From **STEAM** to **STEEM**

Mathematics is for All
if Rooted in PerNumber Economics

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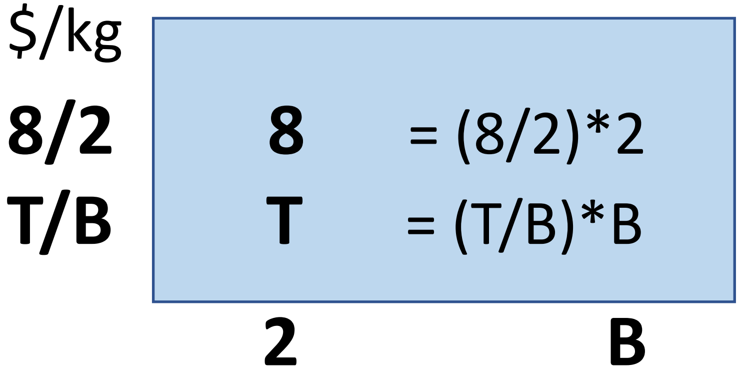
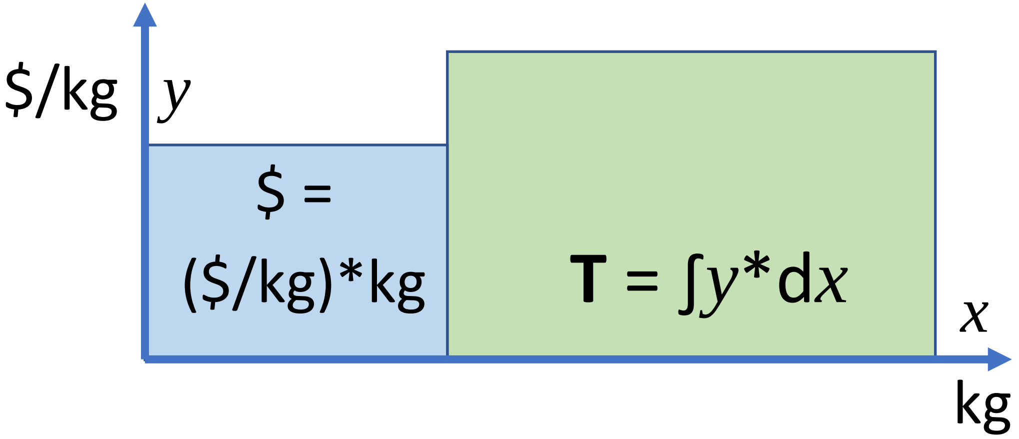




What is the real goal of mathematics education? Inspiration from Grand Theory

PHILOSOPHY

- Is mathematics about **essence** or **existence**? (Heidegger)

SOCIOLOGY

- **Institutions** as solutions to common problems may be tempting, but
- Beware of a **goal displacement** (Bauman) where a means becomes a goal instead, and creates a patronizing **truth regime** (Foucault)

PSYCHOLOGY

- What to learn: essence (Vygotsky), or existence (Piaget)
- Beware of using **fear of exclusion** to keep a **truth regime** in power

From STEM over STEAM to STEEM
From Hard to Easy - if rooting Math in Economics

Math is hard

- Teach STEM. It motivates **reasons** to learn math, before applying it.

Oops, Math is still hard

- Teach STEAM to sugar-coat a **desire** for learning it, before applying it.

Oops, Math is still hard

Philosophy: **Math is hard**, until **existence precedes essence** (existentialism)

- Teach STEEM to **master Math** by ~~applying~~ rooting it in existence.

Let's master Math through its original root, Economics

Let's respect that **Math is a natural science about Many**

How do humans master Many?
Ask a 3year old: how old next time?

The answer is 4, showing 4 fingers I I I I

But, reacting strongly to 4 fingers held together 2 by 2:

“That is not four, that is two twos” II II

<u>Observation 1</u>: Inside, children see what **exists** outside, bundles of **2s**, *in space*; and 2 of them, *in time*. So children use bundle-numbers with units.

<u>Observation 2</u>: The child uses a **full number-language sentence** as in the word-language with a SUBJECT, a VERB, and a PREDICATE:

”That is two twos”, shortened to ”T = 2 **2s**”.

Economics: Sharing what we each produce

To survive, humans must labor, work, and act (Arendt)

So, the human condition is: **eat**, **produce**, and **create**

Production = **productivity** * persons * hours

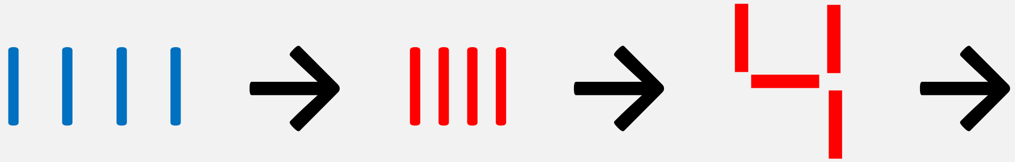
Productivity = what a person produces per hour

Productivity increases with new technology (numbers, calculators, etc.)

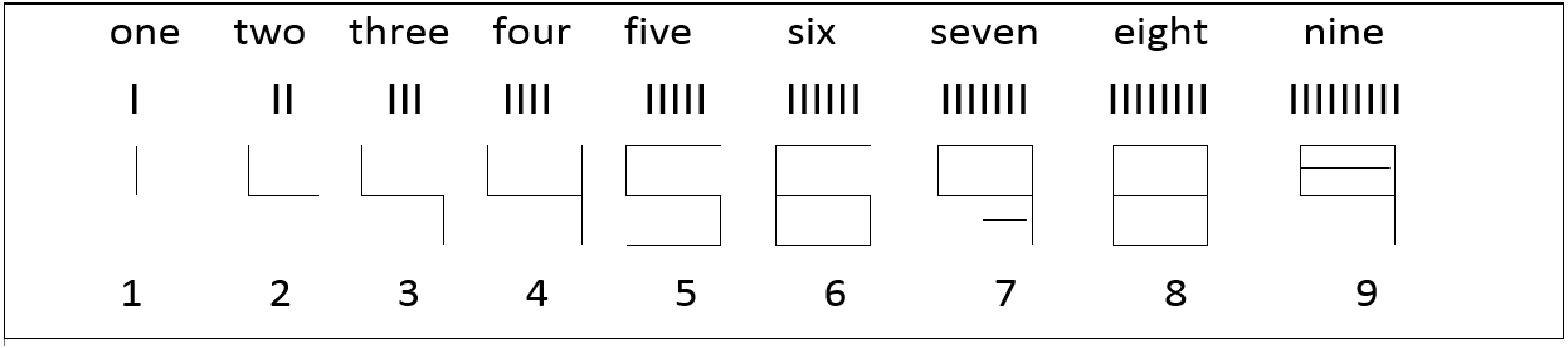
Productivity increases when we produce what **we master best**

But at what **exchange per-number**?

