From evil to good calculus that adds locally constant per-numbers

Allan Tarp

MATHeCADEMY.net, Denmark; Allan.Tarp@gmail.com

Abstract

All can learn to add locally constant per-numbers to later appreciate epsilon-delta calculus, few can do the opposite. Why? Math says it cannot teach adding locally constant per-numbers since they don't exist: Math does not allow units, and local constancy defines piecewise constancy. Sociology and theology have different answers.

Case

A college wanted to lower the number of failing students at their first-year calculus course. So, a math lab was included where tutors could guide students individually or in groups. One tutor wanted to specialize in guiding struggling students to a better understanding of the epsilon-delta definition of continuity and differentiability. Having unsuccessfully tried this for a period, he asked me to design micro curricula to help him. Also, he wanted the curricula to be theoretically founded.

As a theoretical foundation I chose French poststructuralism rooted in Existentialism defined by Sartre as holding that 'existence precedes essence' (Marino, 2004). In Kierkegaard's version this means that what is created by nature should precede what is created by humans. And in Heidegger's version this means that in sentencing is-sentences, that subject should precede the predicate that will always be an individual choice or a social construction that could be different. Later in France, Derrida (1991) agrees by warning against predicates installing instead of enlightening what they describe. Such words should be deconstructed to better represent the existence they are supposed to reflect. Likewise, Foucault (1989, 1995) warns that scientific disciplines may instead exert power by disciplining not only themselves but also their subjects. As a counter measure, concept archaeology might find the original root and reason for concepts. So, micro curricula can be made by de-modeling (Tarp, 2020) traditional concepts (Bressoud et al., 2016) by returning to their original outside roots.

As to the goal of mathematics education a choice must be made. Does mastery of math precede mastery of Many, or the other way around? Existentialism points to Many-before-math, and Essentialism to math-before-Many.

Existentialism accepts that the Arabic word 'algebra' means to re-unite. So, to unite totals in the 'Algebra Square' (Tarp, 2018), multiplication and addition unite constant and changing unit-numbers, and power and integration unite constant and changing per-numbers, that with like units become fractions or percentages, all being operators needing numbers to become numbers. And to split totals, division and subtraction split into constant and changing unit numbers, and the factor-finding root (or the factor-counting logarithm) and differentiation split into constant and changing per-numbers. So, the outside root of calculus is adding or splitting into locally constant per-numbers.

Three micro curricula

The first micro curriculum introduces the difference between 1D line-numbers without, and 2D bundle-numbers with units. Here, line-number math roots 'mathematism' (Tarp, 2018) claiming that 2 + 1 = 3 always despite the fact that 2weeks + 1day = 15days. In contrast, bundle-numbers see 2x3 as an inside description of an outside stack of 2 3s, so that 2+3x4 gives 14 by necessity since it describes 2 1s + 3 4s. This distinction allows calculus to be rooted in next-to addition uniting areas when answering "2 3s + 4 5s = ? 8s", and its reverse.

The second micro curriculum introduces middle school calculus rooted in mixture problems asking "2kg at 3$/kg + 4kg at 5$/kg = (2+4)kg at ? $/kg", and its reverse. We observe that where the unit-numbers 2kg and 4kg add directly, the per-numbers 3$/kg and 5$/kg must multiply to unit-numbers to add, thus becoming areas. So per-numbers add by their areas under the per-number curve.

The third micro curriculum introduces formal definitions for the three kinds of constancy: global, piecewise, and local. In mixture problems, the per-number is piecewise constant. With falling objects, the per-number, meter/second, changes, but may still be called 'locally constant'.

● y is globally constant c it the distance between the two is less than any positive number epsilon.
● y is piecewise constant c if an interval delta exists such that here the distance between the two is less than any positive number epsilon.

Interchanging epsilon and delta then gives a formal definition for local constancy, or continuity. Which differs from the official definition of local constancy, defining piecewise constancy instead.

● $y = f(x)$ is locally constant if an interval delta exists such that here $f(u) = f(v)$ for all u and v.

This allows three formal definitions for linearity, saying that $y = f(x)$ is globally, piecewise, locally linear if the change per-number $\Delta y/\Delta x$ is globally, piecewise, or locally constant.

Locally constant per-numbers p thus add as many small area-strips $p\mathrm{d}x$ that if written as a difference $\mathrm{d}A$ make all middle numbers disappear leaving only one difference between the end and beginning number. This motivates developing a calculus to solve the differential equation $\mathrm{d}A = p\mathrm{d}x$.

Involving the concept of a limit, this may be de-modeled by looking at the length of a circle, and at a 100% rate added continuously: $\pi = n$ x $\tan(180/n)$, and e^$t = (1+t/n)$^n, for n large.

Testing the micro curricula

The three micro curricula were very successful in the math lab with many students being surprised that the epsilon-delta calculus is so easy once you master adding locally constant per-numbers. So, the tutor suggested using the many-before-math approach in the lectures, but the math department refused arguing that they could not teach adding locally constant per-numbers since they don't exist: Math does not allow units, and local constancy defines piecewise constancy instead.

My sociology friend laughed when hearing about it: it is a clear example of a goal replacement in an institution, making itself the goal in order to secure monopoly and employment. (Bauman, 1990). To this, my theology friend replied that, besides a good calculus there also seems to be an evil calculus in need for redemption. Only, if practicing 'the banality of evil' (Arendt, 1963) by just following orders, an individual can get redemption, but not an institution. And, it seems unethical that an unnecessary monopoly should deprive humans of the basic ability to add per-numbers.

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