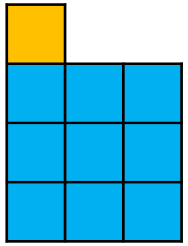
Flexible Bundle-Numbers Develop the Child's Innate Mastery of Many

A Paradigm Shift



from LineNumbers without to BundleNumbers with Units

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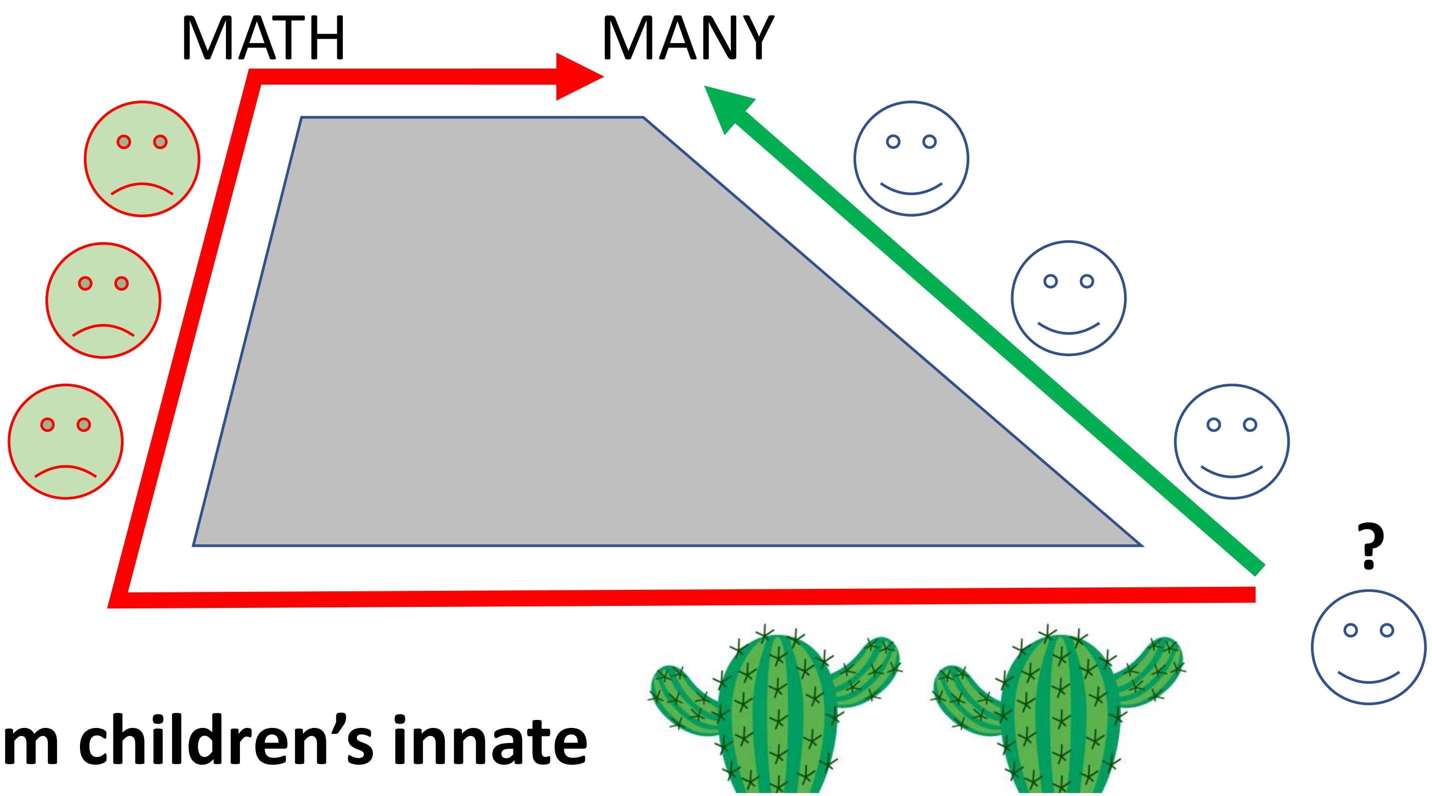
The Goal of Math Education: to master Math or to master Many?

All say:
The goal is to master Math,
to later master Many.
But Math is hard!
Why not first master Many,
to later master Math?



So we ask:

What Math may grow from children's innate mastery of Many, as developed before school?

Grand Theory Questions to Mathematics Education

- Sociology asks

Does mathematics education suffer from a goal displacement?

*What is the goal: to master **Many**, or to master **math**?*

- Philosophy asks

*What to teach: **existence** or **essence***

Is it ethical to teach students if they already know

- Psychology asks

*How to teach: guided interaction with **existence**, or mediated **essence**?*

Why Teach Children if they Already Know?

With education curing un-educatedness, we ask:

To CURE, be SURE

1. The diagnosed is not already cured
2. The diagnose is not self-referring: *teach math to learn math*

Core Questions:

- What Mastery of Many does the child have already?
- What could be a <u>ChildCenteredCurriculum</u> in Mastery of Many?

How do Children Master Many?
Ask a 3-year-old: How old Next Time?

The answer is 4, showing 4 fingers

But, reacting strongly to 4 fingers held together 2 by 2:

“That is not four, that is two twos”

<u>Observation 01</u>: Inside, children see what **exists** outside, bundles of **2s**, *in space*; and 2 of them, *in time*. So, they use, not line-, but **bundle**-numbers with **units**

<u>Observation 02</u>: The child uses a **full number-language sentence** as in the word-language with a SUBJECT, a VERB, and a PREDICATE:

”That is two twos”, shortened to the formula ”T = 2 **2s**”

Adults also use Bundle-Counting with Units to Master Many in Time and Space

Meeting Many in **time** (repetition), and in **space** (extension), we ask: "How Many here", and "How Many in Total?", or Simply "*T* = ?"

To bring Many from **time** to **space**,

- We tally: ~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ ~~IIII~~ II
- And bundle-count in polynomials: *T* = 345

= 3 **ten-ten** 4 **ten** 5

= 3 **Bundle-Bundle** 4 **Bundle** 5

= 3 x ***BxB*** + 4 x ***B*** + 5 x 1

So, digits are not numbers, but operators, needing digits to become numbers.

What is True Always, and What is True Sometimes

Is this true	**Always?**	**Sometimes?**
2 x 3 = 6	2x3 is 2 **3s** that can always be recounted as 6 **1s**	
2 + 3 = 5		Only with the same unit 2xweek + 3xweek = 5xweek 2xweeks + 3xdays = 17xdays
$\frac{1}{2}+\frac{2}{3}=\frac{3}{5}$	 	1 of 2 apples + 2 of 3 apples gives 3 of 5 apples, and not 7 of 6
$\frac{1}{2}+\frac{2}{3}=\frac{7}{6}$		Only if taken of the same total

Inside & Outside the NoUnitMath GreenHouse
Same Question - Different Answers

INSIDE

1+2 = 3

MatheMatism

True inside,

but seldom outside the

NoUnitMath GreenHouse

OUTSIDE

1+2 = ?

depends on the units

1m + 2m = 3m

1km + 2mm = 1km

1week + 2days = 9days

A science must give valid statements
so MatheMatism is no science

2 Math Paradigms: Without and With Units

- an inside '**no-unit-math**' number-line paradigm, where 1+2 = 3 always, and
- an outside '**unit-math**' bundle-number paradigm, where 1+2 = ? needs units

The '**unit-math**' paradigm builds on the philosophy **EXISTENTIALISM** where

EXISTENCE precedes ESSENCE

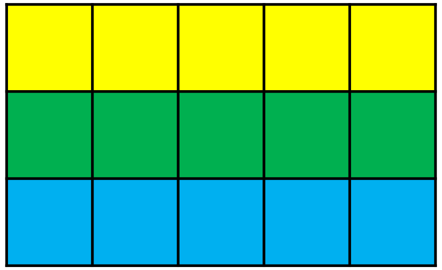
So 'unit-math' describes **EXISTENCE**, and neglects institutionalized ESSENCE

Unit-math is relevant for

- early childhood education
- adult education, and migrant education
- 'bringing back brains' from special education

1, 2, 3, … is a String of Names as Monday, Tuesday, Wednesday

With eyes closed, learning a string of number-names by rote is like learning other strings of names of weekdays, months, etc.



But, in a stack of 3 5s, the 5-bundle exists in space,

and 3, the number of 5s, exists in time.

So, we should include both the 'time-number' and the 'space-number' and the bundle-unit when counting existing things:

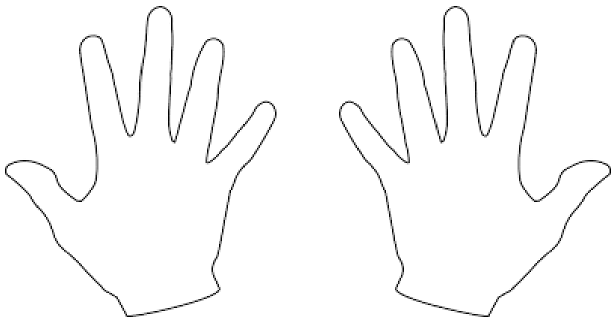
0 bundle 1, 0B2, …, 0B9, 0Bten or 1B0, 1B1,…, 1B9, 1Bten or 2B0,

Teaching 1+2 = 3 without units is falsified by 1week + 2days = 9 days.

Addition is only meaningful with like units according to the distributive law:

T = 0B1 + 0B2 = 0B3.

Bundle-Counting becomes Flexible with Overloads and Underloads



Counting 10 fingers in 5s, we include the unit to describe what exists

0*B*1, 0*B*2, 0*B*3, 0*B*4, 0*B*5, 0*B*6, 0*B*7, 2Bundle less2, 2*B*-1, 2*B*0

Overload	Underload
1*B*0, 1*B*1, 1*B*2,	1*B*3 , 1*B*4

Counting in tens with flexible bundle-numbers:

T = 38 = 3Bundle8 = 2Bundle18 = 4Bundle less2, or short

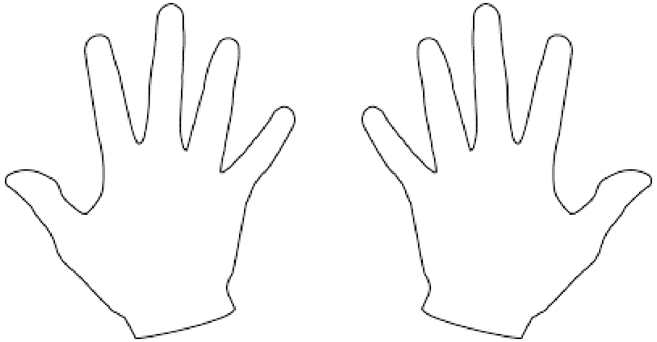
T = 38 = 3*B*8 = 2*B*18 = 4*B*-2, and

T = 347 = 3*BB*4*B*7 = 2*BB*14*B*7 = 1*BB*23*B*17, or

T = 347 = 3*BB*4*B*7 = 3*BB*5*B*-3 = 4*BB*-6*B*7

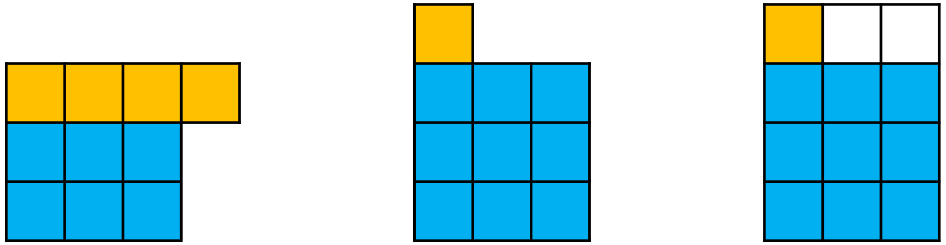
Danish Vikings said:
'en-levn' (elleven, one left)
'twe-levn' (twelve, two left)
'twende-ti' (twenty)

Bundle-Counting Fingers in **3s**



Over-load, Normal, Under-load

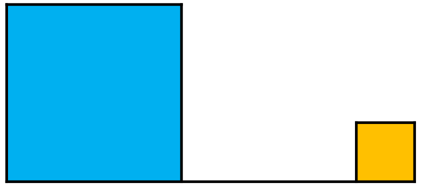
Ten bundle-counted in **3s**:



T = 2**B**4 *over-load*

T = 3**B**1 *normal*

T = 4**B**-2 *under-load*



T = 1**BB** 0**B** 1 = 101 **3s**

Singels, Bundles, Bundle-Bundles

3 **Singles** are 1 **Bundle**

3 **Bundles** are 1 **Bundle-of-Bundles**

So T = 9 = 1**BB 3s**

And T = ten = 3**B**1 **3s** = 1**BB**1 **3s** or

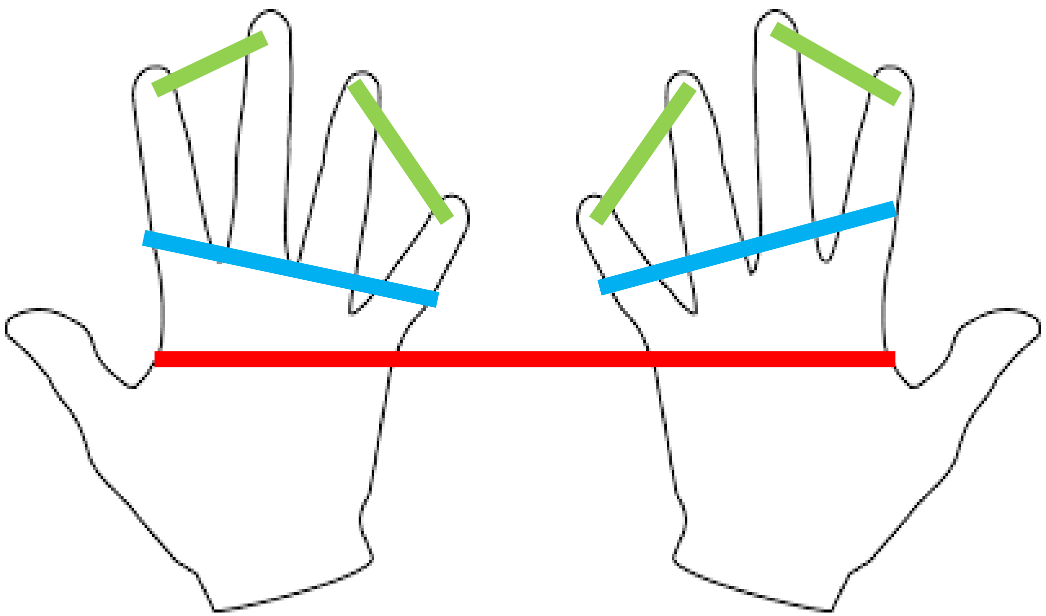
T = ten = 1**BB**0**B**1 **3s** or

T = ten = 101 **3s**

*So with units as **bundles**, and **bundle-of-bundles**, we do not need the place value system.*

Bundle-Counting in 2s, 4 is a bundle-bundle, and 8 is a bundle-bundle-bundle

1	0*B*1	01
2	1*B*0	10
3	1*B*1	11
4	1*BB*0*B*0	100
5	1*BB*0*B*1	101
6	1*BB*1*B*0	110
7	1*BB*1*B*1	111
8	1*BBB*0*BB*0*B*0	1000
9	1*BBB*0*BB*0*B*1	1001
Ten	1*BBB*0*BB*1*B*0	1010



This can be shown with Lego bricks having different colors:

a green 2-brick is ***B***

a blue 4-brick is ***BB***

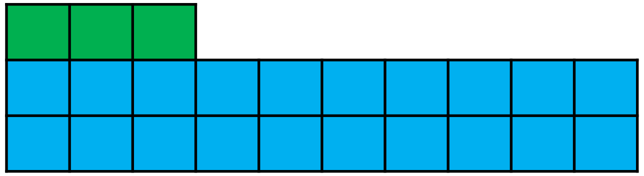
a red 8-brick is ***BBB***

How to see the Unbundled Counted in **tens**?”