Productivity step 1:
Creating icons:



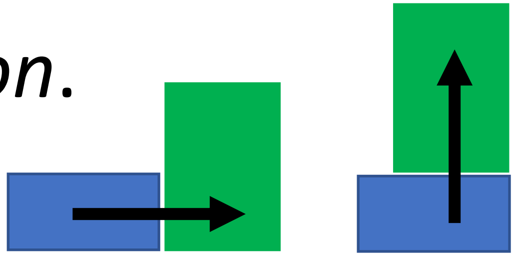
Uniting many 1s into 1 many-icon.
Uniting four ones into one fours creates a 4-icon with four sticks.
An icon contains as many sticks as it represents, if written less sloppy.
Once created, icons become **UNITS** when counting in bundles, as kids do.

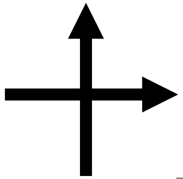


Productivity step 2:
Predict counting with icon-operations

- From 9 **PUSH** away **4s** we write 9/4
  iconizing a broom, called *division*.
- 2 times **LIFT** the **4s** to a stack we write 2x4
  iconizing a lift called *multiplication*.
- "From 9 **PULL** away 2 **4s** to find un-bundled" we write
  9 – 2x4   iconizing a rope, called *subtraction*.

- **UNITE** next-to or on-top we write A+C
  iconizing the two directions, called *addition.*



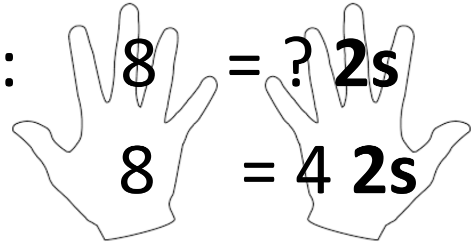


Productivity step 3: Shifting units creates a Recount-formula

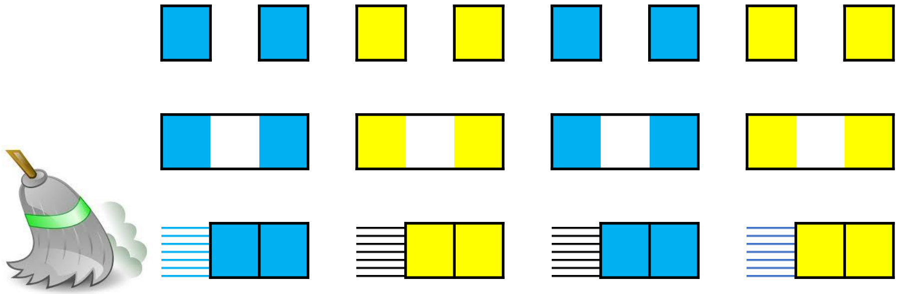
8 = (8/2)x2
T = (*T*/*B*)x*B*

Shift unit from **1s** to **2s**: 8 = ? **2s**

Bundle-counting: 8 = 4 **2s**



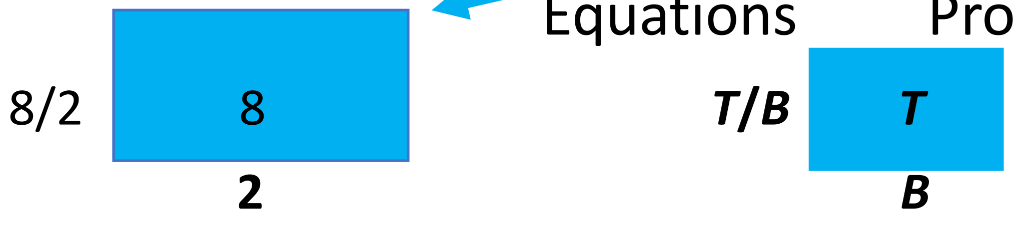
Predict by a calculation: 8/2 = 4



Recount result: 8 = (8/2)x2

Recount-Formula: ***T* = (*T*/*B*)x*B*** *"From **T**, **T/B** times, **B** is pushed away"*

Equations Proportionality



Shifting unit	$y = k * x$
Linearity	$\Delta y = (\Delta y/\Delta x) * \Delta x = m * \Delta x$
Local linearity	dy = (**dy/dx**) * dx = ***y'*** * dx
Trigonometry	*h* = (***h*/*b***) * *b* = **tan*A*** * *b*
Trade	$ = (**$/kg**) * kg = **price** * kg
STEM	meter = (**meter/sec**) * sec = **speed** * sec

Observation: Recounting in Bundle-numbers contains core mathematics & STEM

The Recount-formula solves equations

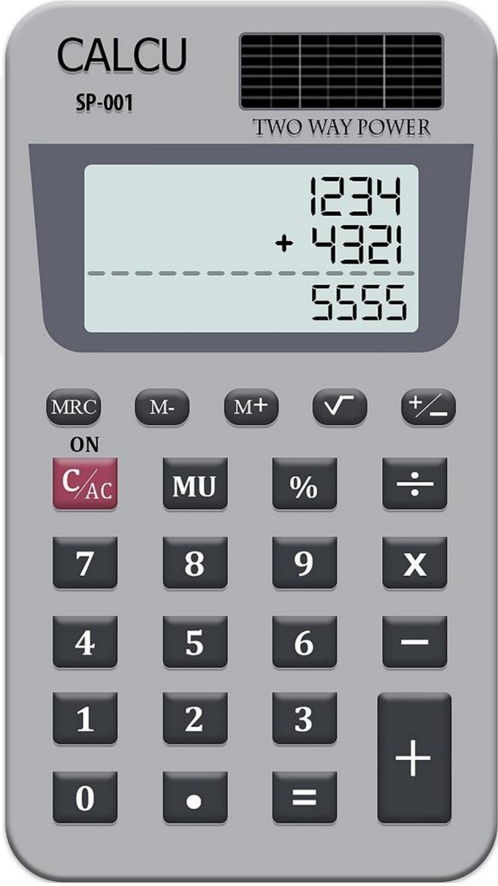
Asking "How many 2s in 8" gives the equation "u*2 = 8"
that is solved by recounting 8 in 2s:

u x 2 = 8 = (8/2) x 2
u = 8/2

Solve equations: MOVE to OPPOSITE side with OPPOSITE calculation sign

$u + 2 = 8$	u x $2 = 8$	u^$8 = 2$	2^$u = 8$
$u = 8 - 2$	$u = 8/2$	$u = \sqrt[8]{2}$	$u = \log_2 8$

The Math-core is seen on a calculator



Numbers: rooted in degrees of many; and bundle-counted in tens

Operations: predict counting results

- 8/2 = 4: **from 8 push away 2** can be done **4** times
- 4x2 = 8: **4 times lift 2s** totals **8**
- 9-4x2 = 1: **from 9 pull away 4 2s** leaves **1**
- 1 **2s** + 2 **3s** = 1**B**3 **5s** or 2**B**2 **3s**: stacks may **unite next-to** and **on-top**
- 3^2 = 9: a **bundle-of-bundles** of 3s totals 9
- $\sqrt{16}$ = 4: a 1.6-by-10 box may be **reshaped** into a 4-by-4 square

How Many did I produce?
Totals become stacks when **B**undle-counted

Twelve: I I I I I I I I I I I I

Viking language: twe-lve = ”twe left”, twen-ty = ”twen-ti”)

