Calculus Conceptually Changed: From Deified to Reified

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Extended Abstract

Statement of the Theoretical Problem

Created to add per-numbers by their areas, integral calculus normally is the last subject in high school, and only taught to a minority of students. But, since most STEM-formulas express proportionality by means of per-numbers, the question is if integral calculus may be taught earlier and to all students. Difference research (Tarp, 2018) may give an answer.

An Account of the Theoretical Proposal Being Made

Reified, integral calculus occurs in grade one when performing next-to addition of bundle-numbers as, e.g., T = 2 3s + 4 5s = ? 8s, leading on to differential calculus as the reverse question: 2 3s + ? 5s = 3 8s, solved by first removing 2 3s from 3 8s, and then counting the rest in 5s, thus letting subtraction precede division, where integral calculus does the opposite by letting multiplication creating areas precede addition.

In middle school adding per-numbers by areas occurs in mixture problems: 3kg at 2$/kg + 5kg at 4$/kg = 8kg at ? $/kg, again with differential calculus coming from the reverse question: 3kg at 2$/kg + 5kg at ? $/kg = 8kg at 4$/kg. Here the per-number graph is piecewise constant c, i.e., there exists a delta-interval so that for all positive epsilons, the distance between y and c is less than epsilon. With like units, per-numbers become fractions thus also added by their areas, and never without units.

In high school adding per-numbers occurs when the distance travelled with a varying per-number P is found as the area under the per-number graph now being locally constant, formalized by interchanging epsilon and delta. Here the area A under the per-number graph P is found by slicing the area thinly. If writing the area strips as differences, addition will make all middle terms disappear and leave only the first and last terms. Alternatively, the last strip represents the change of the area, dA, and may be written as dA = P*dx, thus motivating developing differential calculus to find a formula A with the property that A’ = dA/dx = P. Here, looking at the shadow of an f*g book directly gives the formula (f*g)’/(f*g) = f’/f + g’/g, exemplified, e.g., by (x^2)’ = 2x.

Review of the Relevant Literature

Mathematics education sees its goal as mastering university mathematics, seen as a pure self-supporting science theorizing one-dimensional number sets organised by different operations. And later introducing differentiation and integration as operations on functions seen as set-relations or subsets of set-products where first-component identity implies second-component identity.

As an alternative Kuhnian paradigm, ManyMath sees mathematics as a natural science about the outside fact Many as shown by ‘geometry’ and ‘algebra’ meaning ‘earth-measuring’ and ‘reuniting’ in Greek and Arabic; and developing children’s already existing mastery of Many with two-dimensional bundle-numbers that when added next-to by their areas presents integral calculus before differential, which makes irrelevant the existing calculus literature doing the opposite and neglecting bundle-numbers.

Instead, theoretical guidance comes from seeing mathematics education as an institutionalized goal-directed treatment of human brains, thus being theorized by sociology, philosophy, and psychology. Here however, internal controversies necessitate choices to be made. In sociology, this approach chooses agency over structure by using Bauman, Habermas and Foucault; in philosophy it chooses empiricism over rationalism by using existentialism; and in psychology it chooses nature over culture by choosing Piaget over Vygotsky.

Clarifying the Novel Contribution of this Particular Project

ManyMath education has as goal to master outside Many, where traditional mathematics education has the goal to master inside mathematics so other subjects later may apply it outside.

Accepting and developing the bundle-numbers children create when adapting to Many, ManyMath teach, not numbers, but numbering, using functions from grade one as number-language sentences that, as in the word-language, contain a subject, a verb, and a predicate. The tradition insists on teaching single-numbers, and postpones functions to high school.

In grade one, on-top addition of bundle-numbers leads to proportionality making the units the same, and next-to addition leads to integral calculus by adding areas, and to differential calculus when reversed. In middle school both reappear when adding or subtracting piecewise constant per-numbers by their areas in mixture problems, to be followed in high school by locally constant per-numbers instead, again with integration preceding differentiation. The tradition only teaches on-top addition of single numbers, and teaches fractions as mathematism being added without units. Mixture problems are not illustrated geometrically to show the relationship with calculus, and the concepts of piecewise and local constancy are absent. Finally, differential calculus is taught before integral calculus, and local constancy and linearity is called continuous and differentiable.

Empirical Research that Could Test the Validity of the Theoretical Proposal

Being very costly to change expensive textbooks and long-term teacher education makes testing the validity of Reifying Calculus difficult inside a traditional education, except for where it is stuck, e.g., adding fractions without units. But it may be tested outside: in preschool, special education, home schooling, adult education, migrant or refugee education, or where students choose between different half-year blocks instead of having multi-year compulsory lines forced upon them.

Implication of this Work for Further Theory Development

The MATHeCADEMY.net is designed to provide material for pre- and in-service teacher education using PYRAMIDeDUCATION allowing professional development to take place on the internet in self-controlling groups with eight participants validating predicates by asking the subject itself instead of an instructor. This allows Reifying Calculus to be tested and developed worldwide in small scale design studies ready to be enlarged in countries choosing experiental learning curricula as, e.g., in Vietnam.

Implications for Practice

Institutionalized education systematizes adaption by teaching children about their outside world, and teenagers about their inside talents and potentials. Thus, to develop the students' mastery of Many, outside abstracted ManyMath must replace todays inside derived university mathematics by presenting calculus at all three school levels.

References

Tarp, A. (2013). *Deconstructing Calculus*. https://youtu.be/yNrLk2nYfaY.

Tarp, A. (2018). Mastering Many by counting, re-counting and double-counting before adding on-top and next-to. *Journal of Mathematics Education, 11*(1), 103-117.