Counting in tens, a Total of 2 **tens** & 3 can be described

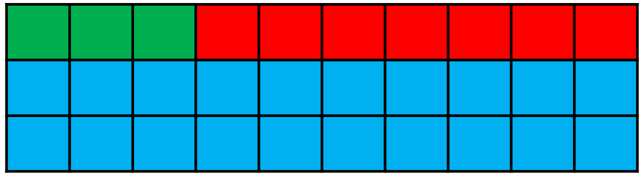
T = 23 if leaving out the unit and the decimal point,

- or as:



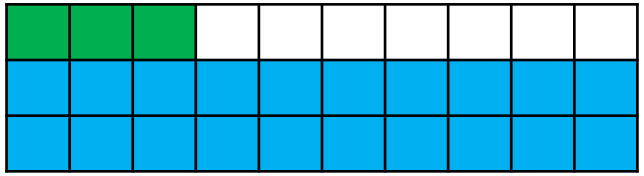
T = 2B3 **tens**

T = 2.3 **tens**



T = 3B-7 **tens**

T = 3.-7 **tens**



T = 2 3/10 B **tens**

T = 2 3/10 **tens**

Bundle-Counting in tens Including Units, 100 becomes a bundle-bundle, BB

1BB0B0	1BB0B1	1BB0B2	1BB0B3	1BB0B4	1BB0B5	1BB0B6	1BB0B7	1BB0B8	1BB0B9	~~1BB0B10~~
~~10B0~~	~~10B1~~	~~10B2~~	~~10B3~~	~~10B4~~	~~10B5~~	~~10B6~~	~~10B7~~	~~10B8~~	~~10B9~~	~~10B10~~
9B0	9B1	9B2	9B3	9B4	9B5	9B6	9B7	9B8	9B9	~~9B10~~
8B0	8B1	8B2	8B3	8B4	8B5	8B6	8B7	8B8	8B9	~~8B10~~
7B0	7B1	7B2	7B3	7B4	7B5	7B6	7B7	7B8	7B9	~~7B10~~
6B0	6B1	6B2	6B3	6B4	6B5	6B6	6B7	6B8	6B9	~~6B10~~
5B0	5B1	5B2	5B3	5B4	5B5	5B6	5B7	5B8	5B9	~~5B10~~
4B0	4B1	4B2	4B3	4B4	4B5	4B6	4B7	4B8	4B9	~~4B10~~
3B0	3B1	3B2	3B3	3B4	3B5	3B6	3B7	3B8	3B9	~~3B10~~
2B0	2B1	2B2	2B3	2B4	2B5	2B6	2B7	2B8	2B9	~~2B10~~
1B0	1B1	1B2	1B3	1B4	1B5	1B6	1B7	1B8	1B9	~~1B10~~
0B0	0B1	0B2	0B3	0B4	0B5	0B6	0B7	0B8	0B9	~~0B10~~

10 = bundle = B 100 = bundle-bundle = BB = B^2 1000 = bundle-bundle-bundle = BBB = B^3

Flexible Bundle-Numbers Ease Operations.
Counting in tens, T = 74 = 7***B***4 = 6***B***14 = 8***B***-6

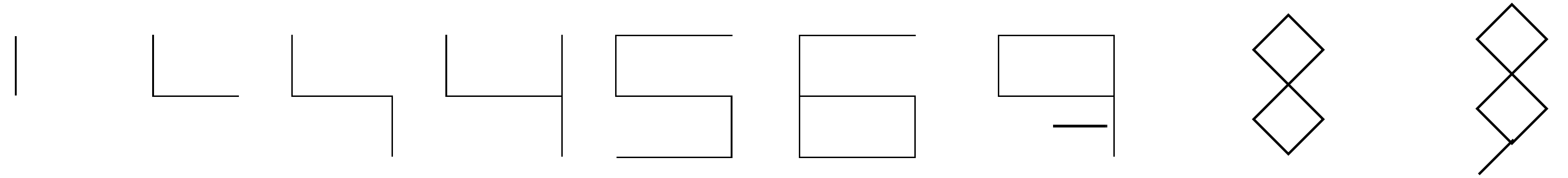
Overload	Underload	Overload	Overload
7 4 + 1 8	7 4 - 1 8	8 x 74	336 /7
7***B***4 + 1***B***8	7***B***4 - 1***B***8	8 x 7***B***4	33***B***6 /7
8***B***12	6***B***-4	56***B***32	28***B***56 /7
9***B***2	5***B***6	59***B***2	4***B***8
92	56	592	48

No need to carry

Digits as Icons: IIII → IIII → 4 →

Children love making number-icons of cars, dolls, spoons, sticks.
Changing four ones to one fours creates a 4-icon with four sticks.
An icon contains as many sticks as it represents, if written less sloppy.
Once created, icons become **UNITS** when counting in bundles, as kids do.

one	two	three	four	five	six	seven	eight	nine
I	II	III	IIII	IIIII	IIIIII	IIIIIII	IIIIIIII	IIIIIIIII



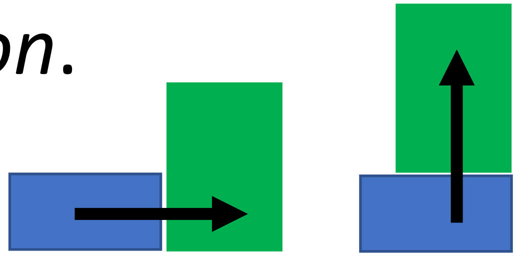
1	2	3	4	5	6	7	8	9

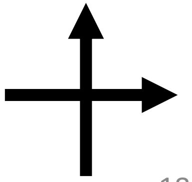
Divide & Multiply & Subtract & Add may be ‘de-modeled’ as Counting-Icons

- From 9 **PUSH** away **4s** we write 9/4 iconizing a broom, called *division*.

- 2 times **LIFT** the **4s** to a stack we write 2x4 iconizing a lift called *multiplication*.
- “From 9 **PULL** away 2 **4s** to find un-bundled” we write 9 – 2x4 iconizing a rope, called *subtraction*.

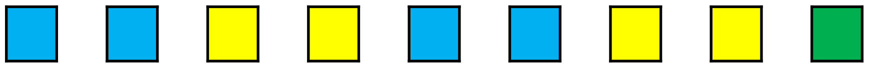
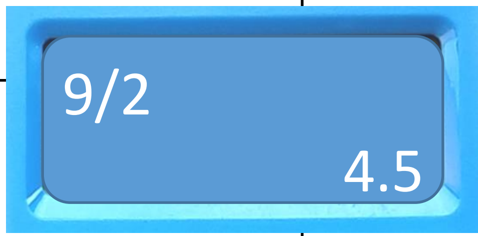
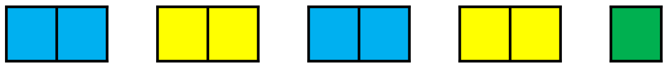
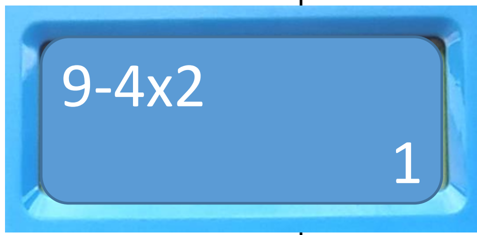
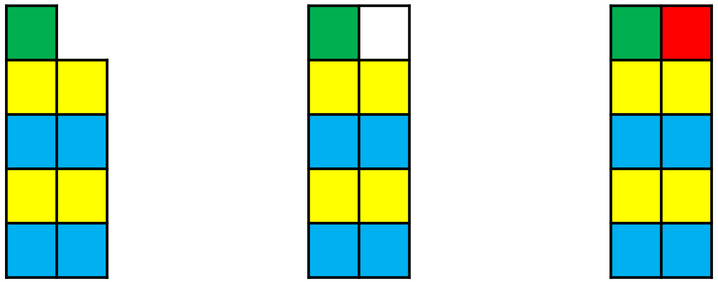
- **UNITING** next-to or on-top we write **B+C** iconizing the two directions, called *addition.*





Calculators Predict Counting Results
Bundle-Counting a Total of 9 in **2s**

9/2 **4.some**
9 – 4x2 **1**

Acting	Predicting
 **9**	From 9, 9/2 times, push away **2s** 
 *bundled in **2s** with 1 unbundled*	From 9, pull away 4 **2s**, leaves 1 Calculator prediction : T = 9 = 4*B*1 **2s** 
 4.1***B*** 4½***B*** 5***B***-1 **2s** *stacked as 4x2 with 1 unbundled placed on-top*	The unbundled are placed on-top • separated by a decimal point, 4.1 **2s** • counted in bundles as 1 = (1/2) x 2 giving 4½***B*** **2s** • counting what is missing in a full bundle, 5***B***-1 **2s** So, inside decimals, fractions, and negatives are rooted in how we outside see the unbundled.

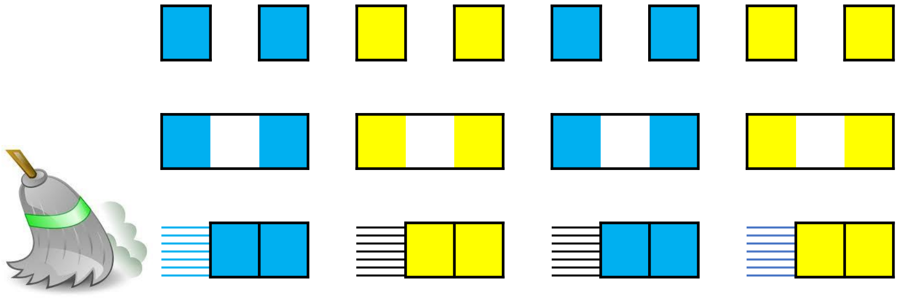
Shifting Units Creates a Recount-Formula

$8 = (8/2)x2$
$T = (T/B)xB$

Shift unit from **1s** to **2s**: 8 = ? **2s**

Bundle-counting: 8 = 4 **2s** = 4x2

Predict by a calculation: 8/2 = 4



Recount result: 8 = (8/2)x2

Recount-Formula: $\boxed{T = (T/B)xB}$ *" T contains T/B Bs"*

ux2 = 8 = (8/2)x2
u = 8/2

Move: OPPOSITE Side & Sign

Equations Proportionality

Shifting unit	y = k * x
Linearity	Δy = (**Δy/Δx**) * Δx = **m** * Δx
Local linearity	dy = (**dy/dx**) * dx = **y'** * dx
Trigonometry	a = (**a/b**) * b = **tanA** * b
Trade	$ = (**$/kg**) * kg = **price** * kg
STEM	meter = (**meter/sec**) * sec = **speed** * sec

The Recount Formula Solves Equations

Asking "How many 2s in 8" gives the equation "u x 2 = 8", that is solved by recounting 8 in 2s:

$$u \times 2 = 8 \qquad = (8/2) \times 2$$
$$u = 8/2$$

To solve equations, we MOVE to OPPOSITE side with OPPOSITE calculation sign
So we don't need the balancing method of abstract algebra

$u + 2 = 8$	$u \times 2 = 8$	u^8 = 2	2^u = 8
$u = 8 - 2$	$u = 8/2$	$u = \sqrt[8]{2}$	$u = \log_2(8)$

FACTOR-finding ROOT FACTOR-counting LOG

Remember Units when Solving Equations

OppoSite Side with OppoSite Sign		Remember Units
$2 + u = 8$ $= (8-2) + 2$	$2 \times u = 8$ $= (8/2) \times 2$	$2 + 3 \times u = 14$ $2\times1 + 3\times u = 14 = 14 - 2 + 2$
$u = 8-2$ *Solved by Splitting*	$u = 8/2$ *Solved by ReCounting*	$u = (14 - 2)/3$, ~~not 14/5~~ *Solved by Including Units*

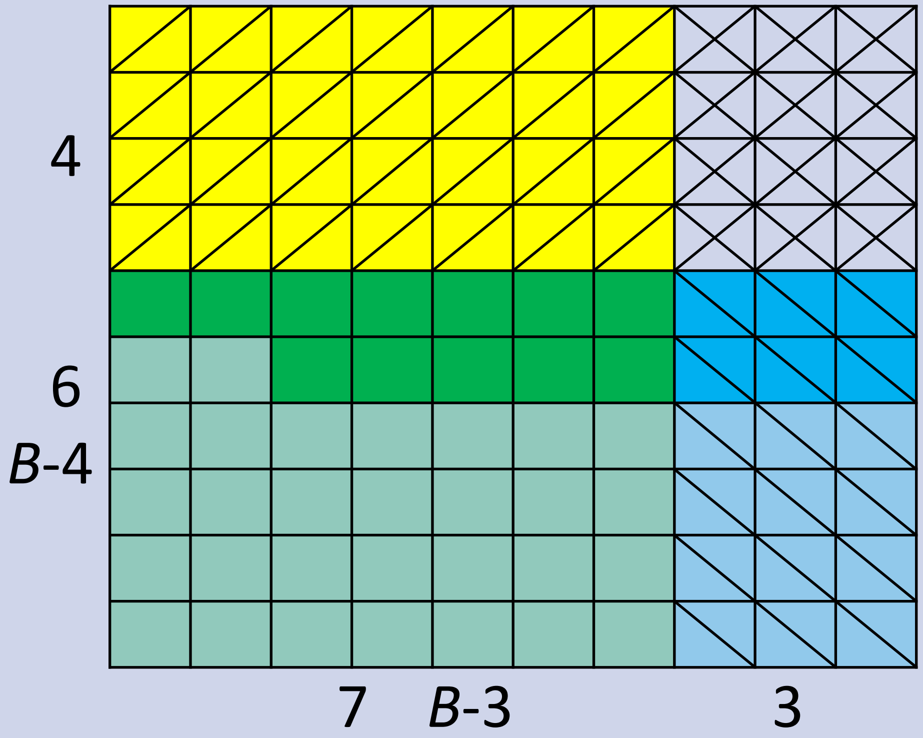
<u>Hymn to Equations</u>

Equations are the best we know,
they are solved by isolation.
But first, the bracket must be placed
around multiplication.

We change the sign and take away
and only x itself will stay.
We just keep on moving, we never give up.
So feed us equations, we don't want to stop!