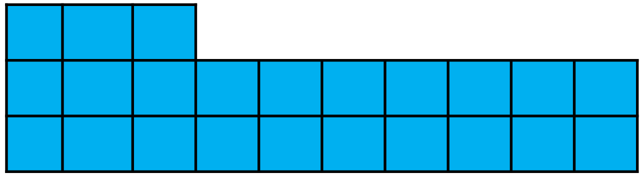
- Fives (hands): T = 2**Bundle**2 = 1**B**7 (*over-load*)= 3**B**-3 (*under-load*)
- Scores: T = 1**B**-8 = ½**B**2
- Dozens: T = 1**B**0 = ½**B**6
- Tens: T = 1**B**2 = 1.2 **B** = 2**B**-8 = ½**B**7 T = 345 = 3**BB**4**B**5 = 2**BB**14**B**5
- Sevens (weeks): T = 1**B**5 = 2**B**-2
- Threes: T = 4**B**0 = 5**B**-3 = 3**B**3 But 3**B** = 1**BB**, so T = 1**BB**1**B**0 = 1**BB**0**B**3
- Twos (pairs): T = 6**B**0 But 2**B** = 1**BB**, and 2**BB** = 1**BBB**, so T = 1**BBB**1**BB**

Counting in tens, how to see the unbundled?

Counting in tens, a Total of 2 **tens** & 3 can be described as T = 23, if

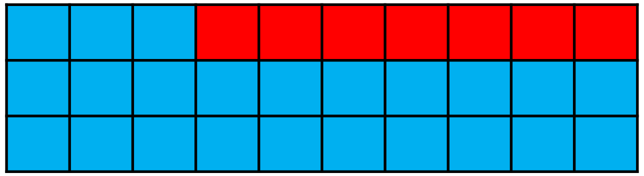
- leaving out the unit and the decimal point,
- teaching a place-value system, silencing that 100 is 1 bundle-bundle

  \- or as:



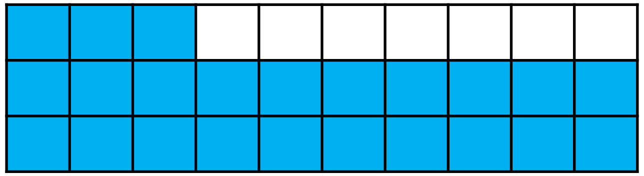
T = 2.3 **tens**

T = 2B3 **tens**



T = 3.-7 **tens**

T = 3B-7 **tens**



T = 2 3/10 **tens**

T = 2 3/10 **B** **tens**

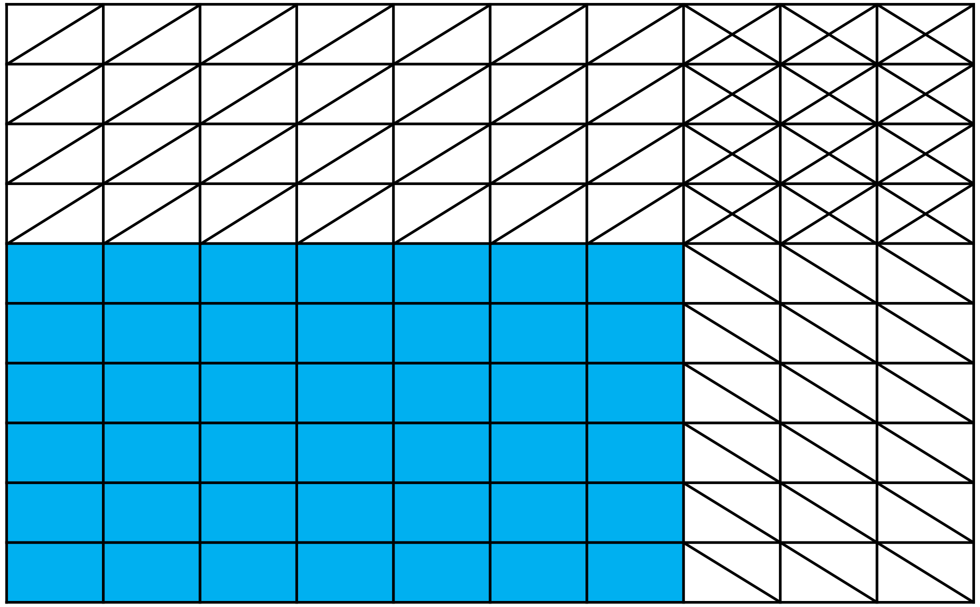
Flexible **B**undle-numbers ease calculations.
Counting in tens, T = 78 = 7**B**8 = 6**B**18 = 8**B**-2

Overload	Underload	Overload	Overload
6 5 + 2 7	6 5 - 2 7	7 x 48	336 /7
6**B**5 + 2**B**7	6**B**5 - 2**B**7	7 x 4**B**8	33**B**6 /7
8**B**12	4**B**-2	28**B**56	28**B**56 /7
9**B**2	3**B**8	33**B**6	4**B**8
92	38	336	48

No need to carry
No place-values

Recounting in tens: Multiplication tables

6 7s = ? tens



This roots the algebraic formula

(a – b) x (c – d) = a x c – a x d – b x c + b x d

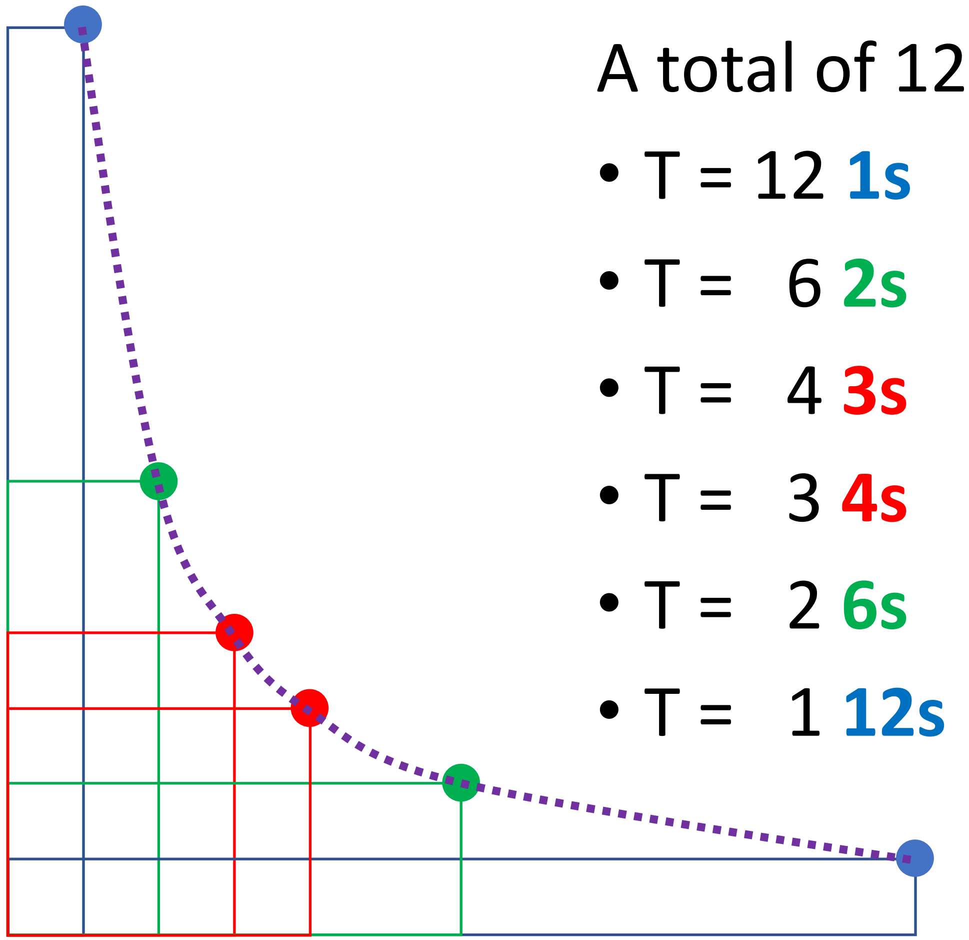
On a pegboard, 2 rubber bands form a ten-by-ten square,
and 2 bands form a 6-by-7 block.
With less-numbers, 6 **7s** is ten **tens**,
less 4 **tens**, less 3 **tens**,
plus 3 **4s** that is removed twice:

