



Difference-Research

Powering PISA Performance:

Count and ReCount before you Add

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Teaches Teachers to Teach MatheMatics as ManyMath, Tales of Many
a Heidegger-inspired VIRUS-Academy: To learn, ask the subject, not the instructor

Full 31 page article: <http://mathecademy.net/difference-research/>

Poor Pisa Performance in Scandinavia

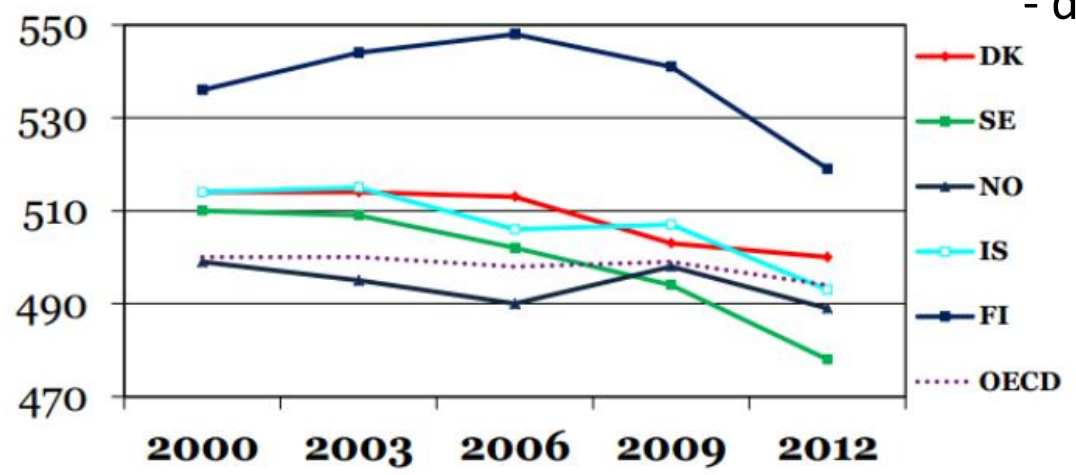


Improving Schools in Sweden:
An OECD Perspective



www.uvm.dk/~media/UVM/Filer/Udd/Folke/PDF13/Dec/131203%20PISA%20Resultatnotat.pdf

Figur 2. Udvikling i matematikresultaterne i nordiske lande (2000-2012).



Ser man bort fra Finland (519 point), er Danmark det eneste af de nordiske lande, som er placeret i gruppen, der ligger signifikant over det internationale gennemsnit. Eleverne i Island (493 point) og i Norge (489 point) præsterer omkring gennemsnittet, mens den svenske score (478 point) er signifikant lavere end gennemsnittet. I tabel 1 nedenfor vises tallene bag figur 2.

Tabel 1. Gennemsnit for nordiske lande 2003-2012

	2003	2006	2009	2012	2012-2009	2012-2003
Finland	544	548	541	519	-22	-25
Danmark	514	513	503	500	-3	-14
Island	515	506	507	493	-14	-22
Norge	495	490	498	489	-9	-6
Sverige	509	502	494	478	-16	-31
OECD	500	498	499	494	-5	-6

All go down, Sweden especially
- despite increased research funding
*Can Difference-Research
make a Difference
by finding a Difference?*

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följ NCM & Nämrenen!

Kängurutävlingen-16
Välkommen att anmäla!

Beställningar: Böcker och tidskrifter

Matte talanger

Läs

Nämrenen

NOMAD

2
Tänka, resonera

Denna bok riktar sig till lärare som undervisar i matematik i förskoleklass men är också relevant för lärare i grundskolans låga årskurser

Utforska

Strävorna

NCM:s bibliotek

Matematikverkstad

Månadens

Mer om

- **Programmering i skolan?**
NCM:s seminarier
11 februari
Malin Christerson m fl »»»
- **Adventsproblemen**
fortfarande kvar
Problem & lösningar »»»
- **Nytt för förskolan**
Broar på Hästhoven »»»
Ljus-projekt »»»
- **Månadens problem**
Januariproblemen »»»
Novemberlösningarna »»»

Aktuellt

- Svarta hål och geometri gav Crafoordpriset (19/1)
- Forskningsseminarium i Matematikdidaktik (19/1)
- Japansk matte får barn att tänka (19/1)
- "Pseudoteorier jämföras med etablerad vetenskap" (13/1)
- Prisd forskare ersätter djurforsk med matematik (13/1)
- Ovetenskapliga idéer om hjärnrörelse sprids i skolor (12/1)
- "Bättre kunskaper med externa lärare" (12/1)

Different Differences



Background

- Poor PISA Performance, witnessing 50 years of low-performing Math Education Research

10. Different Education

- Classroom: Half-Year Self-Chosen Blocks versus Multi-Year Forced Lines

20. Different Mathematics

- BottomUp Many-based Math from Below, versus TopDown Set-based Math from Above

30. Different Research

- Ancient Sophism, Renaissance Natural Science, (Post)Modern Existentialism

40. Different Math Education, showing the Beauty of the Simplicity of Math


- To master Many, Count & recount before you Add, Add next-to & on-top, and forwards & backwards



Powering PISA Performance - in a Nutshell

The Greek Sophists: Beware of choice masked as nature.

