 3kg/5kg = 3/5 = 60/100 = 60%. So with 20$ per 100%, 5$ = (5/20)x20$ = (5/20)x100% = 25%.

Economics and trade are first to use the recount-formula when pricing, when changing currency, etc. And to use the algebra-square where plus- and factor-numbers are renamed to unit- and per-numbers. Here, uniting unit-numbers occurs in everyday trade, uniting constant per-numbers occurs in bank interests. And uniting changing per-numbers occurs in mixture tasks as billing 2kg at 3$/kg and 4kg at 5$/kg; and in basic science when mixing cold and hot water, mixing speed, etc.

Geometry is next, eased by observing that using the diagonal to halve a box with base b, and height h, will allow setting up trigonometry formula when mutually recounting the sides: h = (h/b) x b = tan$A$ x b, etc. Here many engineering tasks wait to be solved: building bridges across canyons, designing roads up a steep hill, etc. (Tarp, 2021)

Algebra and geometry unite in coordinate geometry easing design tasks, and in the science of moving objects and rays, e.g., jumping from a ski hill or from a swing, moving through two areas with different speed, as well as predicting intersection points between lines, triangles and circles.

Technology is filled with present and historic examples. Levers, pulleys, equilibriums, centers of gravity, kinematics, dynamics, force diagrams, etc., as in the UK applied mathematics curriculum. Production, distribution, and storage of energy, electrical circuits, power stations, steam engines, etc.

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MASTERING MANY IN TEACHER & ONLINE EDUCATION

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Is the goal of mathematics education to master mathematics to later master Many, or the other way around? The tradition typically chooses the former thus keeping many from reaching the end goal, mastery of Many. So why not turn it around now that forced online education offers new learning opportunities in teacher and student education? So, why not allow inside geometry and algebra to link to their outside roots shown by their Greek and Arabic meanings, earth-measuring and reuniting.

As to algebra, to recount ten cubes in 3s we push away bundles 10/3 times, we pull away the 3x3 stack, and place the unbundled on-top. This links division, multiplication, subtraction, addition, power, fractions, decimals, and negative numbers to outside boxes described inside by 'flexible bundle-numbers' (Tarp, 2018):

T = ten = 2B4 = 3B1 = 3.1 B = 3+1/3 B = 4B-2 = 1BB0B1 = 1BB1B-2 3s.

Likewise, recounting 8 cubes in 2s is inside predicted by 8 = (8/2)x2, or $T = (T/B)\text{x}B$ with unspecified numbers, called a recount-, proportionality- or linearity formula that is used to solve equations: ux2 = 8 = (8/2)x2, so u = 8/2. And that is also used to change units when recounting cubes in dollars, given that there is 3$ per 5cubes called a per-number 3/5 $/cube, used for recounting: 12$ = (12/3)x 3$ = (12/3)x 5 cubes = 20 cubes.

As to geometry, using the diagonal to halve a box with base b and height h will allow setting up trigonometry formula when mutually recounting the sides: $h = (h/b) \text{ x } b = \tan A \text{ x } b$, which then allows setting up a formula for pi = n x tan(180/n).

Flexible bundle-numbers links inside to outside: bundles to boxes, place values to bundles-of-bundles, equations to recounting taking the pain out of measuring, carrying and borrowing to overload and underload, giving sense to multiplication and division:

7x45 = 7x4*B*5 = 28*B*35 = 31*B*5 = 315, and, 315 /7 = 31*B*5 /7 = 28*B*35 /7 = 4*B*5 = 45

The recounting formula opens up most STEM-tasks, typically using proportionality. And allows integration to add bundle-numbers as per-numbers by their areas.

Trigonometry opens up design and engineering tasks to be solved inside on paper or with playing cards, or outside on tiles (Tarp, 2021).

Surprised by the effect of outside-inside linking, teachers may join 7 other teachers in online 'pyramidEDUCATION' at the MATHeCADEMY.net using question- and task-guidance in all 2x4 CATS-modules, Count & Add in Time & Space at both primary and secondary level. Thus, mathematics as communication (Widdowson, 1978) may be learned through counting, recounting, and double-counting before adding Many.

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MIXING DESIGN AND DIFFERENCE RESEARCH WITH EXPERIENTIAL LEARNING CYCLES ALLOWS CREATING CLASSROOM TEACHING FOR ALL STUDENTS

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International tests show that not all students benefit from mathematics education. This poor-performance-problem raises a Cinderella question: is there a hidden difference that can make the Prince dance? If so, design research can create Kolb's experiential learning cycles to adapt a given micro-curriculum so that all students may benefit.

In primary school, difference research searching for hidden differences has identified several alternatives: Digits are icons. Numbers are double-numbers with bundles as units, e.g. T = 2 3s. Flexible bundle-numbers have over- and underloads, e.g.

T = 53 = 5*B*3 = 4*B*13 = 6*B*-7 tens, and ease operations as e.g.

