A TEXT-FREE MATH EDUCATION FOUND BY DIFFERENCE RESEARCH FOR PROTECTION AGAINST ALIEN ARTIFICIAL INTELLIGENCE

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Artificial Intelligence, AI, friend or foe to math education? Some warn that AI develops into an alien intelligence infiltrating all that is text-bound in a library. So, to protect math education from this, we ask if math may be taught and learned in a text-free form out of reach to AI. Having not yet met math in its text-bound form, 3year old children give the answer by using bundle-numbers with units as 2 2s thus seeing what exists in time and space. This discovery allows difference research to use sociological imagination to design text-free curricula giving priority to outside existence over inside essence, and to use flexible bundle-numbers with units in tales about things and actions on a Bundle-Bundle-Board.

PROTECTING MATH AND ITS EDUCATION FROM BEING INFILTRATED BY AI

It seems only natural that mathematics is a core subject in education because of its many important applications in core societal matters within economy, science, technology, engineering, etc. So, all we need are universities to define and develop mathematics, and to teach teachers how to teach it to students in mathematics classes that it may later be applied in other classes. It is as simple as that. And of course, it goes without saying that first mathematics must be learned to be applied later.

However, a core part of mathematics is geometry, in Greek meaning to measure earth. As well as algebra, in Arabic meaning to reunite changing and constant unit- and per-numbers with addition, multiplication, integration and power. Both thus indicate that inside mathematics has outside roots. So instead of teaching the abstract before its concrete roots, maybe it should be the other way around as suggested by existentialist philosophy (Sartre, 2007) holding that outside existence precedes inside essence that might be power-charged by being not natural but socially constructed (Foucault, 1972)?

Peer-reviewed research may give an answer. But can it be trusted, John Bohannon asked in his 2013 article "Who's Afraid of Peer Review?". And if nobody teaches existence before essence, then peer-review might reject all articles about this arguing they don't discuss or extend established knowledge. So 'difference research' searching for differences making a difference (Tarp, 2018) typically has its papers rejected at conferences' peer-reviews performed by the other contributors. Until now where AI, Artificial Intelligence, with its access to the library may write research articles also in huge numbers. In May 2023, 350 leading scientists and notable figures signed a common statement warning against AI by saying that "Mitigating the risk of extinction from AI should be a global priority alongside other societal-scale risks such as pandemics and nuclear war". A similar warning is found on the YouTube video "AI and the future of humanity" given by Yuval Harari at the Frontiers Forum, May 2023.

To protect math education from infiltration by an alien intelligence seems almost impossible since both mathematics and education are text-bound. So, we may ask: "Can mathematics be taught and learned in a different text-free version?" Let us see what difference research may offer here.

To look for a different version we listen to brains that have not yet been exposed to books, young preschool children. So, we ask a 3year old child "How many years next time?" Typically, the answer is four showing four fingers. But presented by four fingers held together two by two, the child protests: "That is not four, that is two twos". The child thus sees what exists in space and time: Bundles of twos in space, and two of them when counted in time. These rectangular bundle-numbers with units are different from the textbook's linear number-line numbers without units.

Based on this discovery, difference research now uses sociological imagination (Mills, 1959) to design text-free curricula giving priority to outside existence over inside essence, and using bundle-numbers.

Designing micro curricula

Looking at our 5 fingers we observe that bundle-numbers may be flexible when bundle-counting. If we shorten 'Total' to T and 'Bundle' to B we have: T = 0B 5 = 1B 3 = 2B 1 = 3B -1 2s, or T = 1BB 0B 1 = 1B^2 0B 1 = 101 2s if we leave out the units. Here 1B 3 may be called an overload, and 3B -1 may be called an underload. Counting all ten fingers, we get T = 2BB 0B 2 = 1BBB 0BB 1B 0 = 1010 2s. Counting them in 3s, we get T = 3B 1 = 1BB 0B 1 = 101 3s. We notice that with units, the place value system becomes redundant, and that power is the first operation we meet.