Recounting from Icons to Tens, Asking 6x7 = ?, we Meet Algebra in a Bundle-Bundle Square

With no 10-button, multiplication directly gives 6 7s as 4B2 tens

<table>
<tr><th>Action</th><th>Prediction</th></tr>
<tr><td>


</td><td>
<u>T = 6 7s = ? tens</u>
= 6 x 7
= (B-4) x (B-3) = $\left(\begin{array}{c}B-4\\B-3\end{array}\right)$
= BB - 3B - 4B - - 4 x 3
= 3B + 1B2
= 4B2 = 42, so 6 7s is 4B2 tens
- - is + since it is pulled away twice
</td></tr>
</table>

Recounting into tens with Overloads in List- & Block- & Cross Multiplication

We see that 8 16s is 12B8 tens, and that

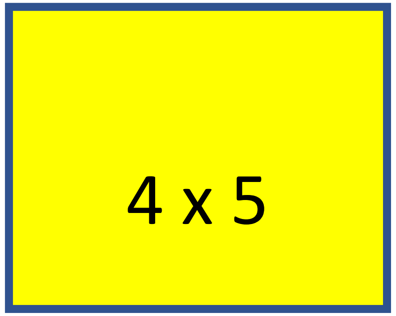
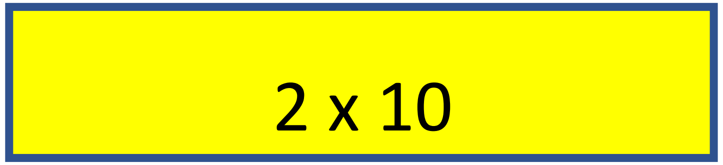
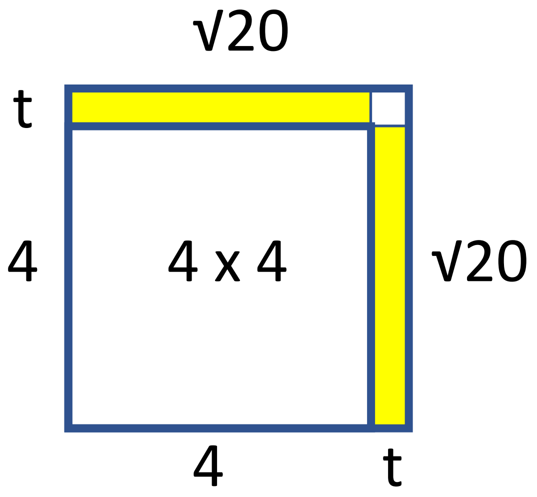
1 x 16 = 1 x 1B6	1B6	1B6 =	16
2 x 16 = 2 x 1B6	2B12	3B2 =	32
3 x 16 = 3 x 1B6	3B18	4B8 =	48
4 x 16 = 4 x 1B6	4B24	6B4 =	64
5 x 16 = 5 x 1B6	5B30	8B0 =	80
6 x 16 = 6 x 1B6	6B36	9B6 =	96
7 x 16 = 7 x 1B6	7B42	11B2 =	112
8 x 16 = 8 x 1B6	8B48	12B8 =	128
9 x 16 = 9 x 1B6	9B54	14B4 =	144
10 x 16 = 10 x 1B6	10B60	16B0 =	160

34 56s is 190B4 tens

	5B	6	
4	20B	24	
3B	15BB	18B	
	15BB	38B	24
	15BB	40B	4
	19	**0B**	**4**

3	B	4
5	B	6
15BB	(18+20)B	24
15BB	40B	4
19	**0B**	**4**

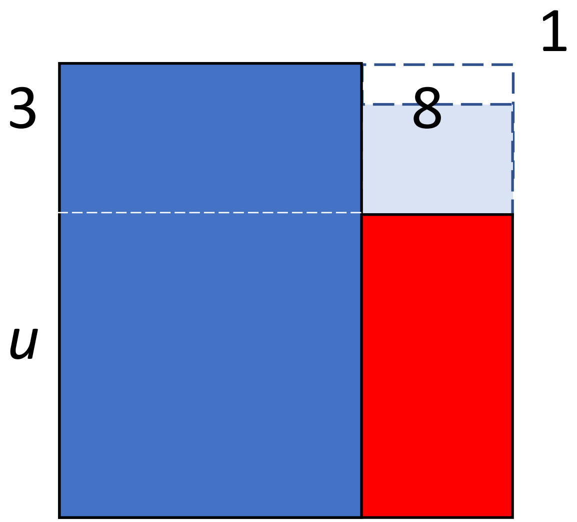
Increasing the Base, we Decrease the Height.
So a Total may be BOX'ed, TEN'ed or SQUARE'd

BOX'ed	TEN'ed	SQUARE'd
		 20 – 4 x 4 = 4 that shared by the two 4 x t boxes gives t = 0.5 So √20 ≈ 4.5 Calculator: √20 = 4.472

Bundle-bundles as Squares Eases Algebra

Quadratic Equations with 2 Cards

$u^2 + 6u + 8 = 0$



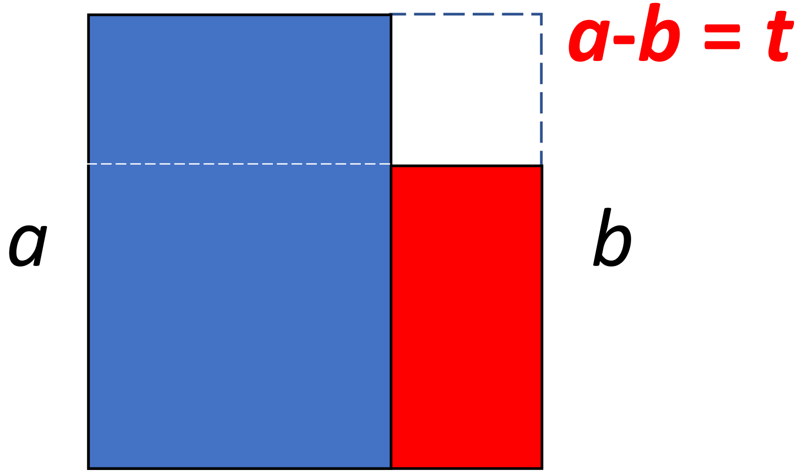
$(u+3)^2 = u^2 + 6u + 8 + 1$

$(u+3)^2 = 0 + 1$

$u = -3 \pm 1$ **$u = -4$ & $u = -2$**

Quadratic Rule with 2 Cards

$(a - b)^2 = ?$

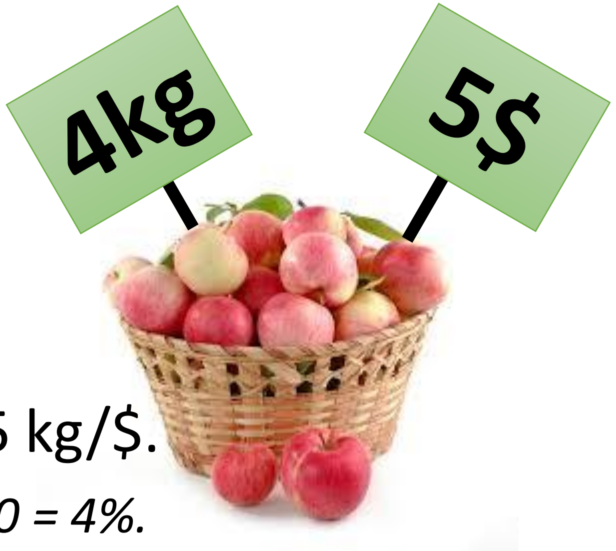


Corner = $(a-b)^2 = a^2 - 2 \text{ cards} + b^2$

So $(a-b)^2 = a^2 - 2 \times a \times b + b^2$

And $(t+b)^2 = t^2 + 2 \times t \times b + b^2$

ReCounting in two Units creates PerNumbers & Proportionality



ReCounting in kg & $, gives a PerNumber 4kg **per** 5$ = 4kg/5$ = 4/5 kg/$.

With like units, per-numbers become fractions: 4$/5$ = 4/5, and 4$/100$ = 4/100 = 4%.

With 4kg linked to 5$, we simply recount in the per-number.

(Or we recount the units directly. Or we equate the per-numbers. Or we use the before 1900 'Rule of 3' (Regula de Tri) alternating the units, and, from behind, first multiply, then divide.)

Questions: 12kg = ?$	**20$ = ?kg**
12kg = (12/4) x 4kg = (12/4) x 5$ = 15$	20$ = (20/5) x 5$ = (20/5) x 4kg = 16kg
$ = ($/kg) x kg = 5/4 x 12 = 15	kg = (kg/$) x $ = 4/5 x 20 = 16
u/12 = 5/4, so *u* = 5/4 x 12 = 15	*u*/20 = 4/5, so *u* = 4/5 x 20 = 16
If 4kg is 5$, then 12kg is ?$; answer: 12x5/4 = 15	If 5$ is 4kg, then 20$ is ?kg; answer: 20x4/5 = 16

Proportionality shows the Flexibility of Unit-Math

Proportionality, **Q1**: "2kg costs 5\$, what does 7kg cost"; **Q2**: "What does 12\$ buy?"

➔1) Regula de Tri (rule of three)

Re-phrase with shifting units, the unknown at last. From behind, first multiply then divide.

Q1: '2kg cost 5\$, 7kg cost ?\$'. Multiply-then-divide gives the \$-number 7x5/2 = 17.5.

Q2: '5\$ buys 2kg, 12\$ buys ?kg'. Multiply-then-divide gives the kg-number 12x2/5 = 4.8.

➔2) Find the unit

Q1: 1kg costs 5/2\$, so 7kg cost 7x(5/2) = 17.5\$. **Q2**: 1\$ buys 2/5kg, so 12\$ buys 12x(2/5) = 4.8kg

➔3) Cross multiplication

Q1: 2/5 = 7/*u*, so 2*u = 7*5, *u* = (7*5)/2 = 17.5. **Q2**: 2/5 = *u*/12, so 5*u = 12*2, *u* = (12*2)/5 = 4.8

➔4) 'Re-counting' in the 'per-number' 2kg/5\$ coming from 'double-counting' the total T.

Q1: T = 7kg = (7/2)x2kg = (7/2)x5\$ = 17.5\$; **Q2**: T = 12\$ = (12/5)x5\$ = (12/5)x2kg = 4.8kg.

Proportionality also shows the Inflexibility of Greenhouse Mathematics

➔ 5) <u>Modeling</u> with linear functions using group theory from abstract algebra.

- A linear function $f(x) = c*x$ from the set of positive kg-numbers to the set of positive \$-numbers, has the domain $DM = \{x \in R \mid x>0\}$.
- Knowing that $f(2) = c*2 = 5$, this equation is solved by multiplying with the inverse element to 2 on both sides, and applying the associative law, and the definition of an inverse element, and of the neutral element under multiplication:
  $c*2 = 5$ ● $(c*2)*½ = 5*½$ ● $c*(2*½) = 5/2$ ● $c*1 = 5/2$ ● $c = 5/2$.
- With $f(x) = 5/2*x$, the inverse function is $f^{-1}(x) = 2/5*x$.
- With 7kg, the answer is $f(7) = 5/2*7 = 17.5$\$.
- With 12\$, the answer is $f^{-1}(12) = 2/5*12 = 4.8$kg.

Recounting gives per-numbers in STEM Multiplication Formulas I

STEM typically contains multiplication formulas with per-numbers coming from recounting.

Examples:

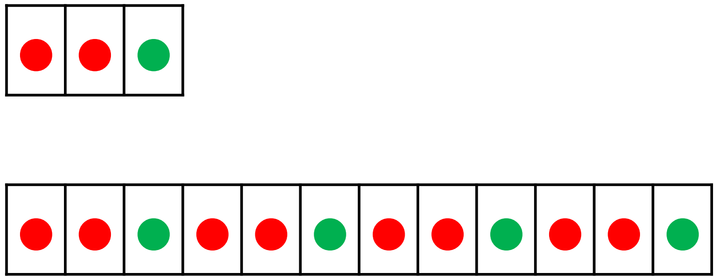
- kg = (**kg/cubic-meter**) x cubic-meter = **density** x cubic-meter
- force = (**force/square-meter**) x square-meter = **pressure** x square-meter
- meter = (**meter/sec**) x sec = **speed** x sec
- energy = (**energy/sec**) x sec = **Watt** x sec
- energy = (**energy/kg**) x kg = **heat** x kg

Recounting gives per-numbers in STEM and Economics Multiplication Formulas II

Extra examples from STEM, and from economics:

- gram = (**gram/mole**) x mole = **molar mass** x mole;
- Δ momentum = (**Δ momentum/sec**) x sec = **force** x sec;
- Δ energy = (**Δ energy/ meter**) x meter = **force** x meter = work;
- **energy/sec** = (**energy/charge**) x (**charge/sec**) or **Watt** = **Volt** x **Amp**;
- dollar = (**dollar/hour**) x hour = **wage** x hour;
- dollar = (**dollar/meter**) x meter = **rate** x meter
- dollar = (**dollar/kg**) x kg = **price** x kg.

With like Units, PerNumbers become Fractions, both Operators Needing Numbers to Become Numbers

Trial	Prediction
In a box with 2 red per 3 apples, re-counting reds and apples gives the FRACTION **2/3** reds/apples. How many red apples among 12 apples? 	Q: ? red in 12 apples. A: Recount 12 in **3s** (the per-number) $T = 12\ a = (12/3) \times 3a$ gives $(12/3) \times 2r = 8$ red apples Or, we equal the per-numbers: $u/12 = 2/3$; so $u = 2/3 \times 12 = 8$ *Moving 12 to opposite side with opposite sign*

Enlarging or Shortening Fractions

Taking 2/3 of 12 means taking 2 per 3 of 12.

With 2 bridged to 3, we recount 12 in **3s**, 12 = (12/3) x 3 = 4 x 3 = 4 **3s**

So 4 times we can take 2, i.e. 8 of the 12. Thus 2 per 3 = 8 per 12.

This may be used for enlarging or shortening fractions by inserting or removing the same unit above and below the fraction line:

$$\frac{2}{3} = \frac{2\ \mathbf{4s}}{3\ \mathbf{4s}} = \frac{2*4}{3*4} = \frac{8}{12} \quad \bullet \quad \frac{8}{12} = \frac{2*4}{3*4} = \frac{2\ \mathbf{4s}}{3\ \mathbf{4s}} = \frac{2}{3} \quad \bullet \quad \frac{12abc}{8a} = \frac{3*4*a*b}{2*4*a} = \frac{3*b\ \mathbf{4as}}{2\ \mathbf{4as}} = \frac{3b}{2}$$

Recounting Sides in a Block Halved by its Diagonal gives Trigonometry and Pi

A block cut by its diagonal creates a right triangle with three sides: the base *b*, the height *h*, and the cut *c*. They connect with the angle *A* by per-number formulas recounting the sides pairwise.