A Number-Language Sentence (a Tale of Many): **the Total is five, T = 5**


T = 5 = 1Bundle3 2s = 2B1 2s = 3B-1 2s, or 1B2 3s = ...

The predicate can be different (choice with alternatives)

The subject cannot be different (nature without alternatives)

One Goal - many Means; Goal Displacement: When a Means becomes the Goal

Difference-Research, unmasking Means masked as Goals:

Use Full Sentences, if not, predicates become subjects and a means the goal

Education & Mathematics & Research



Education: a Social Institution

- In sociology, Bauman warns against 'the danger of so-called *goal displacement*. The survival of the organization, however useless it may have become in the light of its original end, becomes the purpose in its own right.'



Mathematics & Research produce truth claims.

But, are the truths just choices masked as truth, creating a 'truth-regime', we all have to serve?

- In philosophy, Sartre says: 'In existentialism, existence precedes essence.'
- In philosophy, Heidegger warns against true sentences with a subject & verb & predicate: 'Trust the subject; but doubt the predicate, it could be different.'
- In counter-philosophy, the Greek sophists said: 'Beware of choice masked as nature.'

Difference-Research asks 1 Question only: find a Difference that makes a Difference
- to unmask claimed goals, existence, subjects, nature as masked means, essence, predicates, choice.

Difference-Research, Main Finding: Math as Tales of Many shows the Simplicity of Math



Meeting Many we ask: 'How Many in Total'

- To answer, we math. *Oops, sorry, math is not an action word but a predicate.*
- Take II. To answer, we **Count & Add**. And report with Tales of Many (Number-Language sentences): $T = 2 \ 3s = 2*3$



Three ways to Count: BundleCount & ReCount & DoubleCount

- Bundle-Count gives units. Re-Count changes units. Double-count bridges units by per-numbers as 2\$/3kg

Recount to & from tens: Multiplication & Equations come before Addition

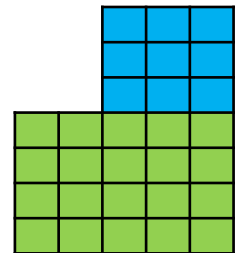
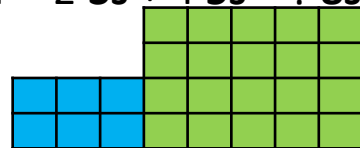
- To tens: $T = 5 \ 7s = ?$ Tens = $5*7 = 35 = 3.5$ tens . From tens: $T = ? \ 7s = u*7 = 42 = (42/7)*7 = 6 \ 7s$ (ReCount-Formula)

Counting gives changing or constant unit- or per-numbers, to be Added in 4 ways

- Addition & multiplication unites changing & constant unit-numbers.
- Integration & power unites changing & constant per-numbers.

Adding NextTo & OnTop roots Early Childhood Calculus & Proportionality

- Early Childhood Calculus: $T = 2 \ 3s + 4 \ 5s = ? \ 8s$. Early Childhood Proportionality: $T = 2 \ 3s + 4 \ 5s = ? \ 5s$



Difference-Research, Main Recommendation: Visible and Tangible BUNDLES in Tales of Many



To improve PISA Performance, the Outsider (Child, Migrant) must touch & see & write the BUNDLE and use full number-language sentences in Tales of Many. (Bundles = units)

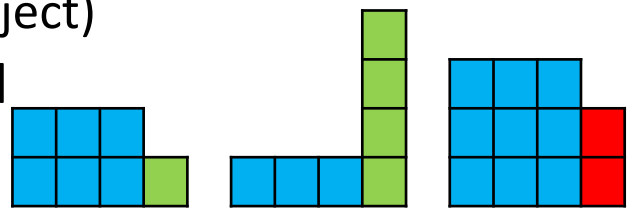
And must Count & Recount before Adding.