T = 6 **7s** = 6 x 7 = (**B**-4) x (**B**-3)
= 10**B** – 4**B** – 3**B** + 4 **3s** removed twice
= 3**B**12 = 4**B**2 = 42

Interesting: negative x negative = positive!

*6 x 7 gives the **less-numbers** 4 & 3. So from 100, we subtract their sum, and add their product.*

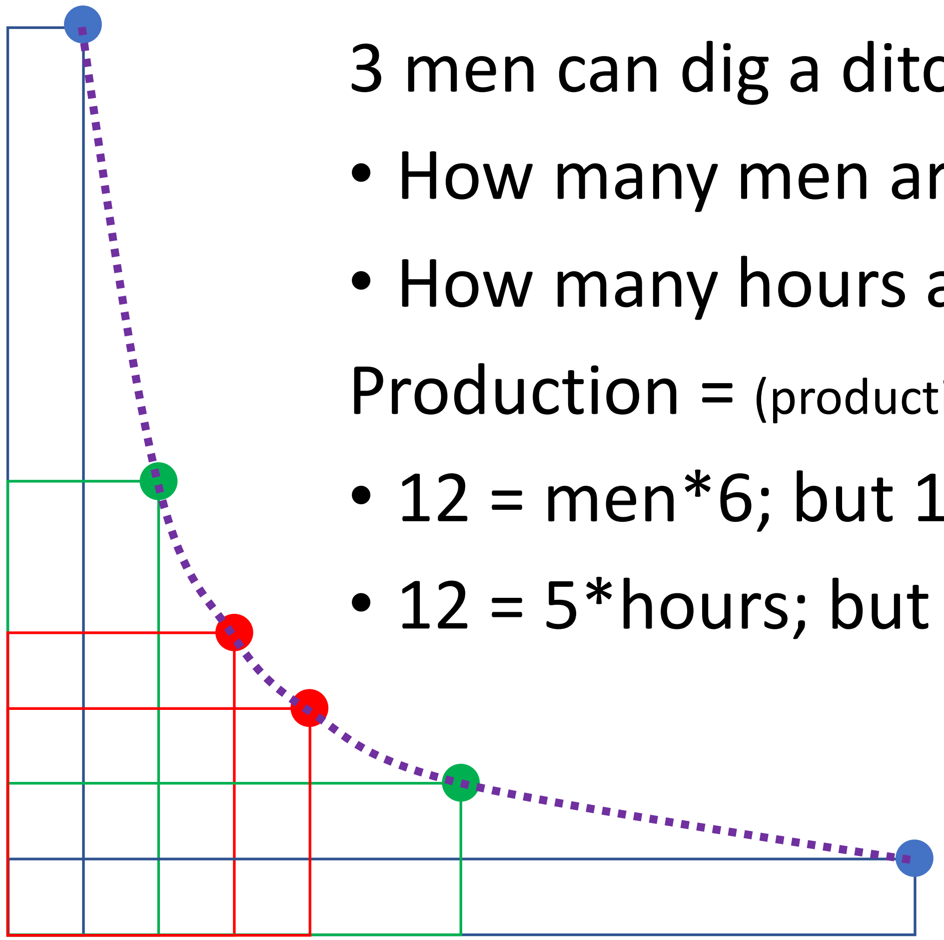
Internal exchange in other units



A total of 12 may be recounted in other units

- T = 12 **1s**
- T = 6 **2s**
- T = 4 **3s**
- T = 3 **4s**
- T = 2 **6s**
- T = 1 **12s**

Working together



3 men can dig a ditch in 4 hours.

- How many men are needed for 6 hours?
- How many hours are needed with 5 men?

Production = (productivity)*men*hours = 3*4 = 12 man-hours

- 12 = men*6; but 12 = (12/6)*6, so men = 12/6 = 2
- 12 = 5*hours; but 12 = (12/5)*5, so hours = 12/5 = 2.4

External exchange with a per-number

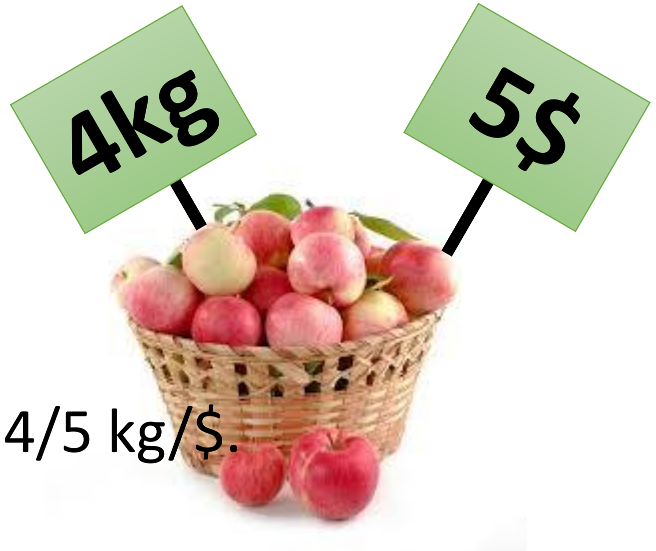
I have 8 3s, how many 4s can I buy?

- **Exchange per-number: 4 3s per 3 4s**
- **So I count my 3s in 4s**
- **T = 8 3s = (8/4)*4 3s = (8/4)*3 4s = 6 4s**

I have 8 apples, how many pears can I buy?

- **Exchange per-number: 4 apples per 3 pears**
- **So I count my apples in 4s**
- **T = 8 apples = (8/4)*4 apples = (8/4)*3 pears = 6 pears**

ReCounting in two units creates PerNumbers & Proportionality



ReCounting in kg & $, we get a PerNumber 4kg **per** 5$ = 4kg/5$ = 4/5 kg/$.

