FROM LOSER TO USER, FROM SPECIAL TO GENERAL EDUCATION, LEARNING INSIDE MATHEMATICS THROUGH OUTSIDE ACTIONS

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Although eager to begin school, some children soon fall behind and are sent to special education teaching the same at a slower pace. Wanting mathematics education to be for all leads to the question: Is this so by nature or by choice? Can it be otherwise? Observing how children communicate about Many before school, this paper asks what kind of mathematics can be learned if accepting the bundle-numbers as 2 3s that children bring to school. Using Difference Research, it turns out that accepting numbers with units means that counting, recounting and solving equations come before adding on-top or next-to introduce integral and differential calculus as well as proportionality in early childhood education. So, it is possible to institute an ethical mathematics education that transforms loser to users returning to general education as stars teaching fellow students and teachers how to master Many.

INSIDE, CHILDREN ADAPT SMOOTHLY TO THEIR OUTSIDE WORLD

It is glad to see how children vividly communicate about Many before school. And it is sad to see how they stop after beginning school, and how more and more are excluded from general education and sent to special education. A day inside a classroom tells you why. The students no more communicate about Many, instead a textbook mediated by a teacher teaches them about what they need in order to communicate: multidigit numbers obeying a place value system, first to be added then subtracted with no respect to their units. Later, also fractions are added without units, thus disregarding the fact that both digits and fractions are not numbers, but operators needing numbers to become numbers.

In a special education class, the same is taught but at a slower pace. Which makes you wonder: With their preschool foundation, can children learn number-language through communication as with the word-language (Widdowson, 1978)? And, can this mastery of Many lead to mastery of mathematics later, if needed? In which case, mastery of mathematics wouldn't be the only way to master Many.

Looking for differences, Difference Research (Tarp, 2018) searching for differences that might make a difference may inspire micro curricula to be tested using, e.g., Design Research (Bakker, 2018).

MEETING MANY, CHILDREN COUNT WITH BUNDLES AS UNITS

How children adapt to Many can be observed from preschool children. Asked "How old next time?", a 3year old will say "Four" and show 4 fingers; but will react strongly to 4 fingers held together 2 by 2, "That is not four, that is two twos", thus describing what exists: bundles of 2s, and 2 of them.

Inside, children thus adapt to outside quantities by using two-dimensional bundle-numbers with units. And they also use full sentences as in the word-language with a subject 'that', and a verb 'is', and a predicate '2 2s', which abbreviated shows a formula as a number-language sentence 'T = 2 2s'.

A BUNDLE-NUMBER CURRICULUM FOR SPECIAL EDUCATION

Listening to special education students helps understanding when and why they fall behind. Inspired by this we may design question guided micro curricula, MC, to further develop the number-language and mastery of Many they acquired before school, allowing them an comeback to general education.

MC01. Digits

The tradition presents both digits and letters as symbols. A difference is letting students experience themselves digits as not symbols, but icons with as many sticks or strokes as they represent if written less sloppy (Tarp, 2018). In this way students see inside icons as linking directly to outside degrees of Many. And that ten has no icon since, as a bundle it becomes the unit, so that two-digit numbers really are two countings of bundles and unbundled singles.

A guiding question can be “There seems to be 5 strokes in a 5-digit if written less sloppy. Is this also the case with other digits?” Outside material could be sticks, a folding ruler, cars, dolls, spoons, etc.

Discussing why numbers after ten has no icon leads on to bundle-counting.

MC02. Bundle-Counting Sequences

Using a place value system, the tradition counts without bundles. A difference is to practice bundle-counting in tens, fives, and threes. In this way students may see that including bundles in number-names prevents mixing up 31 and 13. And they may also be informed that the strange names ‘eleven’ and ‘twelve’ are Viking names meaning ‘one left’ and ‘two left’, and that the name ‘twenty’ has stayed unchanged since the Vikings said ‘tvende ti’; and that English roughly is a mixture of Viking words labeling concrete things and actions, and French words labeling abstract ideas. The Viking tradition saying ‘three-and-twenty’ instead of ‘twenty-three’ was used in English for many years (see, e.g., Jane Austen). Now it stops after 20. The Vikings also counted in scores: 80 = 4 scores, 90 = half-5 scores.

A guiding question can be “Let’s use the word bundle when bundle-counting in tens, in 5s and in 3s.”

Outside material could be fingers, sticks, cubes, and a ten-by-ten abacus. Using fingers and arms we can count to twelve, also called a dozen. Using cubes, the bundles are stacked on-top of each other.