Flexible bundle-numbers may also be used with ten as bundle-size: T = 68 = 6B 8 = 5B 18 = 7B -2 tens. This eases standard operations and makes also carrying and borrowing redundant:

T = 23+59 = 2B 3+5B 9 = 7B 12 = 8B 2 = 82; and T = 83 – 59 = 8B 3 – 5B 9 = 3B -6 = 2B 4 = 24

T = 3*59 = 3*5B 9 = 15B 27 = 17B 7 = 177; and T = 84 /3= 8B 4 /3 = 6B 24 / 3 = 2B 8 = 28

Here we met Many in space. In time we also include the unit in the counting sequence: 0B 1, 0B 2, …, 0B 9, 0B ten or 1B 0, 1B 1 etc., enjoying that 'eleven' comes from the Vikings saying '1 left, 2 left'.

We now look at the counting process by asking "How many 2s in 8?". To answer, first we push-away the 2s, which allows division to be iconized as a broom, 8/2. Then 4 times we stack the 2s, which allows multiplication to be iconized as a lift, 4x2. We may now write the result as a 'recount-formula":

8 = 4 2s = 8/2 2s = (8/2) x 2, or $T = (T/B) \times B$ with unspecified numbers.

So, with bundle-counting changing the units from 1s to bundles we get the proportionality formula directly. Also, we meet a formula or function as a number-language sentence with an outside subject, a verb, and an inside predicate as in word language sentences. Also we meet solving equations since our question could be reformulated as ux2 = 8 where recounting 8 in 2s gives 8 = (8/2)x2. The equation thus is solved by u = 8/2, i.e., by 'moving to opposite side with opposite sign'. Which also follows from the formal definition saying that "8/2 is the number u that multiplied with 2 gives 8, ux2 = 8".

Likewise, the equation u+2 = 5 is solved by moving over as u = 5-2 since u is a placeholder for a number that with 2 added gives 5, thus found by reversing the action and pulling-away the 2 again.

Solving equations by 'opposite side & sign' is a difference to the traditional balancing method 'do the same to both sides' introduced to motivate teaching teachers the abstract algebra concept 'group'.

When bundle-counting, we also meet decimals, fractions, and negative numbers to account for the unbundled singles: First we pull-away the stack which allows subtraction to be iconized as a rope, e.g., 9 – 4x2 = 1. Then we place the unbundled on-top of the stack, as a decimal number, 9 = 4B1 2s, or as a fraction when counted in 2s also as 1 = (1/2) x 2, 9 = 4 ½ 2s, or as a negative number showing in space what is missing for the next bundle, or what have been pulled away from it in time, 9 = 5B -1.

Above we saw that recounting from tens to icons solves equations: *u**6 = 30 = (30/6)*6, so *u* = 30/6.

Recounting from icons to tens gives multiplication tables that may be seen on a ten-by-ten Bundle-Bundle-Board, a BBBoard where 6*7 may be seen as 6 7s or as '(B-4)*(B-3) which leads to early algebra since the 6 7s are left when from the ten Bs we pull-away 4Bs and 3Bs, and then add the 4 3s we pulled-away twice: T = 6 7s = 6*7 = (B-4)*(B-3) = 10B - 3B - 4B + 4 3s = 3B + 1B 2 = 4B 2 = 42.