With like units, per-numbers become fractions: 4$/5$ = 4/5, and 4$/100$ = 4/100 = 4%.

With 4kg linked to 5$, we simply recount in the per-number.

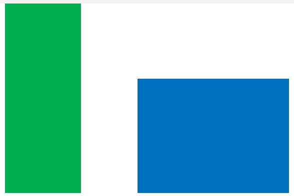
(Or we recount the units directly. Or we equate the per-numbers. Or we use the before 1900 'Rule of 3' (Regula de Tri) alternating the units, and, from behind, first multiply, then divide.)

Questions:

12kg = ?$	**20$ = ?kg**
12kg = (12/4) x 4kg = (12/4) x 5$ = 15$	20$ = (20/5) x 5$ = (20/5) x 4kg = 16kg
$ = ($/kg) x kg = 5/4 x 12 = 15	kg = (kg/$) x $ = 4/5 x 20 = 16
u/12 = 5/4, so *u* = 5/4 x 12 = 15	*u*/20 = 4/5, so *u* = 4/5 x 20 = 16
If 4kg is 5$, then 12kg is ?$; answer: 12x5/4 = 15	If 5$ is 4kg, then 20$ is ?kg; answer: 20x4/5 = 16

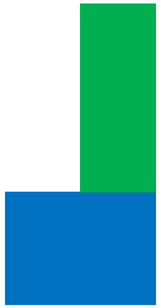
Taking stock roots proportionality & calculus

With the two stacks, 5 2s and 3 **4s**,
what is the total stock?



<u>Add on-top</u>

Counting in **4s**, we shift units by recounting the 5 2s in **4s**
Shifting unit = **proportionality, linearity**

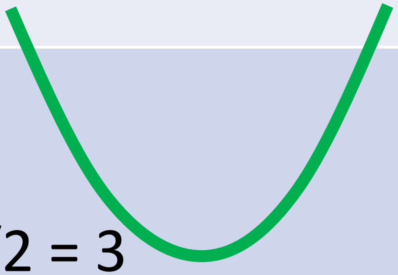


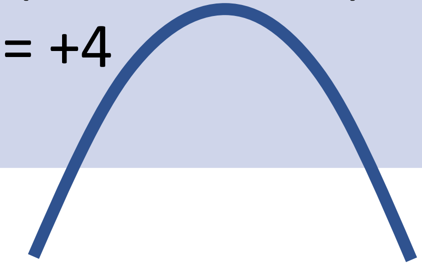
<u>Add next-to</u>

Counted in **6s**, we add by areas, called integral **calculus**,
becoming differential calculus when reversing the question:
5 **2s** + ? **4s** = 3 **6s**

Counting roots linear and quadratic formulas

Linear	Quadratic
T = 3**B**1 = 3***B** + 1 y = **3*x + 1**	T = 3**BB**2**B**-1 = 3***B**^2 + 2***B** – 1 y = 3*x^2 + 2*x – 1
Wher does a quadratic formula turn? T = x^2 + b*x + c T = x^2 + 2*b/2*x + c T = x^2 + 2*b/2*x + (b/2)^2 + c - (b/2)^2 T = (x + b/2)^2 - D = -D for x = -b/2 T turns where ***x* = -*b*/2**	Examples • T = **x^2 - 6*x + 8** T turns where x = 6/2 = 3 • T = **-x^2 + 8*x + 5** = -(x^2 - 8*x - 5) T turns where x = +8/2 = +4



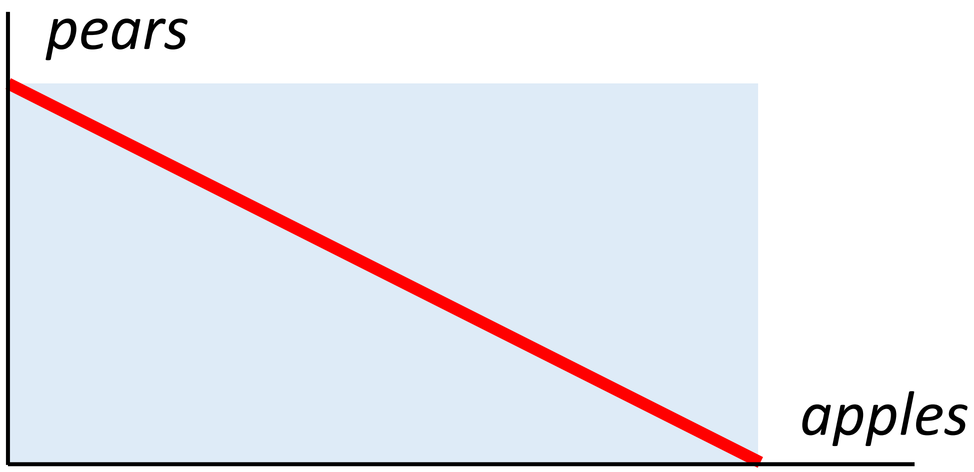


Investing with budget-lines

I invest 100$ in apples at 2$/kg, and in pears at 4$/kg

- I buy x kg apples. It leaves 100 - 2*x $ to buy pears:
- I buy (100 - 2*x)/4 = 25 - 1/2*x kg pears

So, the linear budget-line cuts the axes in 25kg pears and 50kg apples.



Market price, where supply & demand meet

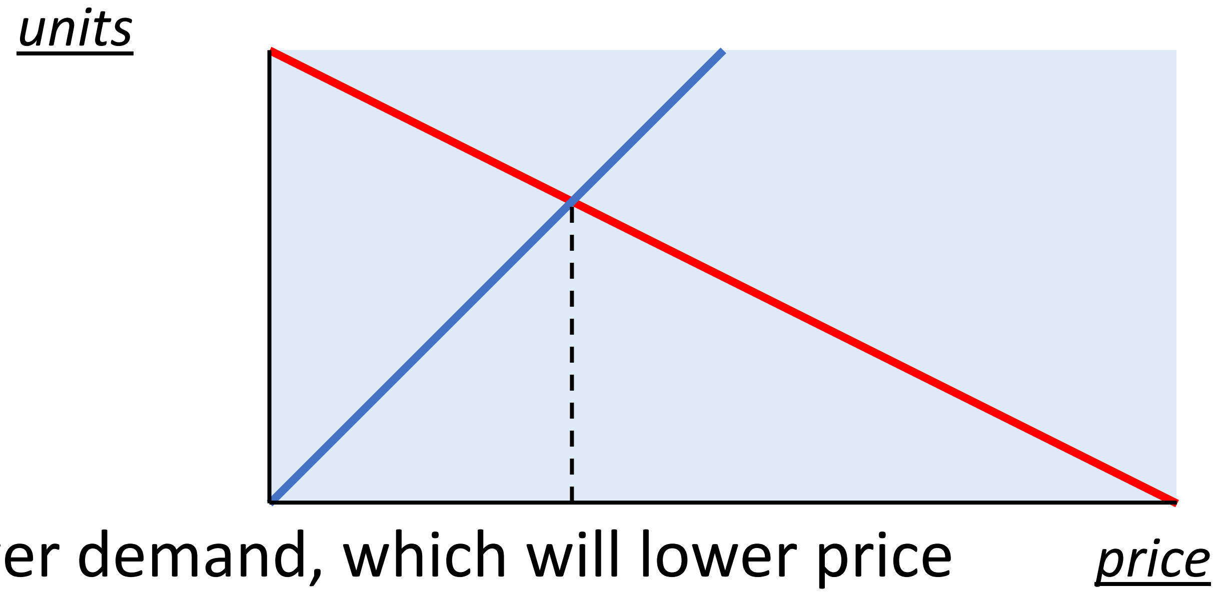
Rising the unit-price *p* will rise **supply** and lower **demand**