- Several counting sequences:

$T = \text{|||||} = 7 = B-3$ (BUNDLE less 3) = $\frac{1}{2}B\&2$ (The Total is the goal, the subject)

- Recount in the same unit, 3s, to create/remove over- or underload

$T = 7 = \text{|||||} = \text{||| |||} = 2B1$ or $T = \text{||| |||} = 1B4$ or $T = \text{||| ||| ||} = 3B-2$



Seeing $T = 47 = 4B7 = 3B17 = 5B-3$ makes a difference in multiplication tables:

$T = 2*7 = 2*(\frac{1}{2}B\&2) = B\&4 = 14,$ or $T = 2*7 = 2*(B-3) = 20-6 = 14$

- A calculator predicts by the RecountFormula, where the operations ($/, *, -$) are icons for bundling & stacking & removing stacks to find unbundled: $T = 7 = (\frac{7}{3}) * 3 = 2B1\ 3s$

$\frac{7}{3}$	2.some
$7 - 2*3$	1



Difference-Research, Main Warning: The 3x3 Goal Displacements in Math Education

Primary	Numbers	Could: be icons & predicates in Tales of Many, $T = 2 \ 3s = 2*3$; show Bundles, $T = 47 = 4B7 = 3B17 = 5B-3$; $T = 456 = 4*BB + 5*B + 6*1$ Instead: are changed from predicates to subjects by silencing the real subject, the total. Place-values hide the bundle structure
	Operations	Could: be icons for the counting process as predicted by the RecountFormula $T = (T/B)*B$, from T pushing Bs away T/B times Instead: hide their icon-nature and their role in counting; are presented in the opposite order (+ - * /) of the natural order (/, *, -, +).
	Addition	Could: wait to after counting & recounting & double-counting have produced unit- and per-numbers; wait to after multiplication Instead: neglects counting & next-to addition; silences bundling & uses carry instead of overloads; assumes numbers are ten-based
Middle	Fractions	Could: be per-numbers coming from double-counting in the same unit; be added by areas (integration) Instead: are defined as rational numbers that can be added without units (mathe-matism, true inside, seldom outside classrooms)
	Equations	Could: be introduced in primary as recounting from ten-bundles to icon-bundles; and as reversed on-top and next-to addition Instead: Defined as equivalence relations in a set of number-names to be neutralized by inverse elements using abstract algebra
	Proportionality	Could: be introduced in primary as recounting in another unit when adding on-top; or by double-counting producing per-numbers Instead: defined as linear functions, or as multiplicative thinking supporting the claim that fractions and ratios are rational numbers
High	Trigonometry	Could: be introduced in primary as mutual recounting of the sides in a right-angled triangle, seen as a block halved by a diagonal Instead: is postponed till after geometry and coordinate geometry, thus splitting up geometry and algebra.
	Functions	Could: be introduced in primary as formulas, i.e. as the number-language's sentences, $T = 2*3$, with subject & verb & predicate Instead: are introduced as set-relations where first-component identity implies second-component identity
	Calculus	Could: be introduced in primary as next-to addition; and in middle & high as adding piecewise & locally-constant per-numbers Instead: differential calculus precedes integral calculus, presented as anti-differentiation

11. Different Educational Systems

EU: Line-organized & Office-directed Education



From secondary school, continental Europe uses **line-organized** education with forced classes and forced schedules making teenagers stay together in age groups even if girls are two years ahead in mental development.

The classroom belongs to the class. This forces teachers to change room and (in lower secondary school) to teach several subjects outside their training.

Tertiary education is also **line-organized** preparing for offices in the public or private sector. This makes it difficult to change line in the case of unemployment, and it forces the youth to stay in education until close to 30 making reproduction fall to 1.5 child/family, making the European population dying out very quickly by decreasing it to 25% in 100 years.

12. Different Educational Systems

US: Block-organized & Talent-directed Education



Alternatively, North America uses **block-organized** education saying to teenagers: “Welcome, inside you carry a **talent**! Together we will uncover and develop your personal talent through daily lessons in self-chosen half-year blocks, academical or practical, together with 1subject teachers. If successful the school will say ‘**good job**, you have a **talent**, you need some more’. If not, the school will say ‘**good try**, you have **courage** to try out the unknown, now try something new’”.

The classroom belongs to the teacher teaching one subject only.

Likewise, college is **block-organized** easy to supplement with additional blocks in the case of unemployment.

At the age of 25, most students have an education, a job and a family with three children, 1 for mother, 1 for father, and 1 for the state to secure reproduction.