$h = (h/b) \times b = \tan\boldsymbol{A} \times b$

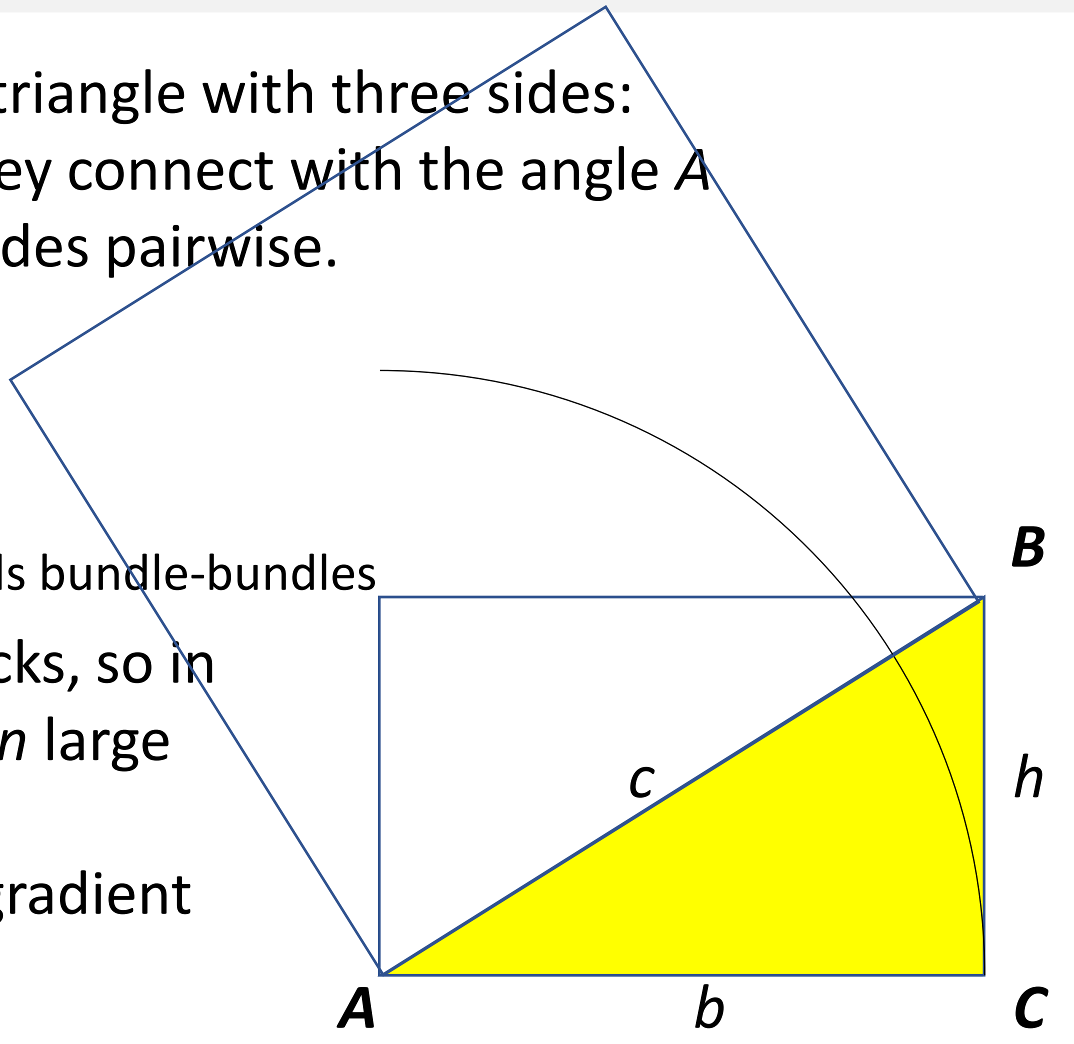
$h = (h/c) \times c = \sin\boldsymbol{A} \times c$

$b = (b/c) \times c = \cos\boldsymbol{A} \times c$

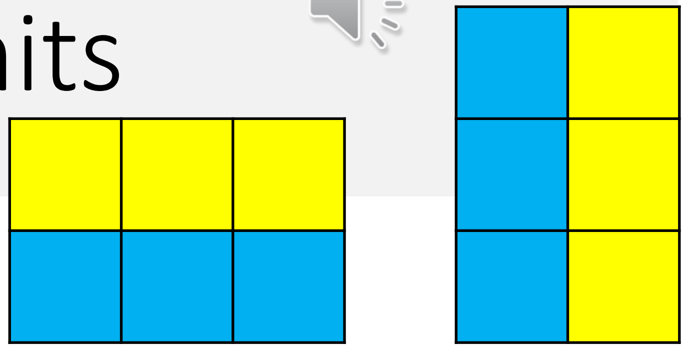
$\underline{h \times h + b \times b = c \times c}$, so the diagonal adds bundle-bundles

A circle contains very many small half-blocks, so in the circumference: $\boldsymbol{\pi = n \times \tan(180/n)}$ for n large

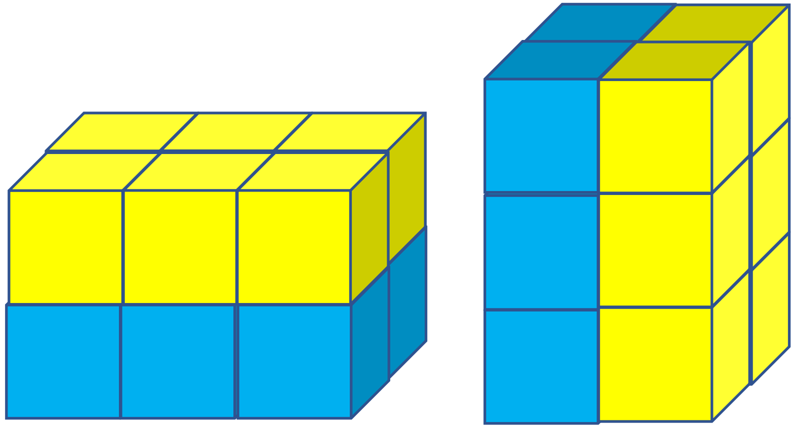
$\tan\boldsymbol{A} = h/b = \Delta y/\Delta x$ = rise/run = diagonal gradient



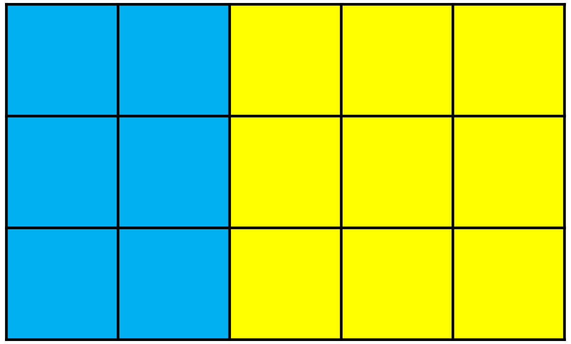
Switching, Uniting & Splitting Units



- Turning a 2D block will change the unit
  T = 2 **3s** = 2x3 → T = 3 **2s** = 3x2,
  So T = 2x3 = 3x2 *(The Commutative law)*

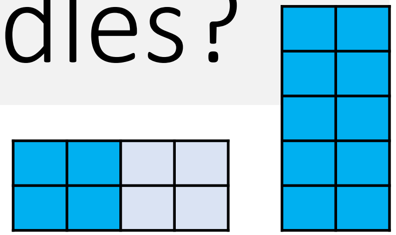


- Turning a 3D block will also change the unit
  So T = 2x(2x3) = (2x2)x3 *(The Associative law)*



- A block may split into two parts
  T = 3 **5s** = 3 **2s** + 3 **3s**
  So T = 3x5 = 3x(2+3) = 3x2 + 3x3 *(The Distributive law)*

Prime Numbers & Foldable Numbers
When can Blocks be Folded in like Bundles?



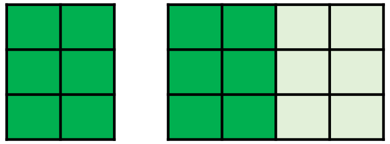
The block T = 2 **4s** = 2 x 4 has 4 as the bundle-unit.

Turning over gives T = 4 **2s** = 4 x 2, now with 2 as the bundle-unit.

4s can be folded in another bundle as 2 **2s**, whereas **2s** cannot.

(1 is not a bundle, nor a unit since a bundle-of-bundles stays as 1).

We call 2 a **prime unit number,** and 4 a **foldable unit number**, 4 = 2 **2s**.



A block of 3 **2s** cannot be folded in like bundles.

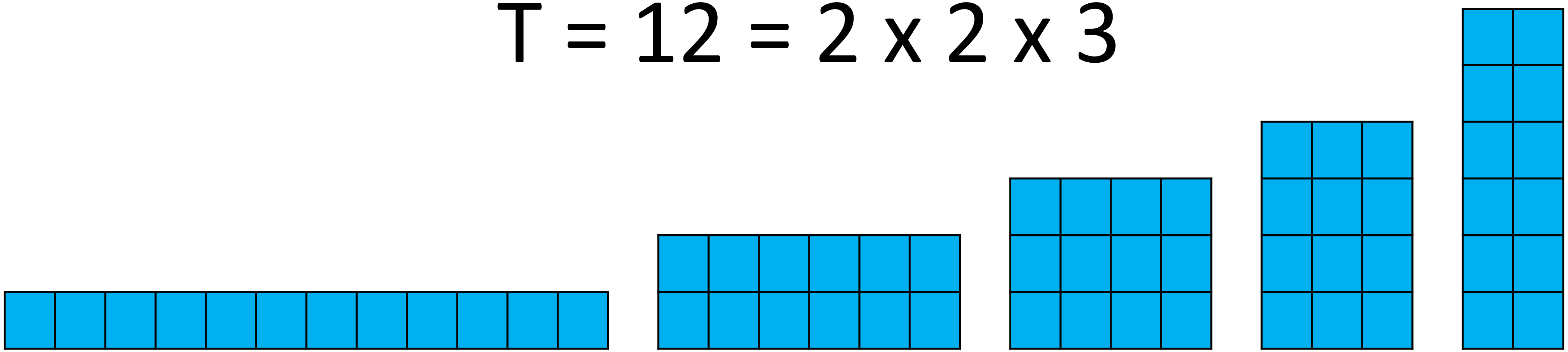
A block of 3 **4s** can be folded: T = 3 **4s** = 3 x (2 x 2) = (3 x 2) x 2 = 2 **3x2s**.

A number is called **even** if it can be written with 2 as the unit, else **odd**.

Finding Possible Units
What are Possible Units in T = 12?

Units come from folding in prime units:

T = 12 = 2 x 2 x 3

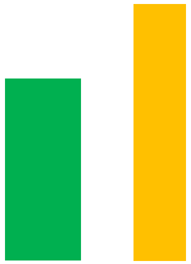


T = 12 = 2 **2x3s** = 3 **2x2s** = 2x2 **3s** = 2x3 **2s**

T = 1 **12s** = 2 **6s** = 3 **4s** = 4 **3s** = 6 **2s**

Recounting Large Numbers in or from Tens:
Same Size, but a new Form

Recounting 6 **47s** in **tens**	Recounting 476 in **7s**
T = 6 **47s** = 6 x 47	T = 476 = 47.6 **tens**
6 x 4**B**7	= 47**B**6
= 24**B**42	= 42**B**56
= 28**B**2	= 6x7**B**8x7
= 28.2 **tens**	= 68 x 7
	= 68 **7s**



Operators become Areas Before Adding

Digits are operators, needing a number to become a number.

And, per-numbers and fractions are also operators.

So, to add, operators must first be multiplied to become a number

But multiplying creates areas.

Digits and per-numbers and fractions thus add as areas, i.e., as integral calculus

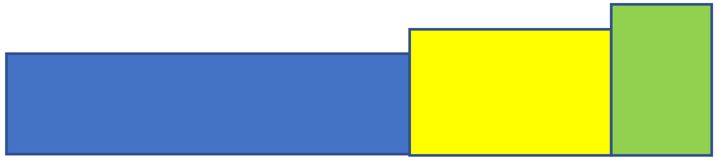
So, calculus comes in three versions

- Primary school calculus: adding digits as bundle-number blocks
- Middle school calculus: adding constant per-numbers and fractions
- High school calculus: adding changing per-numbers

In Unit-Math, BLOCKS are Fundamental

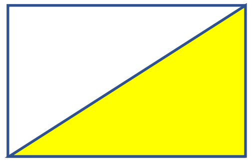
Children see Many as blocks with a number of bundles, and use flexible numbers with units and with over- or underloads

- in numbers: 456 = three blocks



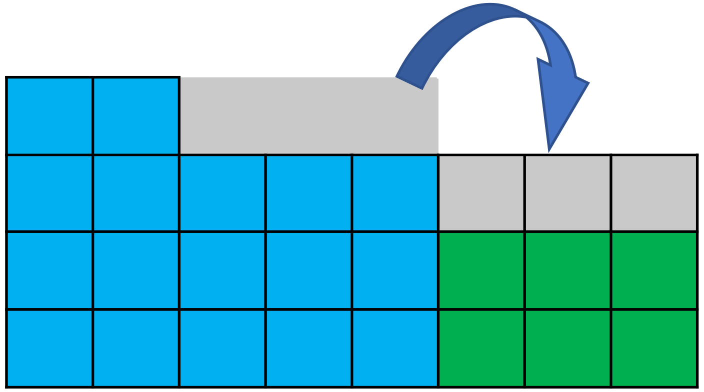
- in algebra: adding blocks next-to or on-top

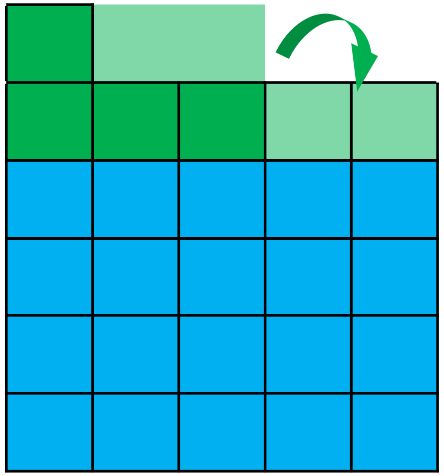
- in geometry: trigonometry recounts half-blocks



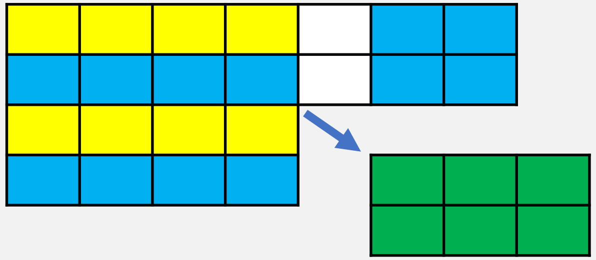
Once Counted & Recounted, Totals may Add

BUT: **NextTo**	**or** **OnTop**
4 **5s** + 2 **3s** = 3**B**2 **8s**	4 **5s** + 2 **3s** = 5**B**1 **5s**
The areas are integrated *Adding areas = Integration*	The units are changed to be the same *Change unit = ReCounting = Proportionality*





Reversing next-to addition



"If T1 = 2 **3s** and T2 add next-to as 4 **7s**, what is T2?"

We pull away the initial block T1 before recounting the rest in **4s**.

The recount formula predicts the result:

T2 = (T2/**B**) x **B**

= ((4x7 – 2x3)/4) x 4 = 5.2 **4s**

(4x7 – 2x3)/4	**5.some**
(4x7 – 2x3) – 5x4	**2**

Since reversed next-to addition finds area-differences, it is called differential calculus. Here subtraction precedes division; which is natural as reversed integration.