Supply = 0.2**p*, and Demand = 6 - 0.1**p*

Met at : Supply = Demand

- 0.2**p* = 6 - 0.1**p*
- 0.3**p* = 6 = (6/0.3)*0.3
- *p* = 6/0.3

So, they meeet at the unit-price 20



This price is stable:

Rising price will rise **supply**, and lower demand, which will lower price

Lowering price will lower **supply**, and rise demand, which will rise price

Saving money at home

Bo has 10$, and saves 3$/day.

So after *n* days, **T = 10 + 3*n**

- <u>How much after 7 days ?</u>

T = 10 + 3*7 = 31

- <u>When 100$?</u>

100 = 10 + 3*n,	but, 100 = (100 – 10) + 10, so
3*n = 100 – 10 = 90	but, 90 = (90/3)*3, so
n = 90/3 = 30	

Comparing savings

Al saves 5$/day. Starting 4 days earlier, Bo saves 3$/day.

After *n* days:

- Al. T = $5*n$
- Bo. T = $3*n + 3*4 = 3*n + 12$,

The same:

$3*n + 12 = 5*n$	but, $5*n = (5*n - 3*n) + 3*n = 2*n + 3*n$, so
$2*n = 12$	but, $12 = (12/2)*2$, so
$n = 12/2 = 6$	

Saving money in a bank

200$ + 12% = ?   Oops, unlike units, so we exchange 200$ = 100%

100% + 12% = 112%, changed back to $:

T = 112% = (112/100)*100% = (112/100)*200$ = 1.12 * 200$

- So, to add 12% means to multiply with 112%, or 1.12.

After *n* years, we have T = 200$*112%^*n*, or **T = 200 * 1.12^*n***

- After 2 years: *R* = (1+ 12%)^2 = 1.12^2 = 1.254,
  so 12% 2 times = 24% + 1.4% extra interest (compound interest)
- Doubling, 1.12^*n* =2, is predicted by the **factor-counting logarithm**
  n = $\log_{1.12}(2)$ = 6.1
- Asking which percent gives a 10year doubling leads to the equation
  (1+x)^10 = 2 predicted by the **factor-finding root** 1+x = $\sqrt[10]{2}$ = 1.072, so x = 7.2%

Instalment plan:
a race between a debt and a saving

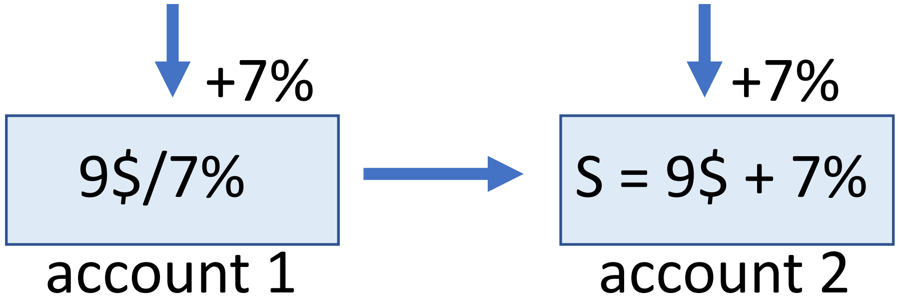
- I borrow 100\$ at 7 % per year. So, as interest I pay 7% of 100\$ = 7\$.

I pay back 7\$ + 2\$ = 9\$ per year.

This gives a saving *S* after *n* years:

$S = R*9/7\%$, with $1+R = (1+7\%)^n$, *or*

$\boldsymbol{S/9 = R/7\%}$



- Balance when Debt = Saving:

$100*1.07^n = (1.07^n - 1)*9/7\%$

Reducing to $1.07^n = 9/2$ (the 'speed-factor')

Solved by $n = 22.2$ years

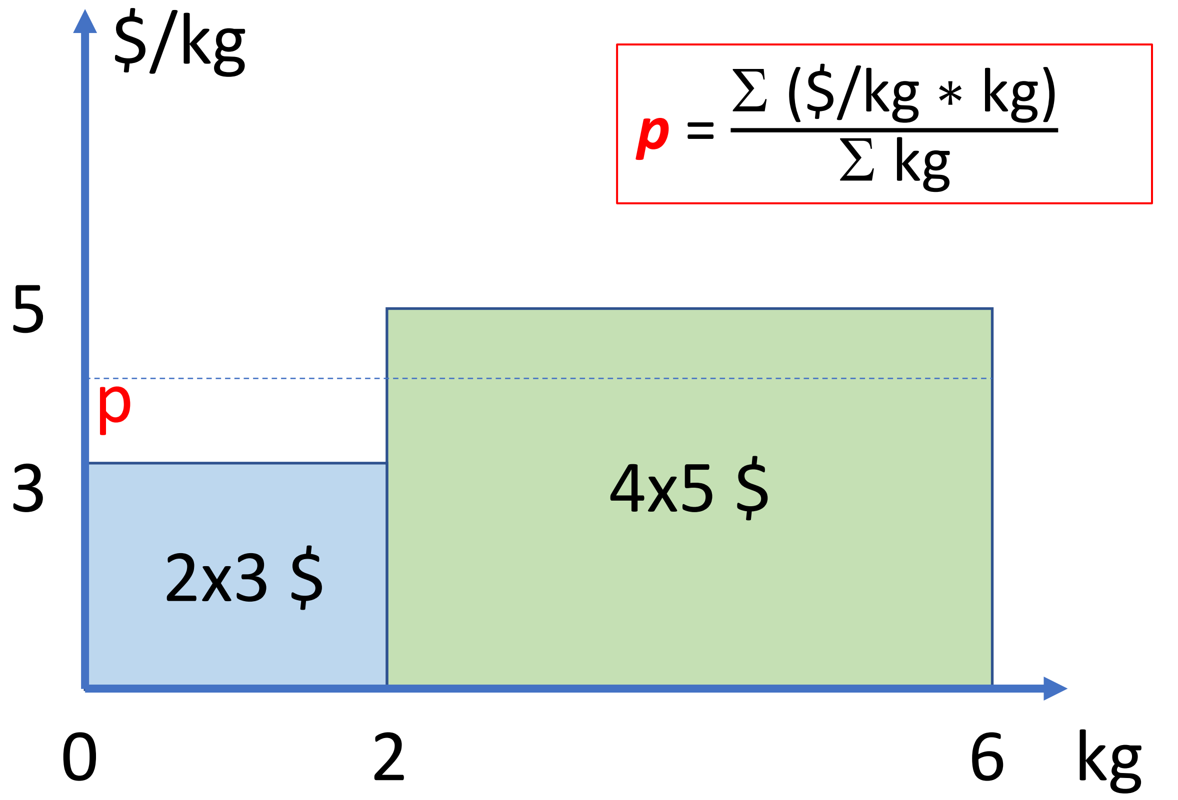
Per-numbers add as areas (integral calculus)
"2kg at **3$/kg** + 4kg at **5$/kg** = 6kg at **?** **$/kg**?"