20. Different Mathematics

The Beauty of the Simplicity of Mathematics



21. The Goal & Means of Mathematics Education

22. Totals as Blocks. Digits as Icons. Operations as BundleCounting Icons

23. ReCounting solves Equations

24. Multiplication tables Simplified by ReCounting

25. DoubleCounting in different & same units creates PerNumbers & Fractions

26. Once Counted, Totals can be Added. But counting and double-counting gives 4 number-types (constant & changing unit-numbers & per-numbers) to add in 4 ways

27. How Different is the Difference? Set-based versus Many-based Mathematics

21. Different Mathematics

The Goal and Means of Mathematics Education



The Set-based Tradition:

- Mathematics exists as a collection of well-proven statements about well-defined concepts, all derived from the mother concept SET
- Mathematics is surprisingly useful to modern society
- Consequently, mathematics must be taught and learned before it can be applied

The Many-based Difference:

- Many exists; to master Many we develop a number-language with Tales of Many, a 'ManyMath'.
- Many-math defines concepts from below as abstractions from external examples
- 'Meta-math' defines concepts from above as examples of internal abstractions
- Many-math adds with units, 'mathe-matism' adds without units ($3+4$ **IS** 7, no matter the units)

How do humans master Many?

Ask a 3year old: how old next time?

The answer is 4, showing 4 fingers



But, reacting strongly to 4 fingers held together 2 by 2:

“That is not four, that is two twos”












Observation 01: Inside, children see what **exists** outside, bundles of **2s**, *in space*; and 2 of them, *in time*. So, children use Bundle-numbers with **units**

Observation 02: The child uses a **full number-language sentence** as in the word-language with a SUBJECT, a VERB, and a PREDICATE:

“That is two twos”, shortened to “T = 2 **2s**”.

22a. Different Mathematics
 Digits as Icons. Totals as Blocks
 that are Bundle-Counted

one	two	three	four	five	six	seven	eight	nine
I	II	III	IIII	IIIII	IIIIII	IIIIIII	IIIIIIII	IIIIIIII
								
1	2	3	4	5	6	7	8	9

Icon-numbers. A folding ruler shows: digits are, not symbols as the alphabet, but sloppy writings of icons having in them as many sticks as they represent. Thus, there are four sticks in the 4-icon, etc.



Counting-sequences. A total of a dozen sticks counted in 5s gives different counting sequences:

'1, 2, 3, 4, Bundle, 1B1, ..., 2 Bundles, 2B1, 2B2', or

'01, 02, 03, 04, 10, 11, ..., 22' , or

'1, .2, .3, .4, 1., 1.1, ..., 2.2' , or

'1, 2, Bundle less 2, B-1, Bundle, B&1, B&2, 2B-2, 2B-1, 2Bundles, 2B&1, 2B&2.'

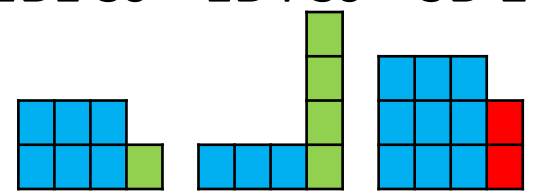
Bundle-Counting. With a cup for the bundles, a total can be 'Bundle-counted' in 3 ways,

Normal or with Overload or with Underload:

$$T = 7 = 2\mathbf{B}1\ 3s = 1\mathbf{B}4\ 3s = 3\mathbf{B}-2\ 3s$$

Or, counting in tens with inside bundles &

outside singles: $T = 37 = 3\mathbf{B}7\ \text{tens} = 2\mathbf{B}17\ \text{tens} = 4\mathbf{B}-3\ \text{tens}$



22b. Different Mathematics

Operations as BundleCounting Icons

$7/3$	2.some
$7 - 2*3$	1



We count by bundling and stacking: $T = | | | | | | | = \text{##} \text{##} | = \text{##} | = 2B1 \text{ 3s} = 2.1 \text{ 3s}$

Thus, to count 7 in **3s** we push away 3 many times, iconized by an uphill stroke showing the broom pushing away the 3s. With $7/3 = 2.\text{some}$, the calculator predicts that 3 can be pushed away 2 times.



To stack the 2 **3s** we use multiplication, iconizing a lift, $2x3$ or $2*3$, transforming the bundles into a stack



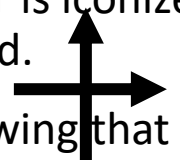
To look for unbundled singles, we pull away the stack of 2 **3s** iconized by a rope: $7 - 2*3 = 1$.



