DEVELOPING CHILDREN'S INNATE MASTERY OF MANY BY ACCEPTING THEIR FLEXIBLE BUNDLE-NUMBERS WITH UNITS

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It is tempting to say: the goal of Math education is to master Math, as a means to later master Many. But since mastery of Math is hard to obtain, we ask instead: What Math grows from developing further the mastery of Many children get before school? With Math and Many as core concepts we first see how well-defined they are. Many is defined by, e.g., our fingers. Mathematics is typically defined by its numbers and operations, and by truth statemenets claiming, e.g., that the statements 2+3 = 5 and 2x3 = 6 are always true. And indeed, stating that 2 3s may be recounted as 6 1s, 2x3 = 6 is always true. However, 2+3 = 5 only holds with like units since, e.g., 2 weeks + 3 days = 17 days.

With multiplication holding and addition folding outside the class, math has two paradigms, with and without units: an outside 'unit-math' paradigm where 1+1 = ?, depending on the units; and an inside 'no-unit-math' paradigm where 1+1 = 2 always, thus creating a 'mathe-matism' valid inside only.

Before school, children live outside the 'no-unit-math' paradigm or greenhouse. In the workshop we investigate the math growing here, using philosophically existentialism where existence precedes essence. So we will work with real concrete existence, and neglect institutionalized abstract essence.

But, how do children master Many before school? Seeing 4 fingers 2 by 2, pre-school children often protest: 'That is not 4. That is 2 2s', thus insisting on describing what exists, bundles of 2s in space, and 2 of them in time. Meeting Many, children thus develop a number-language with full sentences including a subject, a verb, and a predicate as in the word-language, as well as 2-dimensional bundle-numbers with units, i.e., area-numbers different from the school's 1-dimensional line-numbers.

And in fact, children's bundle-numbers with units resonates with adult's numbers, as seen when writing out fully numbers as polynomials, e.g., *T* = 345 = 3x*B*^2 + 4x*B* + 5x1. Here we see that a digit is, not a number, but a factor needing a factor to become a number. Externally, Many esxist in time and space. To internalize Many, we deal with time first, then space (Tarp, 2021).

We bundle-count in time with a sequence. Here, we may use overloads where 34, or 3 bundle 4 tens, is counted as 2*B*14 tens; or underloads if couted as 4*B*less6 = 4*B*-6 tens (Tarp, 2018); and where 347 is 3 bundle-bundles 4 bundles 7 unbundled, or 3*BB*4*B*7. Flexible bundle numbers ease operations where 8x74 = 8x7*B*4 = 56*B*32 = 59*B*2 = 592; and where 336/7 = 33*B*6 /7 = 28*B*56 /7 = 4*B*8 = 48.

Bundle-counting ten fingers in 3s involves bundle-bundles where ten = 3*B*1 3s = 1*BB*0*B*1 3s.

Bundle-counting ten fingers in 2s involves also bundle-bundle-bundles where

ten = 5*B*0 2s = 4*B*2 2s = 2*BB*1*B*0 2s = 1*BBB* 0*BB* 1*B* 0 2s, or 1010 2s in short.

In space, iconizing transforms many 1s to 1 digit with as many sticks as it represents if written less sloppy. Once created, the digits may be used as units when bundle-couting.

Bundle-counting 8 in 2s, we push away bundles by a broom iconized as division, then iconized as multiplication we lift up bundles into a block, then we pull away the block by a rope iconized as subtraction to find unbundled singles that placed on-top become decimals, fractions or negatives.

Now, bundle-counting 8 in 2s may be predicted by the calculation 8/2 = 4. So we can write

8 = 4 2s = 8/2 2s, or 8 = (8/2)x2, or *T* = (*T*/*B*)x*B* with unspecified numbers.

By changing units, this recount-formula expresses proportionality all over STEM: in linearity as $\Delta y = (\Delta y/\Delta x) \times \Delta x = m \times \Delta x$; in science as meter = (meter/second) x second = speed x second, etc.

Recounting also solves equations. Asking "How many 2s in 8" gives the equation "u x 2 = 8", which is solved by recounting 8 in 2s: u x 2 = 8 = (8/2) x 2, so u = 8/2. So to solve equations, we move to opposite side with opposite calculation sign. Thus we don't need the balancing method of abstract algebra. So, recounting from tens to icons gives multiplication-equations solved by moving across.

Conversely, recounting from icons to tens gives multiplication tables, and algebra in a bundle-bundle square when using less-numbers. Here on a pegboard, 6x7 is 6 7s or (B-4) x (B-3) giving a total of T = 10B-3B-4B + 4 3s, added since pulled away twice. Increasing the base will decrease the height. So a total may be block'ed, ten'ed, or square'd allowing to find the square root.

Recounting in physical units creates per-numbers bridging the units. With 5$ per 4 kg, the price for 12 kg is found by recounting in the per-number: 12kg = (12/4) x 4kg = (12/4) x 5$ = 15$. With like units, pwer-numbers become fractions; both are not numbers, but factors needing factors to become numbers. Recounting the sides in a block halved by its diagonal creates trigonometry and pi.

Adding blocks next-to means integrating areas thus becoming integral calculus. Subtracting blocks then becomes differential calculus. Adding blocks on-top, the units become like by proportional recounting. As factors, per-numbers add by their areas under the per-number graph.

Inside the 'no-unit-math' paradigm, numbers subtract serial next-to on the number line. The result comes when counting backwards. Outside, numbers subtract parallel on-top. Here T = 9 – 6 = 3; and T = 6 – 9 = less3. Likewise, inside the 'no-unit-math' paradigm, numbers add serial next-to on the number line. The result comes when counting on from 6 or 9. Outside, numbers add parallel on-top. We see that T = 6 + 9 = 2B3 6s = 2B-3 9s = 2B-5 tens = 1B5 tens.

Change by adding and change by multiplying give linear change $y = b + a$ x n, and exponential change, $y = b$ x a^n, which reversed gives the factor-finding root, and the factor-counting logarithm.

A number-formula, or polynomial, T = 345 = 3BB4B5 = 3xB^2 + 4xB + 5 shows the 4 ways to unite: +, x, ^, and next-to block-addition called integration. Here, addition and multiplication unite changing and constant unit-numbers. Integration and power unite changing and constant per-numbers. The 4 uniting operations each has a reverse splitting operation: Addition and multiplication has subtraction and division. Power has the factor-finding root, and the factor-counting logarithm. Finally, integration has per-number finding, called differentiation. We might call this beautiful simplicity the 'Algebra Square' since in Arabic, algebra means to reunite.

Conclusion. The outside 'unit-math' paradigm provides the same mathematics as the inside 'no-unit-math' paradigm, only in a different order. Meeting Many, first we count and recount before we add. And recounting uncovers core mathematics as concrete existence ready for abstracting essence. Furthermore, the outside paradigm avoids the inside paradigm's mathematism with falsifiable addition-claims. So, to become a full science, mathematics should leave its 'no-unit-math' greenhouse, and accept that, of course, numbers cannot add without units. Counting and adding Many using children's bundle-numbers with units will allow a communicative turn in the number language as the one taking place in the 1970's foreign language education. And of course, an ethical quality education will not force children inside a 'no-unit-math' greenhouse that slowly strangels their innate number-language by using line-numbers to learn no-unit addition that folds outside; when their innate mastery of Many just waits to be developed by flexible bundle-numbers available at their fingertips.

REFERENCES

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