329/ 7 = 32*B*9/ 7 = 28*B*49 /7 = 4*B*7 = 47, or 23*8 = 2*B*3*8 = 16*B*24 = 18*B*4 = 184.

Operations are icons also where division is a broom pushing away bundles, multiplication a lift stacking bundles, subtraction a rope pulling away stacks to find unbundled, and addition the two ways to unite stacks, on-top and next-to.

Changing units may be predicted by a recount-formula $T = (T/B)*B$ coming from recounting 8 in 2s as 8 = (8/2)*2, or, and used to solve the equation $u*2 = 8$ by recounting 8 in 2s to give the solution $u = 8/2$; thus solving most STEM-equations, typically predicting proportionality: meter = (meter/sec)*sec = speed*sec.

In middle school, double-counting leads to per-numbers becoming fractions with like units, and adding by their areas as integral calculus. In algebra, factors are units placed outside a bracket. Trigonometry occurs when mutually double-counting sides in a rectangle halved by its diagonal.

In high-school, redefining inverse operations allows equations to be solved by moving to opposite side with opposite sign. And adding per-numbers by areas allows introducing integral calculus before differential calculus.

Designing and redesigning micro-curricula as experiential learning cycles allows teachers perform design research in their own classroom, to be reported as master projects first, and later perhaps as PhD projects including more details.

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FROM PLACE VALUE TO BUNDLE-BUNDLES: UNITS, DECIMALS, FRACTIONS, NEGATIVES, PROPORTIONALITY, EQUATIONS AND CALCULUS IN GRADE ONE

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Traditionally, a multi-digit number as 2345 is presented top-down as an example of a place value notation counting ones, tens, hundreds, thousands, etc.; and seldom as four numberings of unbundled, bundles, bundle-bundles, bundle-bundle-bundles, etc., to provide a bottom-up understanding abstracted from concrete examples, which would introduce exponents in primary school as the number of bundle-repetitions. Counting ten fingers in 3s thus introduces bundle-bundles:

T = ten = 3*B*1 3s = 1*BB*1 3s.

Stacking bundles, the unbundles singles may be placed as a stack next-to leading to decimals, e.g. T = 7 = 2.1 3s; or on-top of the stack counted as bundles thus leading to fractions, T = 7 = 2 1/3 3s; or to negative numbers counting what is needed for another bundle, T = 7 = 3.-2 3s.

Bundles and negative numbers may also be included in the counting sequence:

0*B*1, 0*B*2, …, 0*B*7, 1*B*-2; 1*B*-1, 1*B*0, 1*B*1, …, 9*B*7, 1*BB*-2, 1*BB*-1, 1*BB*.

Counting makes operations icons: division is a broom pushing away bundles, multiplication is a lift stacking bundles, subtraction is a rope pulling away stacks to find unbundled, and addition is the two ways to unite stacks, on-top and next-to.

Recounting 8 in 2s may be written as a recount-formula:

8 = (8/2)*2, or $T = (T/B)*B$,

used to solve the equation $u*2 = 8$ by recounting 8 in 2s to give the solution $u = 8/2$; thus solving most STEM-equations, typically expressing proportionality.

Once counted, stacks may add on-top after recounting changes the units to the same, or next-to by adding areas as in integral calculus. And reverse addition leads to differential calculus by pulling away the initial stack before pushing away bundles.

At the end of grade one, recounting between digits and tens leads to tables and equations when asking e.g. T = 4 6s = ? tens, and T = 42 = ? 7s.

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SOCIOLOGICAL IMAGINATION DESIGNS MICRO-CURRICULA FOR EXPERIENTIAL LEARNING CYCLES

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Forced by peer review to focus on existing research, many education research articles fail to be validated in the classroom by observing if its educational goal is reached. However, the peer review crisis creates a need for a different research meeting its proper genre demands: reliable data and valid findings to a research question.

To help student brains adapt to the outside world, mathematics education must decide if its goal is to master inside mathematics as a means to later master outside quantity, thus risking what sociology calls a goal displacement (Bauman, 1990) where a means becomes a goal instead; or to master quantity directly if first mastering contemporary university mathematics becomes too difficult to many students.

Many curriculum reforms include competences. But again, we must ask: is the goal to obtain inside mathematical competence, or outside quantitative modeling competence?

A learning-by-doing curriculum calls for experiential learning cycles as described by Kolb's learning theory (Kolb, 1983) being adapted e.g. in the new Vietnamese curriculum; and containing cyclic phases. First micro-curriculum A is taught and validated if meeting its expected goals, next systematic observations gather reliable data as to which goals are met, and which are not, then reflections modifies the micro-curriculum into version B. Then plan B is taught, etc.

Combined with design research (Bakker, 2018), experiential learning cycles allows teachers to become action learners or action researchers in their own classroom reporting their work in master or PhD papers. To meet the genre demands of research, the data gathered must be reliable, and the findings must be tested for validity. In design research, reliability comes by making systematic observations through notes, interviews, questionnaires etc. And testing validity here means holding on to the strong parts of the actual micro-curriculum and changing the weak parts.

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