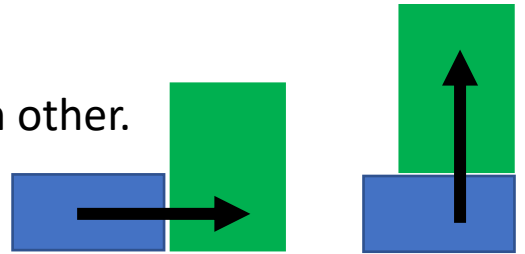
The prediction ' $T = 7 = 2 \text{ 3s} \ \& \ 1 = 2B1 \text{ 3s}$ ' provides the ReCount-formula:

$T = (T/B)*B$ saying 'from T, T/B times B can be pushed away' as e.g. $8 = (8/2)*2 = 4*2 = 4 \text{ 2s}$

To also bundle bundles, power is iconized as a cap, e.g. 5^2 , indicating the number of times bundles themselves have been bundled.



Finally, addition is a cross showing that blocks can be juxtaposed next-to or on-top of each other.



Counting thus provides the number-formula called a polynomial:

$$T = 456 = 4*\text{BundleBundle} + 5*\text{Bundle} + 6*1 = 4*B^2 + 5*B + 6*1$$

So counting creates 3 operations: to divide & to multiply & to subtract.

23. Different Mathematics

ReCounting solves Equations

$4*5/6$	3.some
$4*5 - 3*6$	2



Recounting in the same unit creates overloads & underloads

- $T = \text{IIIIII} = \text{III III I} = 2\text{B}1 \mathbf{3s} = 1\text{B}4 \mathbf{3s}$ (Overload III I I I I) = $3\text{B}-2 \mathbf{3s}$ (Underload III III III II)

ReCounting in different units means changing units (proportionality)

- $T = 4 \text{ 5s} = ? \mathbf{6s}$. Calculator predicts with ReCount-formula $T = (T/B)*B$, $T = 3\text{B}2 \mathbf{6s}$

ReCounting between icons & tens gives multiplication and equations:

ReCounting from icons to tens:

- $T = 5 \mathbf{7s} = ? \mathbf{tens} = 5*7 = 35 = 3.5 \mathbf{tens}$, predicted by multiplication

ReCounting from tens to icons:

- $T = ? \mathbf{7s} = u*7 = 42 = (42/7)*7 = 6 \mathbf{7s}$ with solution $u = 42/7 = 6$.

$u*7 = 42 = (42/7)*7$
$u = 42/7 = 6$

A multiplication equation solved by recounting, i.e. moving to opposite side with opposite sign

Science and STEM are full of multiplication formulas.

24. Different Mathematics

Multiplication Tables Simplified by ReCounting



Geometry: Multiplication means that, recounted in tens, a block increases its width and therefore decreases its height to keep the total unchanged.

Thus $T = 3 * 7$ means 3 **7s** that may be recounted in tens as $T = 2.1 \text{ tens} = 21$.

Algebra: The full ten-by-ten table can be reduced to a small 2-by-2 table containing doubling and tripling, using that 4 is doubling twice, 5 is $\frac{1}{2}$ Bundle, 6 is 5&1 or Bundle less 4, 7 is 5&2 or Bundle less 3, etc.

Beginning with doubling and halving visualized by snap-Cubes

- $T = 2 \text{ 6s} = 2 * 6 = 2 * (\frac{1}{2}B \& 1) = B \& 2 = 12$, or
- $T = 2 \text{ 6s} = 2 * 6 = 2 * (B - 4) = 20 - 8 = 12$.
 - $T = 5 \text{ 7s} = 5 * 7 = 5 * (B - 3) = 5B - 15 = 50 - 15 = 35$
 - $T = 8 \text{ 7s} = 8 * 7 = (B - 2) * (B - 3) = BB - 2B - 3B + 6 = 100 - 20 - 30 + 6 = 56$

25. Different Mathematics

DoubleCounting in 2 units creates PerNumbers (Proportionality)
 DoubleCounting in the same unit creates Fractions



Apples are double-counted in kg and in \$.

With **4kg = 5\$** we have $4\text{kg}/5\$ = 4/5 \text{ kg}/\$ =$ a per-number

$$4\$/100\$ = 4/100 = 4\%$$

Questions:

8kg = ?\$	20\$ = ?kg
$8\text{kg} = (8/4)*4\text{kg}$ $= (8/4)*5\$ = 10\$$	$20\$ = (20/5)*5\$$ $= (20/5)*4\text{kg} = 16\text{kg}$