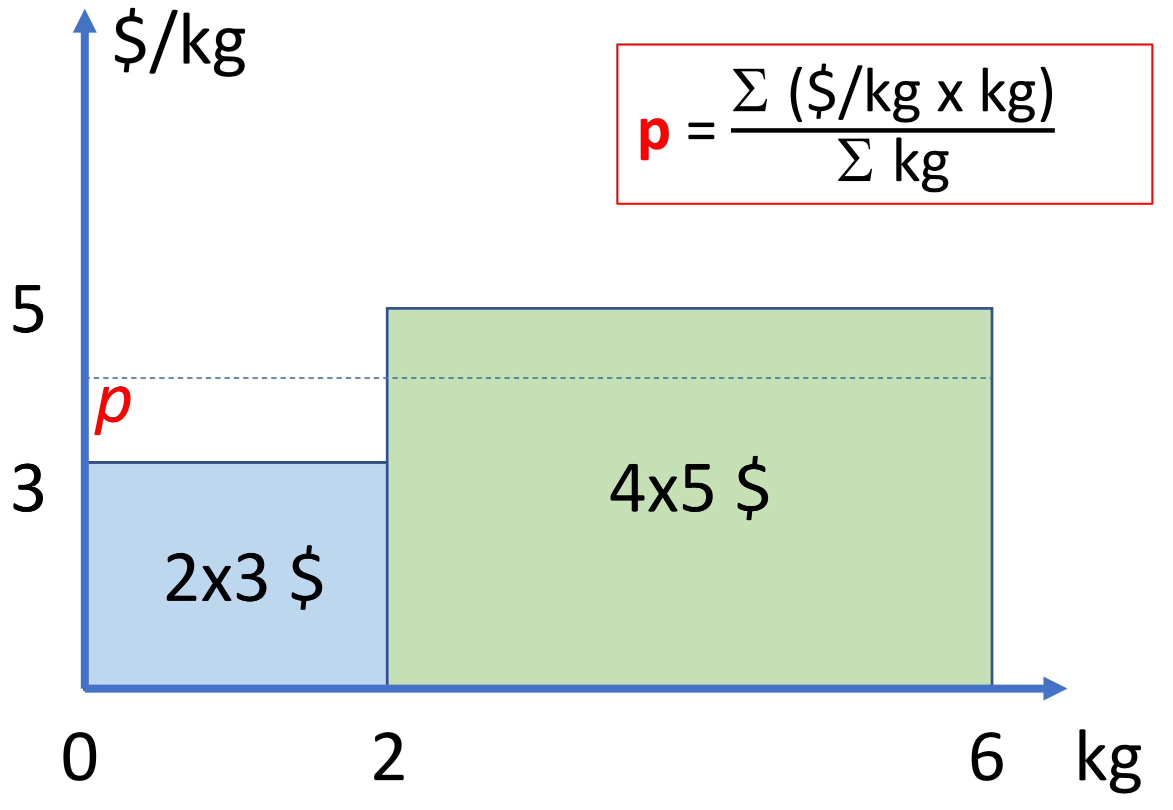
Per-numbers add as Areas (Integral Calculus)

Here, the per-number p is **piecewise constant**, which gives the sum $\Sigma\ (p*\Delta x)$ that becomes $\int p*dx$, if it is **locally constant** , by interchanging epsilon and delta

"2kg at **3$/kg** + 4kg at **5$/kg** = 6kg at **?** **$/kg**?"

2 kg	at	**3 $/kg**
+ 4 kg	at	**5 $/kg**
(2+4) kg	at	***p*** **$/kg**

- Unit-numbers add directly.
- Per-numbers must be multiplied to unit-numbers, thus adding as **areas** under the per-number curve.
- Here, multiplication before addition
- So, per-numbers and fractions are not numbers, but operators needing numbers to be numbers.



Subtracting PerNumbers (Differentiation)

“2kg at **3$/kg** + 4kg at **what** = 6kg at **5$/kg**?”

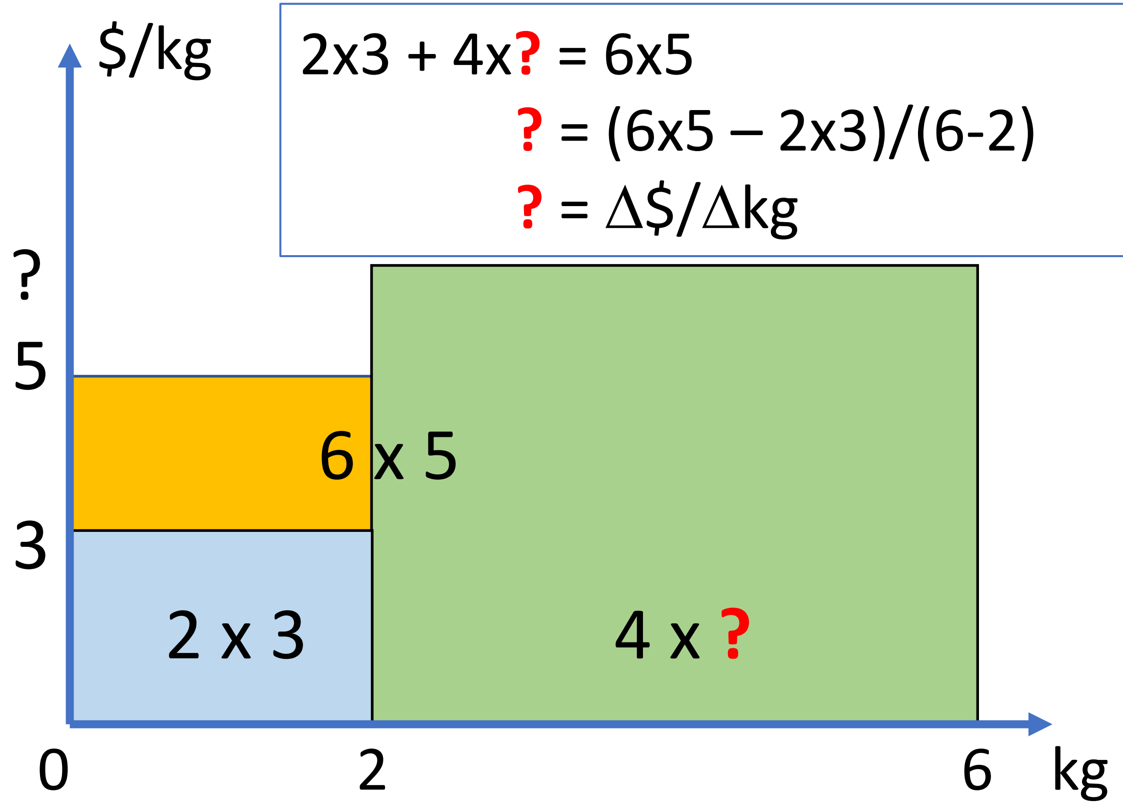
	2 kg	at	**3 $/kg**
+	4 kg	at	**? $/kg**
	6 kg	at	**5 $/kg**

We remove the initial 2x3 block, and recount the rest in 4s to get the per-number.

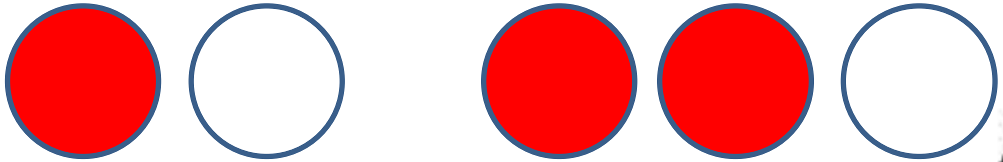
So, reversed per-number addition means subtracting areas to be reshaped, called differential calculus.

Here subtraction (giving a change, Δ) comes before division, the reverse of multiplication before addition in integral calculus.

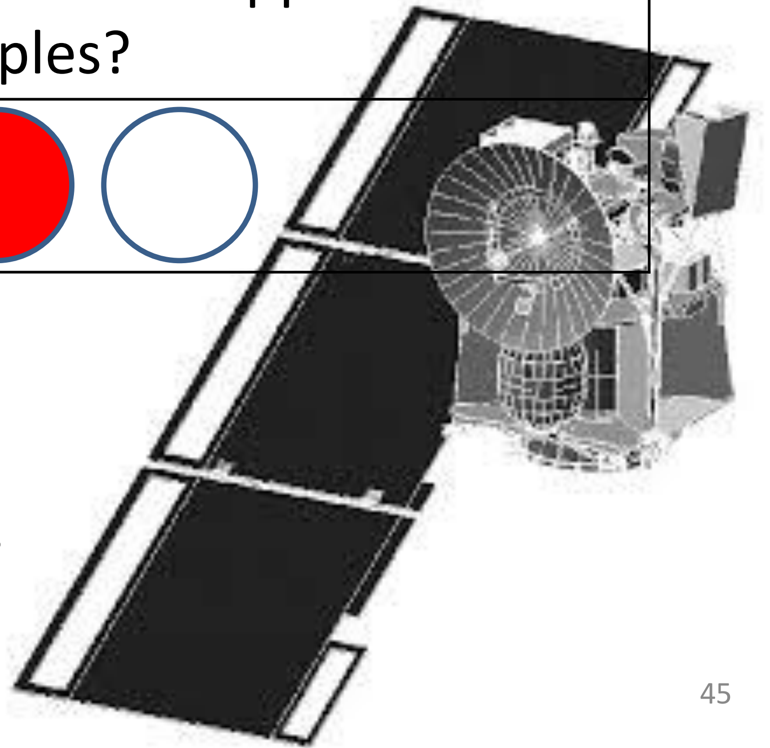
2x3 + 4x? = 6x5
? = (6x5 – 2x3)/(6-2)
? = Δ$/Δkg



Adding Fractions without Units creates a Fraction Paradox

The Teacher	The Students
What is 1/2 + 2/3?	Well, 1/2 + 2/3 = (1+2)/(2+3) = 3/5
No! 1/2 + 2/3 = 3/6 + 4/6 = 7/6	But 1 red of 2 apples + 2 red of 3 apples is 1+2 red of 2+3 apples, i.e., 3 red of 5 apples! How can it be 7 red out of 6 apples?
Inside this classroom 1/2 + 2/3 **IS** 7/6, always!	

Again we see that fractions are not numbers, but operators, needing numbers to become numbers. Adding operators without units, may fold outside the 'no-unit-math' greenhouse.
Mixing English and metric units made NASA’s Mars Climate Orbiter fail in 1999.

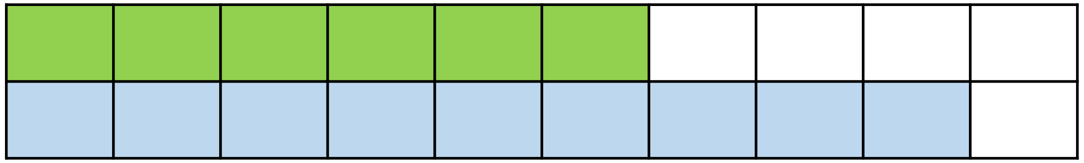


Adding Numbers with Like Units, 6+9

Inside the ‘no-unit-math’ paradigm, numbers add **serial** next-to on the number line. We find the result by counting on from 6 or 9.

Outside, in the ‘unit-math’ paradigm, numbers add **parallel** on-top.

We see that $T = 6 + 9 = 2B3\ 6s = 2B\text{-}3\ 9s = 2B\text{-}5$ tens $= 1B5$ tens $= 15$

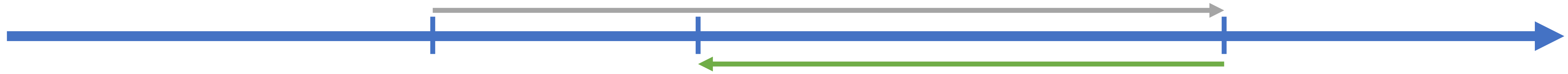


Added directly as less-numbers:

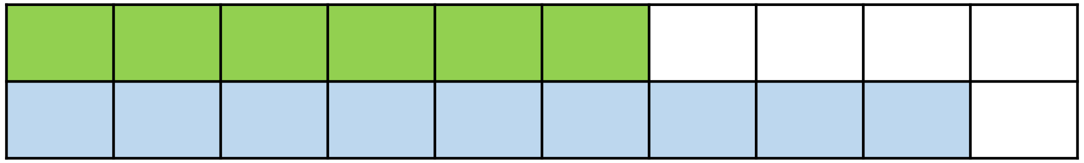
$T = 6 + 9 = B\text{-}4 + B\text{-}1 = 2B\text{-}5 = 15$

Subtracting Numbers with Like Units, 9-6

Inside the ‘no-unit-math’ paradigm, numbers subtract **serial** next-to on the number line. We find the result by counting backwards.



Outside, in the ‘unit-math’ paradigm, numbers subtract **parallel** on-top.
We see that $T = 9 – 6 = 3$; and that $T = 6 – 9 =$ less3, since 6 = 9less3



Subtracted directly as less-numbers:

$T = 9 – 6 = B\text{-}1 – (B\text{-}4) = 0 \text{ -}1 \text{ - -} 4 = \text{-}1 + 4 = 3$,

$T = 6 – 9 = B\text{-}4 – (B\text{-}1) = 0 \text{ -}4 \text{ - -} 1 = \text{-}4 + 1 = \text{-}3$, both showing that - - is +

MATHeCADEMY.net : Math as MANYmath - a Natural Science about MANY

Adding or Subtracting Unspecified Numbers
Only add like units, so how to add $T = 4ab^2 + 6abc$?

With 2x3 as 2 3s, here units come from folding (factoring):

$T = 4ab^2 + 6abc = T1 + T2$

$= 2 \times 2 \times a \times b \times b \quad + \quad 2 \times 3 \times a \times b \times c$

$= 2 \times b \times (2 \times a \times b) + \quad 3 \times c \times (2 \times a \times b)$

$= (2b+3c) \times \mathbf{2ab}$

$= 2b+3c$ $\mathbf{2ab}$s

a factor-filter

T1	2	2	*a*	*b*	*b*
T2	2	3	*a*	*b*	*c*
unit	**2**		***a***	***b***	
T1 left		2			*b*
T2 left		3			*c*

Change by Adding, and by Multiplying

- Change by adding gives a <u>linear change formula</u>:

Adding 3$/day to 2$ gives a total of $T = 2 + 3 \times n$ after n days.
The general formula, $\underline{T = b + a \times n}$, is also called change by adding.

- Change by multiplying gives an <u>exponential change formula</u>:

Adding 3%/day to 2$ gives a total of $T = 2 \times 103\%^n$ after n days since adding 3% means multiplying with 103%.
The formula, $\underline{T = b \times a^n = b \times (1+r)^n}$, is also called change by multiplying,

- Combining the two gives a simple saving-formula, $\underline{A/a = R/r}$; in an installment plan racing with a Debt-formula, $D = Do \times (1+r)^n$.

(a and r is the per-day $ and % input; A and R is the final $ and % output, where $1+R = (1+r)^n$)
Splitting up 100% in n pieces gives the Euler-number $e = (1+1/n)^n$ for n big.

Reversing Change by Adding and Multiplying

Reversing change by adding gives one equation:

- <u>100 = 20 + 5 x *u*</u>, easily solved by splitting, and recounting:

100 = (100–20)+20, so *u*x5 = 100–20 = 80 = (80/5)x5, so *u* = 80/5 = 16.

Reversing change by multiplying gives two equations:

- In <u>20 = *u*^5</u>, 5 factors *u* give 20, predicted by a <u>factor-finding root</u>,

$u = \sqrt[5]{20} = 1.82$.

- In <u>20 = 5^*u*</u>, *u* factors 5 give 20, predicted by a <u>factor-counting logarithm</u>, $u = \log_5(20) = 1.86$.