Inside, outside totals become rectangular stacks as T = 8 4s = 8*4, or squares in the case of bundle-bundles T = 4 4s = 4*4. So, we may ask "How to square a rectangle?", e.g., T = 8 4s = B*B = B^2 where B is called the square root of 32, B = √32, iconized by half a perimeter. We begin by adding half of the excess, ½*(8-4) 4s= 2 4s to both sides of the 4x4 square, which gives a 6x6 square with a total of 36. This is too much since also the upper right corner must be included. So instead we add a number *t* determined by (4+*t*)^2 = 32. On a drawing we see that the square (4+*t*)^2 has four parts, 4^2 and *t*^2 and 2*4**t*, so (4+*t*)^2 = 4^2 + *t*^2 + 8**t* = 32, or *t*^2 + 8**t* + 16 = 32 , or *t*^2 + 8**t* – 16 = 0, called a quadratic now rooted in transforming a 32-rectangle into a '4+*t* square'. Likewise, we may rewrite the quadratic *t*^2 + *b***t* + *c* = 0 as *t*^2 + 2**b*/2**t* + (*b*/2)^2 = (*b*/2)^2 – *c* = *D*/4, or as (*t*+ *b*/2)^2 = *D*/4 where *D* is called a discriminant. This shows that a *D*/4 rectangle may be transformed into a *b*/2+*t* square thus providing the solution to the quadratic as *t* = -*b*/2 ±√(*D*/4).

Changing the unit, the recount-formula roots the proportionality formula *T* = *a***b* recounting *T* in *b*s. Examples may be meter = meter/sec*sec, recounting a distance in seconds, or $ = $/kg*kg, recounting dollars in weight, thus creating 'per-numbers' as meter/sec, $/kg, etc. Or part = part/whole * whole, recounting a part in wholes and becoming fractions with like units. In time, the end value may be recounted in the start-value: end = end/start * start, where end/start is the change-factor, e.g., 105%.

Finally, in a rectangle with a base, *b*, a height, *h*, and a diagonal, *d*, mutual recounting roots trigonometry as per-numbers: *h* = (*h*/*b*)**b* = tangent(Angle)**b* where tan(*A*) is the per-number *h*/*b*. Splitting the circumference of half a unit-circle in *n* parts gives the number π = *n**tan(180/*n*) for *n* big.

Once counted, stacks may be added, on-top after recounting has provided like units, or next-to as areas thus rooting integral calculus, as well as differential calculus when reversing asks "2 3s + ?5s = 4 8s".

Per-numbers add in mixture problems as "2kg at 3$/kg plus 4kg at 5$/kg gives what?". With like units, the unit-numbers 2 and 4 add directly. But per-numbers must be multiplied to unit-numbers before adding as the areas created by the multiplication. So, mixture problems root integral calculus, becoming differential calculus when the problem is reversed. So, integral calculus should be introduced before its inverse differential calculus show that many differences add as one difference.

Fact, fiction & fake, the 3 modeling genres

With mathematics as a number-language for outside things in space and actions in time, its quantitative literature or models needs to be divided into fact, fiction or fake, the same genres used in the word-language for qualitative literature. Fact stories are 'since-then' stories that quantify and predict predictable quantities by using factual numbers and formulas; and that need to be checked for units and correctness. Fiction stories are 'if-then' stories that quantify and predict unpredictable quantities by using assumed numbers and formulas; and that need to be supplied with scenarios with alternative assumptions. Fake stories are 'what-then' stories that quantify and predict unpredictable qualities by using fake numbers and formulas; and that need to be replaced by word stories (Tarp, 2001).

Conclusion, yes we can

So, the answer to our question is: Yes, mathematics may be taught and learned in a version free from text and AI if we follow the advice of existentialist philosophy and let outside text-free existence precede inside text-bound essence; and use children's own flexible bundle-numbers with units instead of the textbook's line-numbers without units (Tarp, 2020-2023). This allows a communicative turn in number-language as the one that took place in the word-language education in the 1970s (Widdowson, 1978). Core mathematics will then be learned automatically as tales about Many on a Bundle-Bundle-Board. As to teacher education and research, the MATHeCADEMY.net offers a corresponding free online teacher education using Bundle-numbers with units where learning takes place through guided activities that allow questions to be answered by the subject in the laboratory instead of by an instructor in a library. The academy also contains many articles showing the possibilities of leaning to master Many before mathematics. They propose that studies of the discovered differences are carried out as design research (Bakker, 2018). And studies on the ability to bring back brains from special education is especially needed in a subject that has deprived children of their own bundle-numbers with units.

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