First, we count in 5s (hands): $0B1$, …, $0B4$, $0B5$ or $1B0$, $1B1$, …, $1B5$ or $2B0$, $2B1$, $2B2$.

Then we count in 3s (triplets): $0B1$, …, $0B3$ or $1B0$, …, $1B3$ or $2B0$, …, $2B3$ or $3B0$, $3B1$, ...

Counting cubes in 3s, 3 bundles is 1 bundle-of-bundles or $1BB$ in writing, so we repeat: $0B1$, …, $2B3$ or $3B0$ or $1BB0B0$, $1BB0B1$, $1BB0B2$, $1BB0B3$ or $1BB1B0$.

Then we count in tens, including the bundles: $0B1$, …, $0B8$, $0B9$, $0B10$ or $1B0$, $1B1$, $1B2$.

Finally, we bundle-count in tens from 0 to 111.

MC03. Bundle-Counting with Underloads and Overloads

Strictly following the place value system, the tradition silences the units when writing ‘two hundred and fifty-seven’ as plain 257. A difference may be inspired by the Romans using ‘underloads’ when writing four as “five less one”, IV; and by overloads when small children use ‘past-counting’: “twenty-nine, twenty-ten, twenty-eleven”.

A guiding question can be. “Let us count with underloads missing for the next bundle. And with overloads as children saying ‘twenty-eleven’. Outside material could be sticks, cubes, and an abacus.

First, we notice that five fingers can be counted in pairs in three different ways

T = 5 = I I I I I = ~~II~~ I I I = 1*B*3, overload

T = 5 = I I I I I = ~~II~~ ~~II~~ I = 2*B*1, normal

T = 5 = I I I I I = ~~II~~ ~~II~~ ~~II~~ = 3*B*-1, underload

Using fingers and arms, first we count using underloads: 0*B*1 or 1*B*-9, 0*B*2 or 1*B*-8, …, 0*B*9 or 1*B*-1, 1*B*0, 1*B*1 or 2*B*-9, 1*B*2 or 2*B*-8. The we count to twelve in 5s (hands): 0*B*1 or 1*B*-4, 0*B*2 or 1*B*-3, …, 0*B*4 or 1*B*-1, 1*B*0, 1*B*1 or 2*B*-4, …, 2*B*2 or 3*B*-3.

And in 3s (triplets): 0*B*1 or 1*B*-2, 0*B*2 or 1*B*-1, 0*B*3 or 1*B*0, 1*B*1 or 2*B*-2, …, 3*B*3 or 4*B*0 or 1*BB*1*B*0. Cup-counting with a cup for bundles, and for bundles-of-bundles: T = 1]1]0 = 4]0 = 3]3 = 2]6 = 1]9.

Then we count in tens from 1 to 111, using past-counting: … 1*B*9, 1*B*10, 1*B*11 or 2*B*1, 2*B*2, …, 2*B*11 or 3*B*1, …, 9*B*9, 9*B*10, 9*B*11 or 10*B*1 or 1*BB*0*B*1.

Then we rewrite totals as ‘flexible bundle-numbers’ with overloads and underloads:

T = 38 = 3*B*8 = 2*B*18 = 1*B*28 = 4*B*-2 = 5*B*-12

MC04. Doing Math with Flexible Bundle-Numbers

The tradition uses carrying when adding and multiplying, and borrowing when subtracting and dividing. Here, a difference is to instead use flexible bundle-numbers.

A guiding question can be. “Let us do inside arithmetic with flexible bundle-numbers.”

Overload	Underload	Overload	Overload
65 + 27	65 - 27	7 x 48	336 /7
6 *B* 5 + 2 *B* 7	6 *B* 5 - 2 *B* 7	7 x 4 *B* 8	33 *B* 6 /7
8 *B*12 9 *B* 2	4 *B*-2 3 *B* 8	28 *B* 56 33 *B* 6	28 *B* 56 /7 4 *B* 8
92	38	336	48

Figure 1: Doing Arithmetic with Flexible Bundle-Numbers

MC05. Talking Math with Formulas

In a number-language sentence as “The total is 3 4s”, the tradition silences all but the calculation 3*4. A difference is to use full sentences with an outside subject, a verb and an inside predicate. And to emphasize that a formula is an inside prediction of an outside action. The sentence “$T = 5*6 = 30$” thus inside predicts that outside 5 6s can be re-counted as 3 tens.

A guiding question can be. “Let us talk math with full sentences about what we calculate and how.”