There are 4 ways to Unite or Split a Total into Unit- and Per-numbers

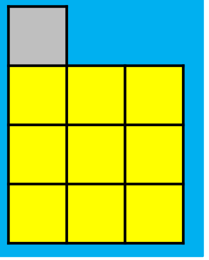
A number-formula (polynomial) T = 345 = 3**BB**4**B**5 = 3x**B**^2 + 4x**B** + 5 shows the 4 ways to unite: +, x, ^, next-to block-addition (integration). Addition and multiplication unite changing and constant unit-numbers. Integration and power unite changing and constant per-numbers. We might call this beautiful simplicity the 'Algebra Square' since in Arabic, algebra means to reunite. ● The 4 uniting operations each has a reverse splitting operation: Addition has subtraction (–), and multiplication has division (/). Power has factor-finding (root, √) and factor-counting (logarithm, log). Integration has per-number finding (differentiation dT/dn = T'). Reversing operations is solving equations, done by moving to **opposite side** with **opposite sign**.

Operations unite / split into	**Changing**	**Constant**
Unit-numbers *$, kg, m, s*	***T = a + n*** *T – a = n*	***T = a x n*** *T/n = a*
Per-numbers *$/kg, m/s, m/(100m) = %*	***T = ∫ a dn*** d*T/*dn = *a*	***T = a^n*** $\log_a T = n$, $\sqrt[n]{T} = a$

8 Competencies Inside the No-Unit Greenhouse only 2 Competences Outside

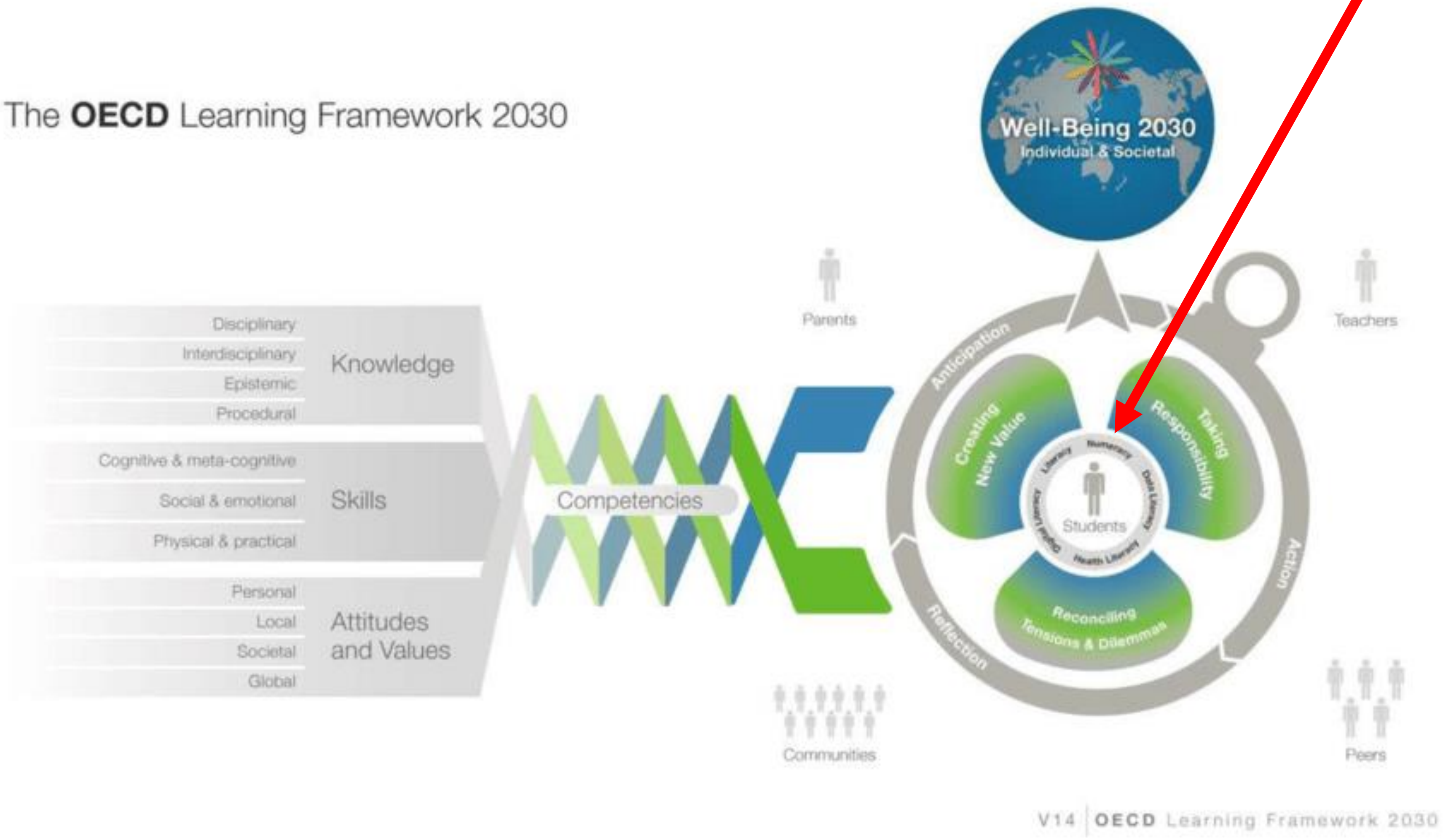
OUTSIDE

To master MANY, first you COUNT then you ADD



To do: Findthe 8 competencies inside the NoUnit Greenhouse. Why are they not needed outside the Greenhouse?

OECD ‘s View is in its Learning Framework 2030 Instead of Mathematics, it points to Numeracy



Concepts have Different Definitions in the 'no-unit-Math' and the 'unit-Math' Paradigm

	Inside 'no-unit-Math'	**Outside the 'no-unit' Greenhouse**
Goal	To master, first Math, later Many	To master first Many, later Math
Basis	Sets, expressions, and functions	Many, counted and added in full sentences
Science	Pure science	Natural science about Many
Self-reference	Self-reference is meaningful	Self-reference is meaningless
Validity	Addition and multiplication are valid	Multiplication is always valid, addition only sometimes
Education	Mediate institutionalized essence	Exploring existing things and actions
Theorist	Vygotsky	Piaget
Competence	8 needed to master Math	2 needed to master Many: Count and Add
Accessible	To few only	To all
Modeling	All models are fiction	Fact and fiction as in the word-language

Concepts have Different Definitions Inside and Outside the ‘no-unit-Math’ Greenhouse II

	Inside ‘no-unit-Math’	Outside the ‘no-unit’ Greenhouse
Digits	Symbols as letters	Icons with as many sticks as they represent
Many digits	One number obeying a place value system	Many numbers numbering unbundled, bundles, bundle-of-bundles, etc.
Numbers	Line-numbers with lengths, without units	Block-numbers with areas, and with units
Types	Cardinal and ordinal numbers	Space and time numbers
Divide, 8/2	Equal sharing, 8 equally shared by 2	Bundle-counting, 8 counted in 2s
Multiply, 6x7	6x7 = 42 = 4.2 tens, recounted in tens,	6x7 = 6 7s, a block that may or not recount in tens
Subtract, 7–2	Take away singles, 7 – 2 = 5	Pull away a block to look for unbundled singles 7 – 2x3 = 1. So, T = 7 = 2*B*1 3s
Add	Unite line-segments 2+3 = 5	Unite areas on-top; or next-to, integral calculus 2 3s + 4 5s = ? 5s; or 2 3s + 4 5s = ? 8s

Concepts have Different Definitions Inside and Outside the 'no-unit-Math' Greenhouse III

	Inside 'no-unit-Math'	Outside the 'no-unit' Greenhouse
PerNumber	Not recognized	Recounting in two units
Fraction	A rational number to add without units	A per-number, both operators needing a number to become a number; so they add by areas, integral calculus
Geometry	Plane, coordinate, trigonometry	Trigonometry, coordinate, plane
Equations	Equivalence relation, solved by using abstract algebraic group theory	Reversed calculation, solved by moving to opposite side with opposite sign
Function	A subset of a set-product where first-comp. identity implies second-component identity	A number-language sentence or formula with an subject, and a predicate or calculation
Graph	A subset in a set-product	Running numbers in a plane
Linear function	A homomorphism	Constant change per-number (gradient, slope)
Differentiable	Meeting an epsilon-delta condition	Locally linear, with a locally constant change per-number
Continuous	Meeting an epsilon-delta condition	Locally constant, as piecewise, only epsilon <-> delta

Concepts have Different Definitions Inside and Outside the ‘no-unit-Math’ Greenhouse IV

	Inside ‘no-unit-Math’	Outside the Greenhouse
Root, log	Irrational numbers	Root is a factor-finder, log is a factor-counter
PreCalculus	Linear and exponential functions	Constant change by adding & multiplying
Statistics	Stochastics and random variables	Unpredictable change predicted by mean & median
Quadratics	Degree 2 polynomials to factorize	Constant changing change and curvature
Calculus	Derivatives and anti-derivatives	Adding and finding locally constant per-numbers
Differential	Derivatives as limits, first	Change predicted by locally constant per-numbers, next
Integral	Finding anti-derivatives, next	Adding locally constant per-numbers by areas, first
Differential equations	Only few have solutions Numerical methods should be avoided	Change formulas, numerically solved by a computer adding the formula’s changes to a total change
Vectors	Vector space: the top algebraic structure	A quick way to do 2D and 3D coordinate geometry

Solving Equations by Recounting, we may Bracket Group Theory from Abstract Algebra

Unit-Math

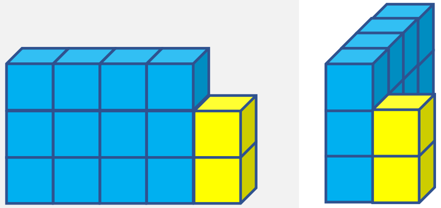
2 x *u* = 8 = (8/2) x 2	Solved by re-counting 8 in 2s
u = 8/2 = 4	Move: **O**pposite **S**ide with **O**ppo**S**ite **S**ign

No-unit-Math (Don't test, but DO remember the bi-implication arrows)

	2 x *u* = 8	Multiplication has 1 as its **neutral element**, and 2 has ½ as its **inverse element**
	(2 x *u*) x (½) = 8 x (½)	Multiplying 2's inverse element ½ to both number-names
⇕	(*u* x 2) x (½) = 4	Applying the **commutative** law to u x 2; 4 is the short number-name for 8 x ½
⇕	*u* x (2 x (½)) = 4	Applying the **associative** law
⇕	*u* x 1 = 4	Applying the definition of an inverse element
⇕	*u* = 4	Applying the definition of a neutral element. *With arrows a test is not needed.*

⇕ Solution set: S = { *u* in R I 2 x *u* = 8} = {4}

The Child's own Unit-Math Curriculum



1) Digits are (sloppy) icons, with as many sticks as they represent.

2) Totals are counted by bundling, giving geometrical multi-blocks, and algebraic bundle-numbers with units, when turned to hide the units behind.

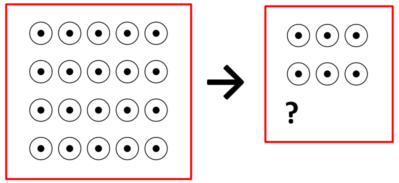
3) Operations are icons showing the counting steps. First PUSH & LIFT bundles. Then PULL stacks to find the unbundled ones.

4) The operation order is division first, then multiplication, then subtraction. Addition next-to & on-top comes later after totals are counted & re-counted.

5) Counting & re-counting is big fun, when predicted by a calculator with the recount formula: **T = (T/B)xB** *(The total T contains T/B times Bs)*

Question: T = 4 ***5s*** *= ?* ***3s*** ● *Answer: T = 4* ***5s*** *= 6B2* ***3s*** ● *Prediction:*

4x5/3	6.some
4x5 – 6x3	2



Differences to the No-Unit-Math Curriculum I

A no-unit-math curriculum operates on specified and unspecified numbers.

- Digits are given directly as symbols, without letting children discover digits as icons with as many strokes or sticks as they represent.
- Numbers are one-dimensional line-numbers with digits respecting a place value system, without letting children discover the thrill of two-dimensional bundling and stacking, counting both <u>singles</u> and <u>bundles</u> and <u>bundles-of-bundles</u> etc., and also including the units.
- Seldom, if ever, 0 is included in the counting sequence as ‘0Bundle1, 0B2, 0B3’, in order to show the importance of bundles as units.

Differences to the No-Unit-Math Curriculum II

- Never children are told that eleven and twelve comes from the Vikings, counting ‘(ten and) 1 left’, ‘(ten and) 2 left’.
- Never children use full number-language sentences, T = 2 **5s**, including both a subject & a verb & a predicate with a unit.
- Seldom children are asked to describe numbers after ten as 1**B**4 **tens** or 1**ten**4 or 1.4 **tens** with a unit and with a decimal point separating bundles and unbundled singles.
- Seldom 17 is recounted as 2**B**-3 tens, or 2.-3 **tens**. Nor is 24 recounted as 1**B**14 **tens**, or 3**B**-6 **tens**.

Differences to the No-Unit-Math Curriculum III

- The tradition never respects the natural order of operations. Instead it turns the order around by giving addition without units priority over subtraction & multiplication & division.
- In short, children never experience the enchantment of counting and re-counting Many before being forced to add on-top only, thus neglecting next-to addition.

Re-enchanting Many therefore is a goal for the unit-math curriculum in Mastery of Many. So, it respects and develops the children's existing mastery of Many, by counting before adding, and by using flexible bundle-numbers with units.

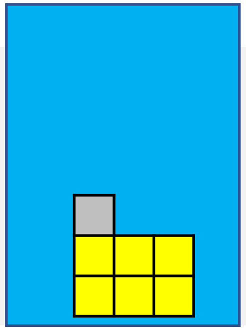
Summary: What is the Core Difference?

		No-unit-math	**Unit-math**
Digits	4	Symbol	Icon with four strokes
Numbers	456	One number	Three numberings, 4**BB** 5**B** 6
Division	8/2	8 split in 2	8 counted in **2s**
Multiplication	6 x 7	42	6 **7s** or 4**B**2 **tens**
Addition	2+3	2+3 = 5	2 **4s** + 3 **5s** = 2**B**3 **9s** 2 **4s** + 3 **5s** = 4**B**1 **5s**
Equations	$3 \times u = 12$	Neutralize $(3 \times u) \times 1/3 = 12 \times 1/3$ $(u \times 3) \times 1/3 = 4$ $u \times (3 \times 1/3) = 4$ $u \times 1 = 4$ $u = 4$	Recount, or simply move to Opposite side with opposite sign $u \times 3 = 12 = (12/3) \times 3$ $u = 12/3 = 4$
Fractions	2/3	Numbers 1/2 + 2/3 **IS** 7/6 always	Per-numbers, i.e., operators, needing numbers to become numbers: 1/2 of 2 + 2/3 of 3 **IS** 3/5 of 5

Different Answers from ESSENCE- & EXISTENCE-based Math Education

	ESSENCE-set-based Math	EXISTENCE-many-based Math
Philo sophy	Line-numbers Addition first, Division last Fractions as numbers Geometry before Trigonometry Late Calculus, Differentiation before Integration	Area-numbers (flexible bundle-numbers with units) Division first, Addition last: push, lift, pull, add Fractions as Per-Numbers, operators - not numbers Trigonometry before Geometry Primary, middle and high school Calc. , Int. before Diff.
Socio logy	Serve institutional knowledge Use thick textbooks Accept goal displacement: Mastery of Mathematics is the goal. Later, others may apply mathematics	Facilitate individual knowledge construction Use short compendia with models Reject goal displacement: Mastery of Many is the goal. Later mastery of essence-based mathematics may be tried, if needed
Psycho logy	Mediate knowledge about expressions. High dropout rate accepted, and explained by theory-based research	Communicate about Many in full sentences with an outside subject, and an inside chosen predicate. Low dropout rate with grounded design research

Conclusion I



The outside ‘unit-math’ paradigm provides the same mathematics as the inside ‘no-unit-math’ paradigm, only in a different order. And the ‘unit-math’ paradigm avoids the inside paradigm’s ‘mathematism’ with falsifiable addition-claims.