Here, the per-number curve is **piecewise constant**

Σ (*p**Δ*x*) becomes $\int$*p**dx, if it is **locally constant** , interchanging *epsilon* and *delta*

2 kg	at	**3 $/kg**
+ 4 kg	at	**5 $/kg**
(2+4) kg	at	***p*** **$/kg**

- Unit-numbers add directly.
- Per-numbers must be multiplied to unit-numbers, thus adding as **areas** under the per-number curve.
- Here, multiplication before addition
- So, per-numbers and fractions are not numbers, but operators needing numbers to be numbers.



Per-numbers are found (differentiation)
"2kg at 3$/kg + 4kg at what = 6kg at 5$/kg?"

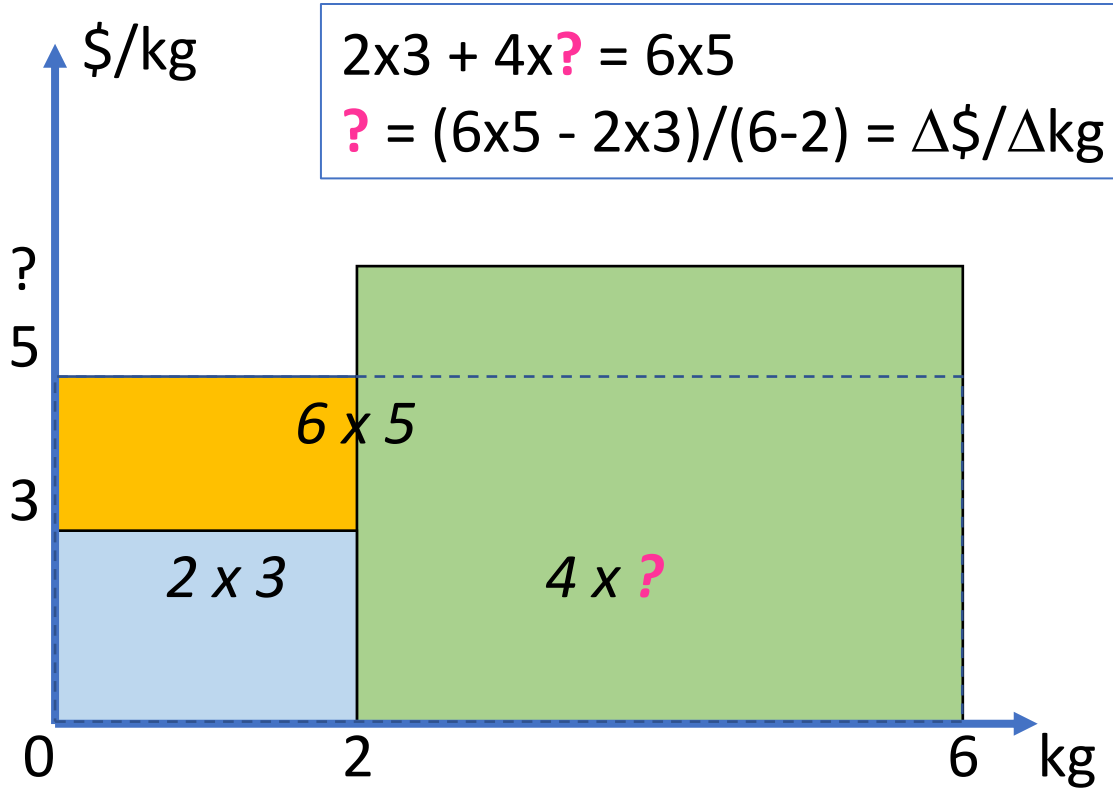
2 kg	at	3 $/kg	
+ 4 kg	at	? $/kg	
6 kg	at	5 $/kg	

Outside, we remove the initial 2x3 block and recount the rest in 4s. Geometrically, reversed per-number addition means subtracting areas to be reshaped, called differential calculus.

Inside, the recount-formula algebraically predicts the result. Here subtraction (giving a change, Δ) comes before division.

2x3 + 4x? = 6x5

? = (6x5 - 2x3)/(6-2) = Δ\$/Δkg



Four ways to unite and split a Total

A number-formula T = 345 = 3**BB**4**B**5 = 3*__B__^2 + 4*__B__ + 5 (a polynomial) shows the 4 ways to unite: +, *, ^, next-to block-addition (integration). <u>Addition</u> and <u>multiplication</u> unite changing and constant unit-numbers. <u>Integration</u> and <u>power</u> unite changing and constant per-numbers. We might call this beautiful simplicity the 'Algebra Square' since in Arabic, algebra means to reunite. ● The 4 uniting operations each has a reverse splitting operation: Addition has <u>subtraction</u> (–), and multiplication has <u>division</u> (/). Power has factor-finding (<u>root</u>, √) and factor-counting (<u>logarithm</u>, log). Integration has per-number finding (<u>differentiation</u> dT/dn = T'). Reversing operations is solving equations, done by moving to **opposite side** with **opposite sign**.

Operations unite / split into	**Changing**	**Constant**
Unit-numbers *m, s, $, kg*	***T = a + n*** *T – a = n*	***T = a * n*** *T/n = a*
Per-numbers *m/s, $/kg, m/(100m) = %*	***T = ∫ a dn*** *dT/dn = a*	***T = a^n*** $\log_a T = n$, $\sqrt[n]{T} = a$

Consumer and product behavior: Never trust totals & averages, always split 'em

Sale: 5 apples to 4 girls & 1 boy; 5 pears to 2 girls & 3 boys. Do girls & boys buy similar? Do apples & pears sell similar? We use 3 cross-tables: 1 with unit-, 2 with per-numbers.

In average, apples & pears sell like; and 60% go to girls.

Totals	**Apples**	**Pears**	**Total**
Girls	4	2	6
Boys	1	3	4
Total #	5	5	**10**

By product	**Apples**	**Pears**	**Total%**	Total #
Girls	4/6 = 67%	33%	100%	**6**
Boys	25%	75%	100%	**4**

Girls are 3 times more likely to buy apples than pears.
Boys are 3 times more likely to buy pears than apples.

By buyer	**Apples**	**Pears**
Girls	4/5 = 80%	40%
Boys	20%	60%
Total %	100%	100%
Total #	**5**	**5**

Never go between per-numbers
Always go via the unit-numbers

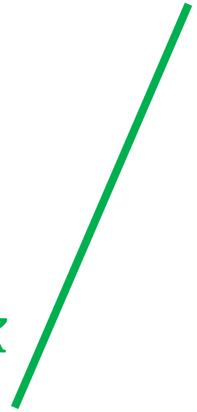
Apples are sold to 4 times as many girls than boys.
Pears are sold to 50% more boys than girls.