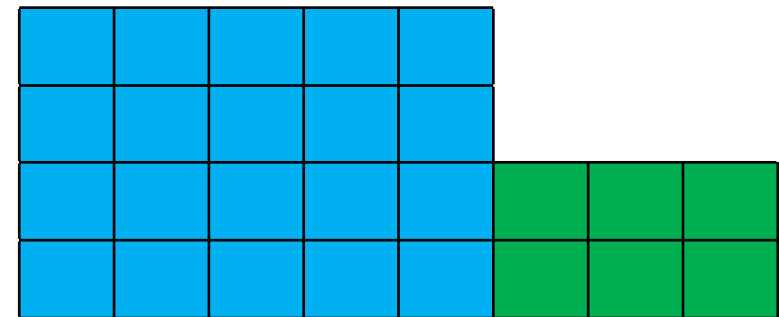
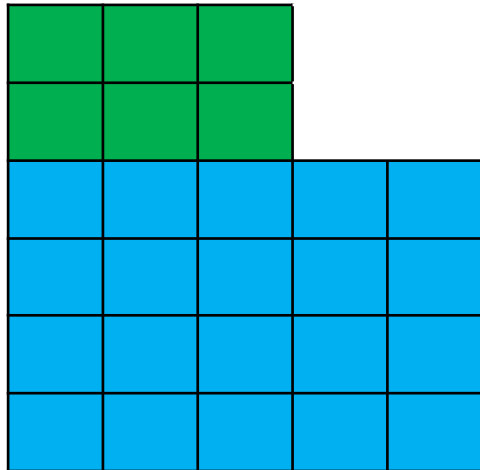
Answer: *To shift units, simply recount in the per-number*

26a. Different Mathematics

Once Counted & ReCounted, Totals can be Added



OnTop	NextTo
$2 \text{ } 3s + 4 \text{ } 5s = 1 \text{ } 10s + 4 \text{ } 5s$ $= 5 \text{ } 10s$	$2 \text{ } 3s + 4 \text{ } 5s = 3 \text{ } 10s + 2 \text{ } 10s$
<p>The units are changed to be the same. <i>Change unit = Proportionality</i></p>	<p>The areas are added. <i>Adding areas = Integral calculus</i></p>



26b. Different Mathematics

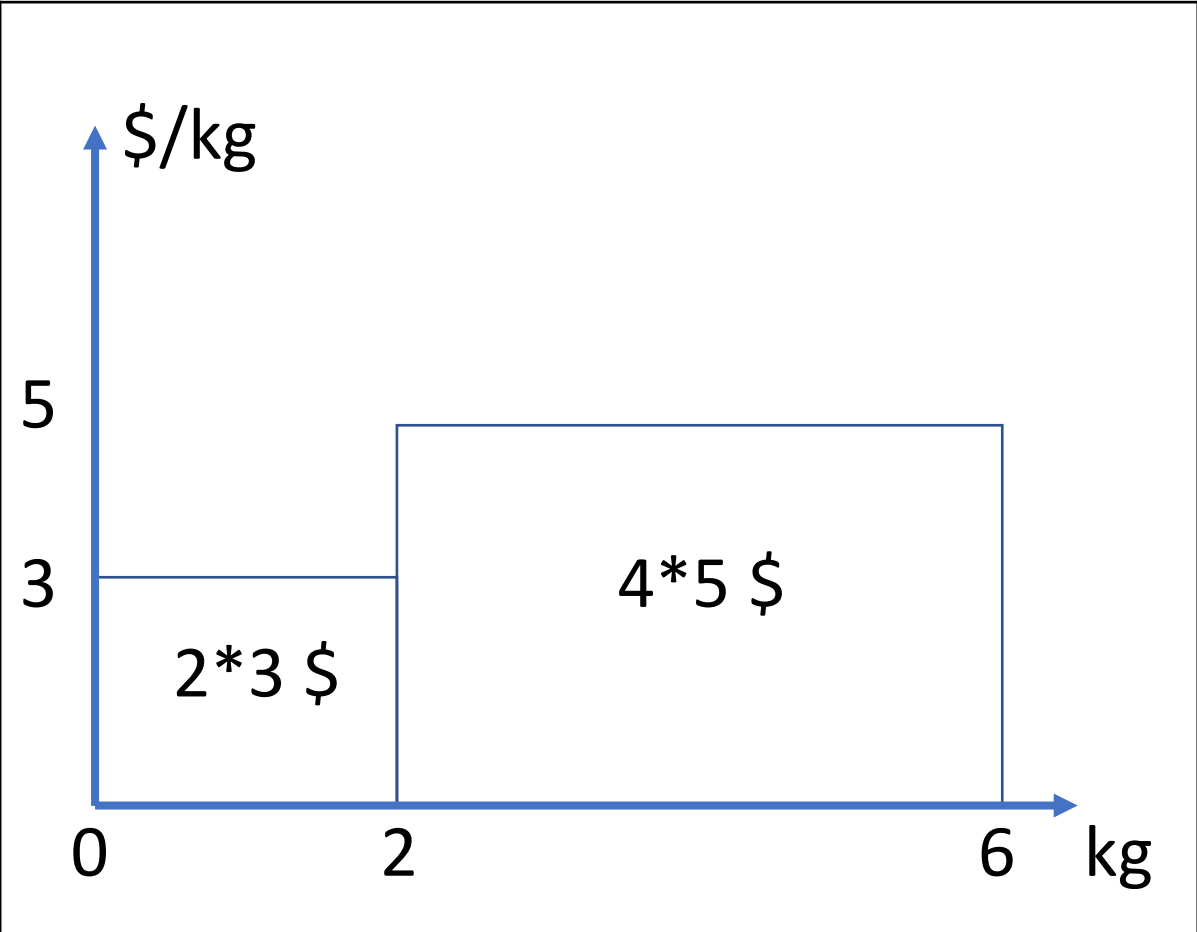
Adding PerNumbers as Areas (Integral Calculus)