So, to become a full science, mathematics should leave its 1 plus 2 is 3 ‘no-unit-math’ greenhouse, and accept that, of course, numbers cannot add without units.

It should teach the outside ‘counting-before-adding’ ‘unit-math’ paradigm where

- Numbers and operations are icons linked directly to existing things and actions
- Totals are bundle-counted to create flexible bundle-numbers that with overloads and underloads include decimals, negatives and fractions for the unbundled singles
- Recounting to change unit creates a recount formula, $T = (T/B)xB$, that solves proportionality equations all over mathematics and STEM
- Adding next-to and on-top leads directly to the math core: calculus and linearity

Conclusion II

- Bundle-counting means pushing away bundles by division, lifting bundles into a block by multiplication, and pulling away the block to find unbundled singles that may be counted as decimals, fractions or negatives
- Recounting a total in another unit creates a recount formula, $T = (T/B)xB$, predicting recounting results and proportionality all over STEM
- Recounting from tens to icons gives multiplication-equations solved by moving known numbers to opposite side with opposite calculation sign
- Recounting from icons to tens roots multiplication tables, and algebra
- Recounting in physical units creates per-numbers, becoming fractions with like units; both are not numbers, but operators needing numbers to become numbers
- Recounting the sides in a block halved by its diagonal creates trigonometry and pi

Conclusion III

- Adding blocks next-to means integrating areas thus becoming integral calculus, subtracting blocks then becoming differential calculus
- Adding blocks on-top, the units become like through proportional recounting
- Per-numbers add by their areas under the per-number graph
- A total may change by adding, or by multiplying. Reversing change by multiplying gives the factor-finding root, and the factor-counting logarithm
- The algebra-square shows the 4 ways to unite parts into a total, and the 5 ways to split a total into parts
- In philosophy, the ‘unit-math’ paradigm resonates with existentialism (where existence precedes essence); and with Piaget psychology (where learning takes place through guided meetings with the subjects, and not by having institutionalized essence mediated)

A Communicative Turn in Language Education

Before 1970, foreign language was taught as an example of its grammar.
Then a reaction came with **The Communicative Turn.**
Halliday: “A functional approach to language means investigating how language is used: trying to find out what are the purposes that language serves for us.”
Likewise, **Widdowson** adopts a “communicative approach to the teaching of language” so more students learn a language by communicating about outside things and actions.

Also mathematics should have its Communicative Turn!

So, language before meta-language, please.

	WORD language	NUMBER language
	Meta-	*Language*
Inside Language	This is a chair	T = 3 4s
Outside world		

Mother language

Mathematics

A Final Question

SUSTAINABLE DEVELOPMENT GOALS

4 QUALITY EDUCATION

Should Ethical Quality Education force children inside a 'no-unit-math' greenhouse that slowly strangles their innate number-language by using line-numbers to learn no-unit addition that folds outside?

Where children's innate mastery of Many just waits to be developed by flexible bundle-numbers available at their fingertips.

BundleCount Before you Add Booklet
Free to Download

Math Dislike CURED

by Flexible BundleNumbers

My Many Math Tears will not Stay – if I Bundle the Stray Away

BundleCOUNT before you ADD

T = 5 = ~~II~~ I I I = 1B3 2s

T = 5 = ~~II~~ ~~II~~ I = 2B1 2s

T = 5 = ~~II~~ ~~II~~ ~~II~~ = 3B-1 2s

3 ways to BundleCount: Overload, Normal, Underload

ReCount 47 in tens: T = 47 = 4B7 = 3B17 = 5B-3 tens

NO, 4x7 is not 28, it is 4 7s = 2B8 = 1B18 = 3B-2 tens
NO, 30/6 is not 30 split by 6, but 3 tens recounted in 6s

BundleWriting tells InSide Bundles from OutSide Singles

• 65 + 27	=	6B5 + 2B7 = 8B12 = 9B2 =	92
• 65 – 27	=	6B5 – 2B7 = 4B-2 = 3B8 =	38
• 7x 48	=	7x 4B8 = 28B56 = 33B6 =	336
• 336 /7	=	33B6 /7 = 28B56 /7 = 4B8 =	48
• 336 /7	=	33B6 /7 = 35B-14 /7 = 5B-2 =	48

MatheMatics as ManyMath - a Natural Science about Many

Makes Math Potentials Blossom in Children, Adults & Migrants

Allan.Tarp@MATHeCADEMY.net

Contents

03. BundleCount in Icons

Job		Do	Calculator
9 in 5s	Line Count Bundle Stack B-write Answer	T = I I I I I I I I I 1, 2, 3, 4, B, 1B1, 1B2, 1B3, **1B4** T = ~~IIIII~~ I I I I T = 1B4 5s = 0B9 5s = 2B-1 5s T = 9 = 1.4 5s	9/5 1.some 9 – 1*5 4 *9 – 0*5 9* *9 – 2*5 -1*
9 in 4s	Line Count Bundle B-write Stack Answer	T = I I I I I I I I I 1, 2, 3, B, 1B1, 1B2, 1B3, 2B, **2B1** T = ~~IIII~~ ~~IIII~~ I T = 2B1 4s = 1B5 4s = 3B-3 4s T = 9 = 2.1 4s	9/4 2.some 9 – 2*4 1 *9 – 1*4 5* *9 – 3*4 -3*
9 in 3s	Line Count Bundle B-write Stack Answer		9/ 9 –
8 in 4s	Line Count Bundle B-write Stack Answer		8 8
8 in 3s	Line Count Bundle B-write Stack Answer		8 8

MATHeCADEMY.net 5

1day free Zoom Seminar:
To Cure Math Dislike, BundleCount before you Add

Action Learning based on the Child’s own Bundle-Numbers with Units

09-11. Listen and Discuss the PowerPointPresentation
Flexible Bundle-Numbers Develops Children’s Innate Mastery of Many, from MetaMatism to ManyMath

- **MetaMatism** = MetaMatics + MatheMatism
- **MetaMatics** presents a concept TopDown as an example, instead of BottomUp as an abstraction
- **MatheMatism** is true inside, but seldom outside the ‘no-unit-math’ greenhouse
- **ManyMath**, a natural science about Many, mastering Many by BundleCounting, ReCounting & Adding NextTo and OnTop.

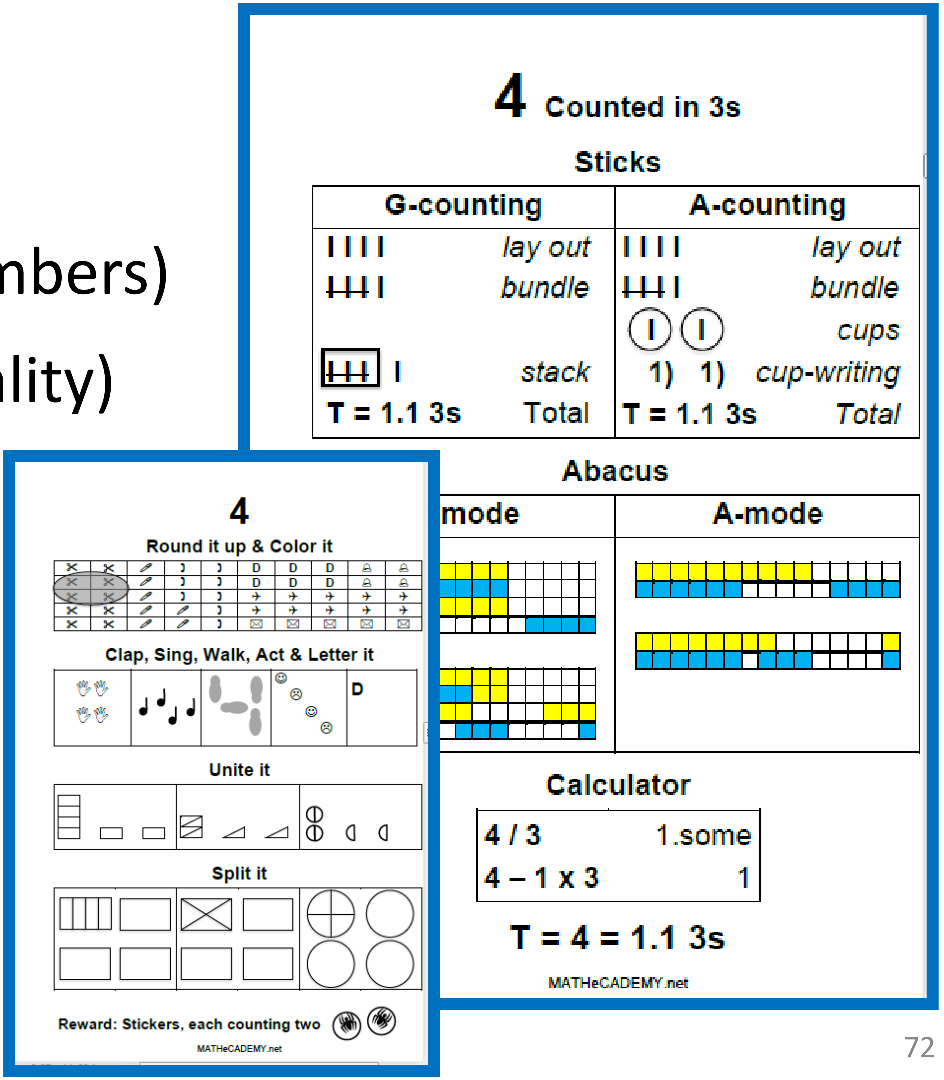
11-13. Zoom Conference. Lunch.

13-15. Workshop in the BundleCount before you Add booklet to experience proportionality & calculus & solving equations as golden LearningOpportunities in BundleCounting & NextTo Addition.

15-16. Coffee. Zoom Conference.

8 MicroCurricula for Action Learning & Research

C1. Create Icons
C2. Count in Icons
C3. ReCount in the Same Icon (Negative Numbers)
C4. ReCount in a Different Icon (Proportionality)
A1. Add OnTop (Proportionality)
A2. Add NextTo (Integrate)
A3. Reverse Adding OnTop (Solve Equations)
A4. Reverse Adding NextTo (Differentiate)



Question Guided Teacher Education

MATHeCADEMY.net

Teaches Teachers to Teach MatheMatics as ManyMath, a Natural Science about MANY.

To learn Math, Count & Add MANY, using the CATS method:

Count & Add in Time & Space

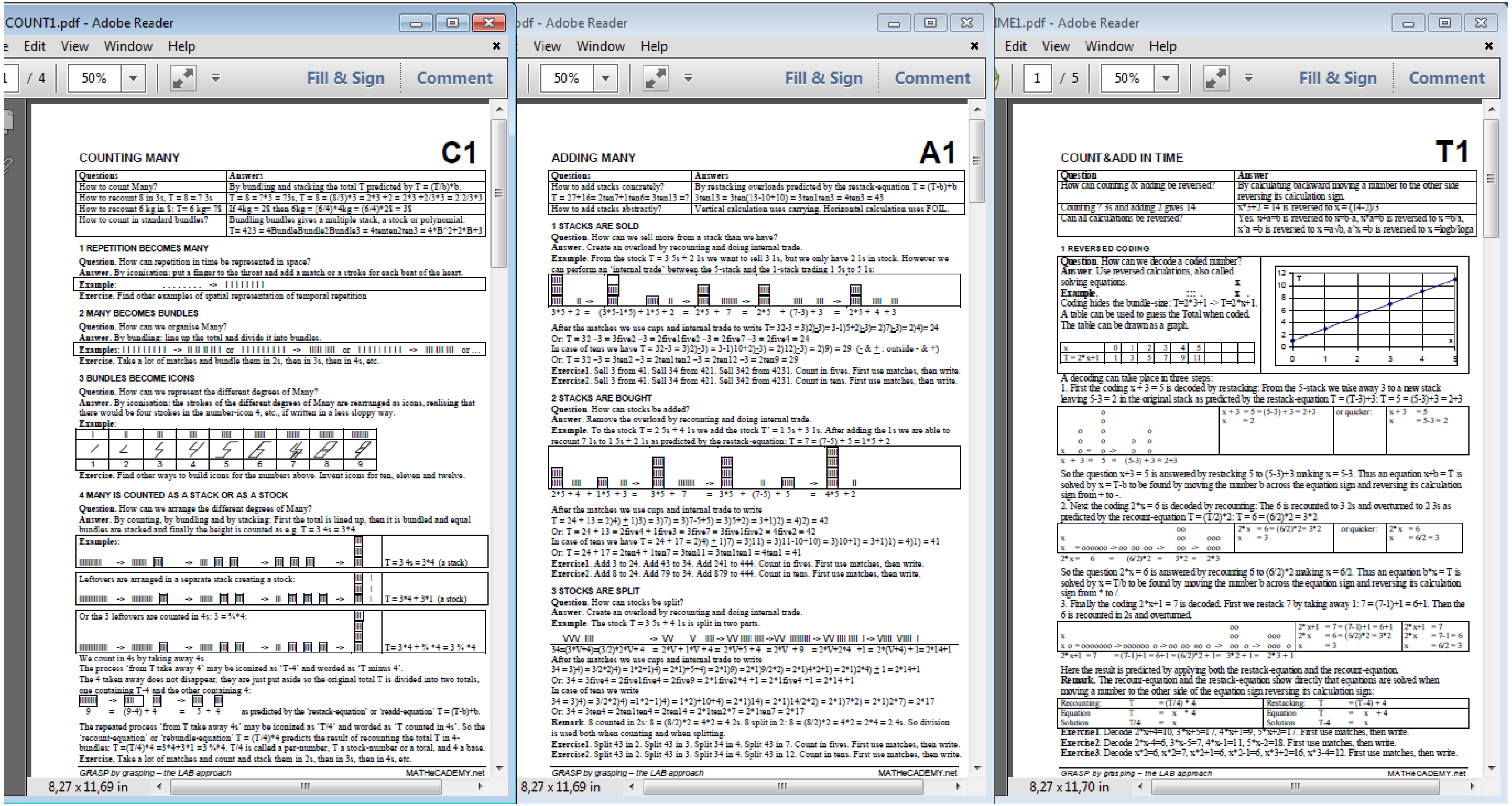
- Primary: C1 & A1 & T1 & S1
- Secondary: C2 & A2 & T2 & S2