Quadratics have constant change-of-change

x ^2 gives the numbers 0, 1, 4, 9, …

The numbers change with +1, +3, +5, …

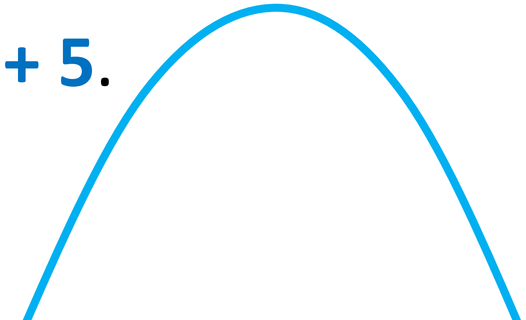
The changes change with +2



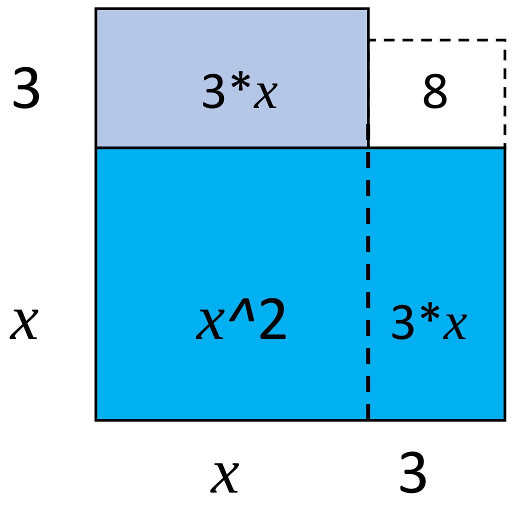
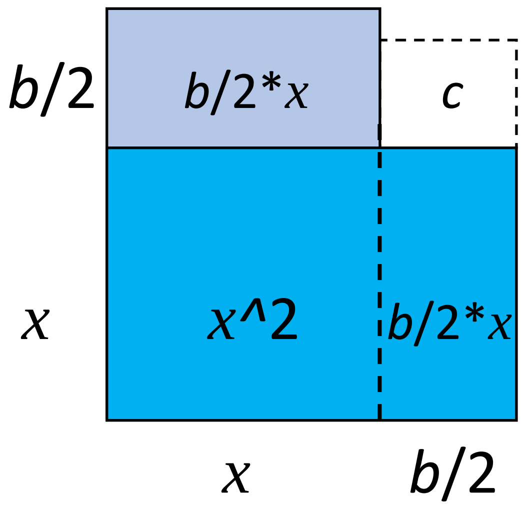
- T may increase daily with a number that is **constant** 7, **T = 5 + 7*x**
- T may increase daily with a number that is **constantly decreasing**

as 7, 5, 3, ... Here the change-of-change is constant, -2

Checking, gives the formula **T = - x^2 + 8*x + 5**.



Outside geometry & inside algebra, should always go hand in hand. Quadratic equations solved with 2 cards

$x^2 + 6*x + 8 = 0$	$x^2 + b*x + c = 0$
	
$(x+3)^2 = x^2 + 6*x + 8 + 1$ $(x+3)^2 = 0 + 1$ $x+3 = \pm\sqrt{1}$ $x = -3 \pm 1$ $\quad$ $\mathbf{x = -4}$ & $\mathbf{x = -2}$	$(x+b/2)^2 = x^2 + b*x + c + [(b/2)^2 - c]$ $(x+b/2)^2 = 0 + D$ $x+b/2 = \pm\sqrt{D}$ $x = -b/2 \pm \sqrt{D}$, $\mathbf{x = -b/2 \pm \sqrt{(b/2)^2 - c}}$

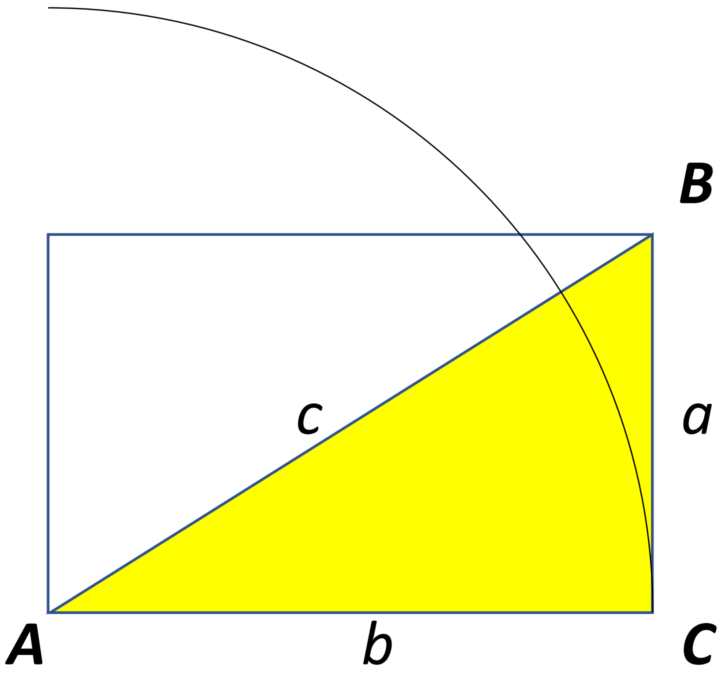
Recounting sides in a box gives trigonometry

In Greek, geometry means to earth-measure. The earth may be divided in triangles; that may be divided in right triangles; that may be seen as a box cut by its diagonal thus having three sides: the base *b*, the altitude *a*, and the cut *c*, connected with the angle *A* by per-number formulas recounting the sides pairwise.

$a = (a/c) \times c = \sin\boldsymbol{A} \times c$

$a = (a/b) \times b = \tan\boldsymbol{A} \times b$

$b = (b/c) \times c = \cos\boldsymbol{A} \times c$



tan***A*** = *a*/*b* = Δ*y*/Δ*x* = gradient

Circle: circum./diam. is π **= *n**tan(180/*n*)** for *n* large

Conclusion

- Yes, core mathematics may be learned through its historic root, economics, describing how humans share what they produce
- Asking "How many did I produce?" roots counting, predicted by division iconizing a broom pushing away bundles, to be stacked by a multiplication-lift, to be pulled away by a subtraction-rope to look for unbundled singles, to be added on-top or next-to, thus rooting decimal and negative numbers
- Recounting in a new unit creates a recount formula, used to solve equations, and to change units as in most STEM formulas
- Uniting stacks on-top or next-to roots proportionality or calculus
- So why make mathematics hard when it may also be easy & meaningful?

Theoretical background

Tarp, A. (2018). Mastering Many by counting and recounting before adding on-top and next-to. *Journal of Math Education, March 2018, 11*(1), 103-117.

Tarp, A. (2020). De-modeling numbers, operations and equations: from inside-inside to outside-inside understanding. *Ho Chi Minh City University of Education Journal of Science 17*(3), 453-466.