2 kg at 3 \$/kg
 + 4 kg at 5 \$/kg

 (2+4)kg at ? \$/kg
 (2+4)kg at $(2*3 + 4*5)/(2+4)$ \$/kg

Unit-numbers add on-top.
 Per-numbers add next-to as **areas**
 under the per-number graph.



26c. Different Mathematics

With 2x2 different number-types we Add in 4 ways
 And solves equations by moving to opposite side with opp. sign



Counting produces changing or constant **unit-numbers** or **per-numbers**

- Addition & Multiplication unites changing & constant unit-numbers
 - Subtraction & division splits into changing & constant unit-numbers
- Integration & Power unites changing & constant per-numbers
 - Differentiation & root/logarithm splits into changing & constant per-numbers

Operations unite / <i>split into</i>	Changing	Constant
Unit-numbers <i>m, s, \$, kg</i>	$T = a + n$ $T - a = n$	$T = a * n$ $T/n = a$
Per-numbers <i>m/s, \$/kg, m/(100m) = %</i>	$T = \int a \, dn$ $dT/dn = a$	$T = a^n$ $\log_a T = n, \sqrt[n]{T} = a$



27. Different Mathematics

How Different is the Difference? Same Questions, but Different Answers

	SET-based Tradition	Many-based Difference
Digits	Symbols as letters	Icons with as many sticks as they represent
Numbers	Line numbers. Never units, but with place-values	Area numbers: unions of blocks of stacked singles, bundles, bundle-bundles etc. Always with units
Number-types	4 types: Natural, Integers, Rational, Real, no units	Positive and negative decimal numbers with units
Operations	Mapping from a set-product to the set	Counting-icons: /, *, -, + (push, lift, pull, and unite)
Order	Addition, subtraction, multiplication, division	The opposite
Fractions	Rational numbers, that add without units	Per-numbers, not numbers but operators needing a number to become a number, so added by integration
Equations	Statement about equivalent number-names	Recounting from tens to icons, reversing operations
Functions	Mappings between sets	Number-language sentences with a subject, a verb and a predicate
Proportionality	A linear function	A name for double-counting to different units
Calculus	Differential before integral (anti-differentiation)	Integration first to add locally constant per-numbers
Goal/Mean	Master Mathematics with few concrete examples	Master Many through 3 actions: count, recount, add

31. Different Research

Ancient Greece: Sophist vs. Philo-Sophists



Difference research began with the Greek controversy between two attitudes to knowledge, called 'sophy' in Greek. To avoid hidden patronization, the sophists warned: 'Know the difference between nature and choice to unmask choice masked as nature.'

To their counterpart, the philosophers, choice was an illusion since the physical was but examples of metaphysical forms only visible to people educated at the Plato academy.

The Christian church transformed the academies into monasteries but kept the idea of a metaphysical patronization by replacing the forms with a Lord using an unpredictable will to choose world behavior.

32. Different Research

Renaissance Natural Science



Background: Viking descendants in UK know how to sail, how to steal Spanish silver, how to follow the moon to go to India on open sea to buy silk and pepper:

How does the moon move?

Tradition: Between the stars. Newton: No, falling as the apple

Why do moons and apples fall?

Tradition : They follow a metaphysical unpredictable will. Newton: No, they follow a physical predictable will, by following formulas.

What is the effect of a will or force

Tradition : Aristotle: a force upholds order . Newton: No, a force changes order.

How to use formulas?