MATHeCADEMY.net
a VIRUSeCADEMY:
ask Many, not the Instructor

SUMMARY

	QUESTIONS	ANSWERS
C1 COUNT	How to count Many? How to recount 8 in 3s: T= 8 = ? 3s How to recount 6kg in $: T=6kg=?$ How to count in standard bundles?	By bundling and stacking the total T predicted by T = (T/b)*b T= 8= ?*3= ?3s, T= 8=(8/3)*3 = 2*3 +2 = 2*3 +2/3*3 = 2 2/3*3 If 4kg = 2$ then 6kg = (6/4)*4kg = (6/4)*2$ = 3$ Bundling bundles gives a multiple stack, a stock or polynomial: T = 423 = 4BundleBundle+2Bundle+3 = 4tenten2ten3 = 4*B^2+2*B+3
C2 COUNT	How can we count possibilities? How can we predict unpredictable numbers?	By using the numbers in Pascal's triangle We 'post-dict' that the average number is 8.2 with the deviation 2.3. We 'pre-dict' that the next number, with 95% probability, will fall in the confidence interval 8.2 ± 4.6 (average ± 2*deviation)
A1 ADD	How to add stacks concretely? T=27+16= 2ten7+1ten6= 3ten13=? How to add stacks abstractly?	By restacking overloads predicted by the restack-equation T= (T-b)+b T = 27+16 = 2 ten 7+1 ten 6 = 3 ten 13 = 3 ten 1 ten 3 = 4 ten 3 = 43 Vertical calculation uses carrying. Horizontal calculation uses FOIL
A2 ADD	What is a prime number? What is a per-number? How to add per-numbers?	Fold-numbers can be folded: 10=2fold5. Prime-numbers cannot: 5=1fold5 Per-numbers occur when counting, when pricing and when splitting. The $/day-number a is multiplied with the day-number b before added to the total $-number T: T2 = T1 + a*b
T1 TIME	How can counting & adding be reversed ? Counting ? 3s and adding 2 gave 14. Can all calculations be reversed?	By calculating backward, i.e. by moving a number to the other side of the equation sign and reversing its calculation sign. x*3+2=14 is reversed to x = (14-2)/3 Yes. x+a=b is reversed to x=b-a, x*a=b is reversed to x=b/a, x^a=b is reversed to x=a√b, a^x=b is reversed to x=logb/loga
T2 TIME	How to predict the terminal number when the change is constant? How to predict the terminal number when the change is variable, but predictable?	By using constant change-equations: If Ko = 30 and ΔK/n = a = 2, then K7 = Ko+a*n = 30+2*7 = 44 If Ko = 30 and ΔK/K = r = 2%, then K7= Ko*(1+r)^n= 30*1.02^7= 34.46 By solving a variable change-equation: If Ko = 30 and dK/dx = K', then ΔK = K-Ko = ∫K'dx
S1 SPACE	How to count plane and spatial properties of stacks and boxes and round objects?	By using a ruler, a protractor and a triangular shape. By the 3 Greek Pythagoras', mini, midi & maxi By the 3 Arabic recount-equations: sinA=a/c, cosA=b/c, tanA=a/b
S2 SPACE	How to predict the position of points and lines? How to use the new calculation technology?	By using a coordinate-system: If Po(x,y) = (3,4) and if Δy/Δx = 2, then P1(8,y) = P1(x+Δx,y+Δy) = P1((8-3)+3,4+2*(8-3)) = (8,14) Computers can calculate a set of numbers (vectors) and a set of vectors (matrices)
QL	What is quantitative literature? Does quantitative literature also have the 3 different genres: fact, fiction and fiddle?	Quantitative literature tells about Many in time and space The word and the number language share genres: Fact is a since-so calculation or a room-calculation Fiction is an if-then calculation or a rate-calculation Fiddle is a so-what calculation or a risk-calculation

Teacher Training in ManyMath, using CATS
Count & Add in Time & Space

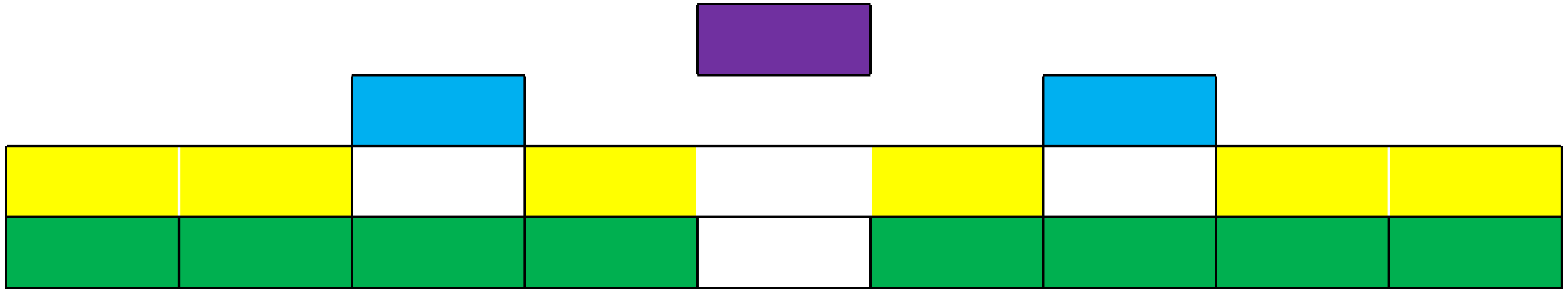


PYRAMIDeDUCATION

In PYRAMIDeDUCATION a group of 8 teachers are organized in 2 teams of 4 choosing 2 instructors and 3 pairs by turn.

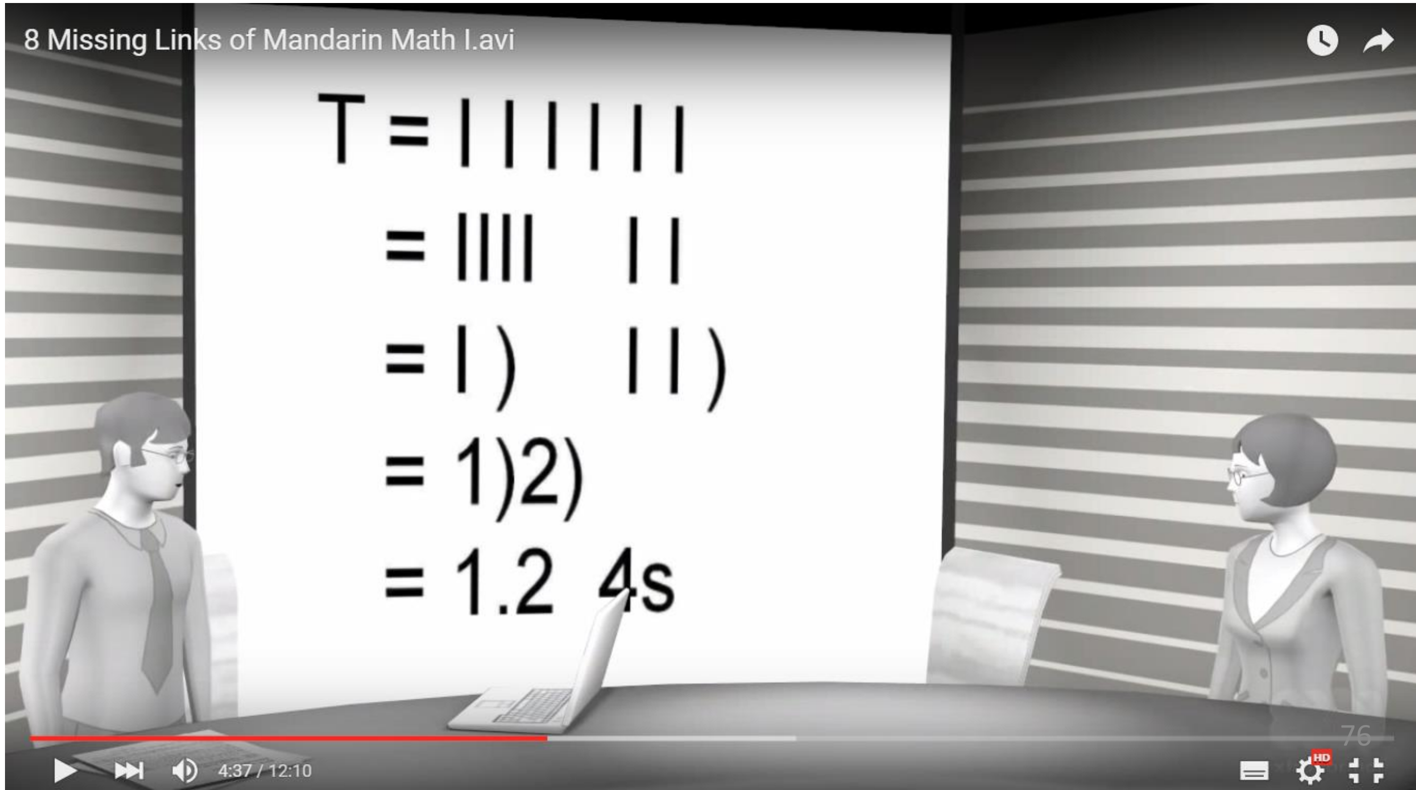
- Each pair works together to solve Count&Add problems.
- The coach assists the instructors when instructing their team and when correcting their Count&Add assignments.
- Each teacher pays by coaching a new group of 8 teachers.

1 Coach
2 Instructors
3 Pairs
2 Teams

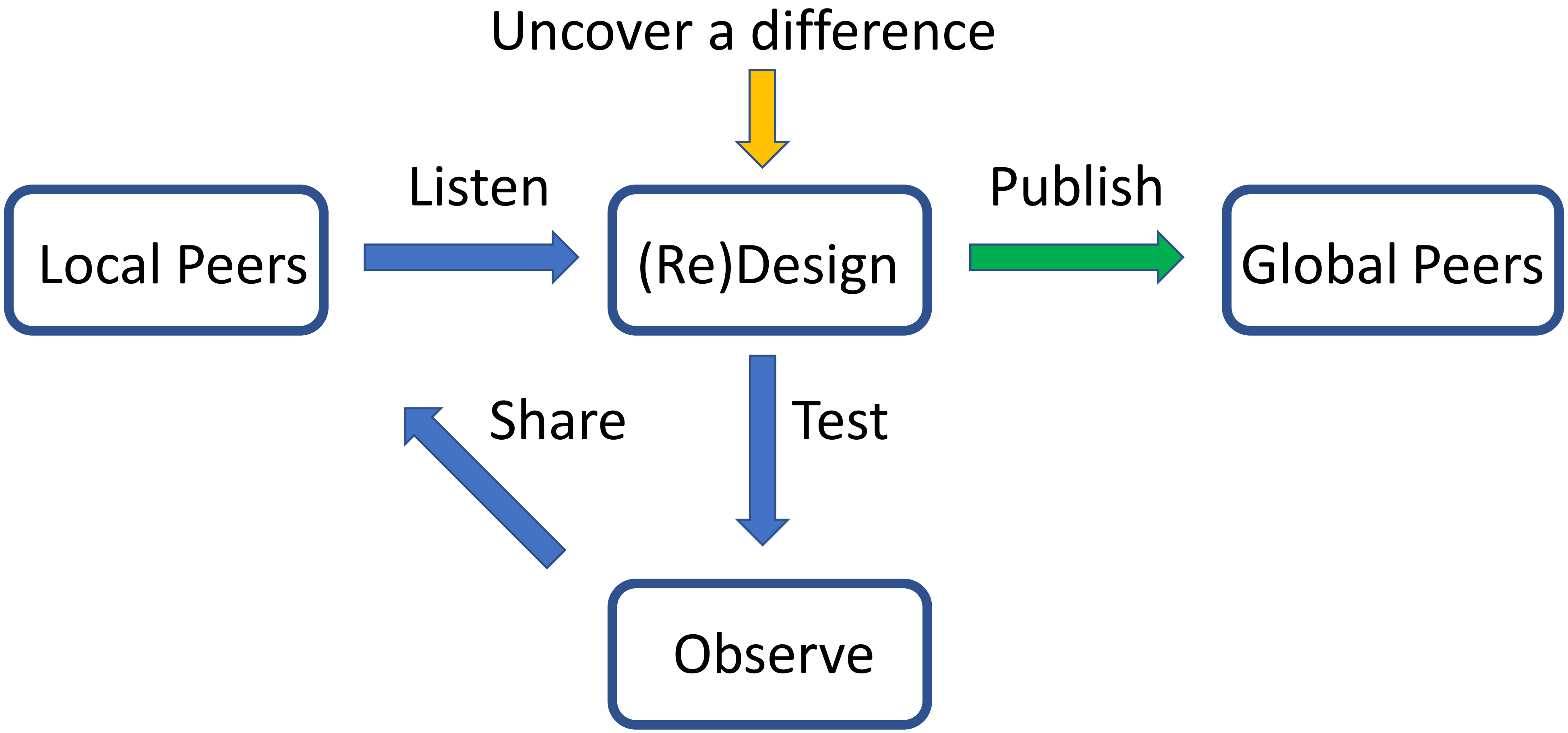


Watch MrAlTarp YouTube Videos

- Postmodern Mathematics Debate
- ReCounting removes Math Dislike
- IconCounting & NextTo-Addition
- PreSchool Mathematics
- Fractions
- PreCalculus
- Calculus
- Mandarin Mathematics
- World History



DifferenceResearch finds Differences making a Difference
Action Learning or Action Research



Recounting looks like Dienes MultiBase Blocks

- "Dienes' name is synonymous with the Multi-base blocks (also known as Dienes blocks) which he invented for the teaching of place value.
- He also is the inventor of Algebraic materials and logic blocks, which sowed the seeds of contemporary uses of manipulative materials in mathematics instruction.
- Dienes' place is unique in the field of mathematics education because of his theories on how mathematical structures can be taught from the early grades onwards using multiple embodiments through manipulatives, games, stories and dance."

(http://www.zoltandienes.com/about/)

Dienes on Numbers and MultiBase Blocks

"The position of the written digits in a written number tells us whether they are counting singles or tens or hundreds or higher powers. This is why our system of numbering, introduced in the middle ages by Arabs, is called the place value system. My contention has been, that in order to fully understand how the system works, we have to understand the concept of power. (..) In school, when young children learn how to write numbers, they use the base ten exclusively and they only use the exponents zero and one (namely denoting units and tens) , since for some time they do not go beyond two digit numbers. So neither the base nor the exponent are varied, and it is a small wonder that children have trouble in understanding the place value convention. (..) Educators today use the "multibase blocks", but most of them only use the base ten, yet they call the set "multibase". These educators miss the point of the material entirely."

(What is a base?, http://www.zoltandienes.com/academic-articles/)

UnitMath turns MetaMath upside down

Dienes teaches the 1D place value system with 3D blocks to illustrate the importance of the power concept.

- Unit-Math teaches decimal numbers with units and stays with 2D to illustrate the importance of bundle-units and of adding areas.

Dienes wants to bring examples of abstractions to the classroom

- Unit-Math wants to build abstractions from concrete examples

Dienes teaches top-down 'MetaMath' derived from the concept Set

- Unit-Math is a bottom-up natural science about the physical fact Many; and sees Set as a meaningless concept because of Russell's set-paradox: The set of sets not belonging to itself, belong to itself if it does not.

Theoretical Background

Tarp, A. (2018). Mastering Many by counting and recounting before adding on-top and next-to. *Journal of Math Education, March 2018, 11*(1), 103-117.

Tarp, A. (2020). De-modeling numbers, operations and equations: from inside-inside to outside-inside understanding. *Ho Chi Minh City Univ. of Education Journal of Science 17*(3), 453-466.

Tarp, A. (2022). Bundles bring back brains from unethical special education. *Journal of Math Education,* submitted.

Flexible Bundle-Numbers Develop the Child's Innate Mastery of Many

A Paradigm Shift from LineNumbers without to BundleNumbers with Units

This presentation and its PDF may be found here:

http://mathecademy.net/Korean-math-ed-2021/

Q&A: Questions and answers are shown here also.

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