Tradition : Arabic algebra. Newton: No, different algebra about change, Calculus

33. Different Research

Enlightenment Century 1700-1800



Newton's physical will inspired the Enlightenment century (Locke) with its two republics

The US: Skepticism towards philosophy, US pragmatism, Symbolic Interactionism, Grounded Theory, Action Learning & Research

The French 5th : post-structuralism inspired by existentialism with Kierkegaard, Nietzsche, Heidegger, Sartre saying "In Existentialism, existence precedes essence." And reacting against

- Counter-enlightenment: Hegel's metaphysical Spirit, the basis for Marxism and EU line-organized office-directed Bildung-education

34. Different Research

French Post-Structuralism



Inspired by Heidegger's: 'In sentences, trust the subject, but doubt the predicate'

Since differences disappear when institutionalizing words, truths, cures, and education

- Derrida: Words can be different (DeConstruction)
- Lyotard: Truth can be different (PostModern skepticism towards meta-narratives)
- Foucault: Truth creates regimes with Curing Institutions (education is really a 'pris-pital' mixing power techniques from a prison and a hospital)
- Bourdieu: Education can be different, and stop using symbolic violence and mathematics especially to create a new knowledge-nobility

35. Different Research

Difference-Research finds Differences making a Difference



Difference-Research, inspired by its skeptical roots,

- Questions traditional words & truths & institutions
- Designs different micro-curricula & macro-curricula
- Reports if a difference makes a difference

Examples

Micro-curricula: MATHeCADEMY.net with YouTube MrAlTarp-videos

Macro-curriculum: 'The Simplicity of Mathematics Designing a STEM-based Core Math Curriculum for Outsiders and Migrants',

<http://mathecademy.net/stem-based-core-math-for-migrants/>

36. Different Research

Difference-Research: For whom?



- For teachers observing problems in the classroom
- For teacher-researchers splitting their time between academic work at a university and intervention research in a classroom.
- For full-time researchers cooperating with teachers both using difference-research, the teacher to observe problems, the researcher to identify differences, together working out a different micro-curriculum, to be tested by the teacher, and reported by the researcher conducting a pretest-posttest study.
- Difference-research begins by observing learning problems and wondering if we could teach differently, e.g. a child saying 'II II, that is not 4, but 2 **2s**', showing that children bring 2dimensional block-numbers to school where 1dimensional cardinal line-numbers then are forced upon them.



Conclusion

The 3x3 Goal Displacements in Math Education

Primary	Numbers	Could: be icons & predicates in Tales of Many, $T = 2 \ 3s = 2*3$; show Bundles, $T = 47 = 4B7 = 3B17 = 5B-3$; $T = 456 = 4*BB + 5*B + 6*1$ Instead: are changed from predicates to subjects by silencing the real subject, the total. Place-values hide the bundle structure
	Operations	Could: be icons for the counting process as predicted by the RecountFormula $T = (T/B)*B$, from T pushing Bs away T/B times Instead: hide their icon-nature and their role in counting; are presented in the opposite order (+ - * /) of the natural order (/, *, -, +).
	Addition	Could: wait to after counting & recounting & double-counting have produced unit- and per-numbers; wait to after multiplication Instead: silences counting and next-to addition; silences bundling & uses carry instead of overloads; assumes numbers as ten-based
Middle	Fractions	Could: be per-numbers coming from double-counting in the same unit; be added by areas (integration) Instead: are defined as rational numbers that can be added without units (mathe-matism, true inside, seldom outside classrooms)
	Equations	Could: be introduced in primary as recounting from ten-bundles to icon-bundles; and as reversed on-top and next-to addition Instead: Defined as equivalence relations in a set of number-names to be neutralized by inverse elements using abstract algebra
	Proportionality	Could: be introduced in primary as recounting in another unit when adding on-top; be double-counting producing per-numbers Instead: defined as linear functions, or as multiplicative thinking supporting the claim that fractions and ratios are rational numbers
High	Trigonometry	Could: be introduced in primary as mutual recounting of the sides in a right-angled triangle, seen as a block halved by a diagonal Instead: is postponed till after geometry and coordinate geometry, thus splitting up geometry and algebra.
	Functions	Could: be introduced in primary as formulas, i.e. as the number-language's sentences, $T = 2*3$, with subject & verb & predicate Instead: are introduced as set-relations where first-component identity implies second-component identity
	Calculus	Could: be introduced in primary as next-to addition; and in middle & high as adding piecewise & locally-constant per-numbers Instead: differential calculus precedes integral calculus, presented as anti-differentiation



To Master Many Count & ReCount before you Add

A full 31 page article is here: <http://mathecademy.net/difference-research/>

This talk has been in 'Anglish', a dialect from the Viking area on the Danish WestCoast

The words 'eleven', and 'twelve', and 'twenty', come from the Vikings, saying

'one-levnt', 'twe-levnt' and 'twende-ti',

where 'levnt' means 'left'

Thank you for listening (Tak do for lytning)

Farewell (Farvel)

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