We begin by counting in ones using a full sentence: “From the total we pull away one to get one.”

We then write what was before and after, using a rope as an icon for pulling away: “$T = (T – 1) + 1$”.

This number-language sentence formulates what we call a formula. It also applies if instead pulling away 2, "$T = (T – 2) + 2$", and if pulling away any unspecified bundle B, "$T = (T – B) + B$". We call this formula a 're-stack formula' since, with the total as a stack, we pull away the bundle from the top and place it next-to as its own stack.

Outside asking "Adding what to 2 gives 5?", inside becomes "? + 2 = 5" in writing. Using the letter u for the unknown number, this becomes an 'equation' "$u + 2 = 5$", easily solved outside by pulling away the 2 that was added, described inside by restacking the 5: $u + 2 = 5 = (5 – 2) + 2$, so $u = 5 – 2$.

So, inside an equation is solved by moving a number to the opposite side with the opposite sign. Also, we see the definition of the number '5-2': "5 minus 2 is the number u that added to 2 gives 5".

We now use full sentences when counting in bundles, e.g., re-counting 8 1s in 2s. We use '/' to iconize a broom brushing away 2s. So '8/2 = 4' is an inside prediction for the outside action "From 8, brush 2 away, 4 times."

Having been brushed away, the bundles of 2s are stacked. This is iconized by an 'x' for lifting the bundles, so '4x2 = 8' is an inside prediction for the outside action "4 times stacking 2s gives 8 1s".

Re-counting 8 in 2s thus gives a 're-count formula' 8 = (8/2) x 2, outside showing a box with the counter 8/2 and the unit 2 on the vertical and horizontal side, and with the total 8 as the area. So, totals add by areas, called integral calculus. With unspecified numbers it says: $T = (T/B) \times B$, or $T = (T/B) * B$, or "From a total T, T/B times, B can be brushed away".

Outside asking "How many 2s in 8", inside is the equation "? * 2 = 8", or "$u * 2 = 8$" easily solved outside by brushing away 2s, described inside by recounting 8 in 2s: $u * 2 = 8 = (8/2) * 2$, so $u = 8/ 2$.

Again, an equation is solved by moving a number to the opposite side with the opposite sign. Also, we see the formal definition of '8/2': "8 divided by 2 is the number u that multiplied to 2 gives 8".

MC06. Naming the Unbundled Singles

Without bundling, the tradition cannot talk about the unbundled singles. A difference is to see them in three different ways when placed on-top of the stack of bundles. A guiding question can be "How to see the unbundled singles?". Outside materials can be cubes or an abacus

Before outside recounting 9 in 2s, inside we let a calculator predict the result: Entering 9/2 gives '4.some' predicting that "from 9, brush away 2s can be done 4 times". To find unbundled singles, outside we pull away the 4-by-2 stack, inside predicted by entering '9 – 4 * 2' giving 1. So, inside the calculator predicts that 9 recounts as 4*B*1 2s, which is also observed outside.

Recounting 9 cubes in 2s, the unbundled can be placed on-top of the stack. Here it can be described inside by a decimal point separating the bundles from the unbundled: T = 4*B*1 2s = 4.1 2s. Likewise, when counting in tens: T = 4*B*2 tens = 4.2 tens = 4.2 * 10 = 42.

Seen as part of a bundle, inside we can count it in bundles as a 'fraction', 1 = (1/2) * 2 = 1/2 2s; or we can count what is missing in a full bundle, 1 = 1*B*-1 2s.

Again, we see the flexibility of bundle-numbers: T = 4*B*1 2s = 4 ½ 2s = 4.1 2s = 5.-1 2s.

Likewise, when counting in tens: T = 4*B*2 tens = 4 2/10 tens = 4.2 tens = 5.-8 tens.

MC07. Changing Number Units

Always counting in tens, the tradition never asks how to change number units. A difference is to change from one icon to another, from icons to tens, or from tens to icons, or into a square.

A guiding question can be "How to change number units?". Outside materials can be an abacus.

Asking '3 4s = ? 5s', we inside predict the result by entering on a calculator the 3 4s as 3*4, to be counted in 5s by dividing by 5. The answer '2.some' predicts that from 3 4s, 2 times, 5 can be brushed away. To find unbundled singles, outside we pull away the 2 fives from the 3 4s; inside we predict this by entering '3*4 – 2*5'. The answer '2' predicts that 3 4s can be recounted as 2 5s & 2, or 2*B*2 5s.

Asking "40 = ? 5s", we predict the result by solving the equation "u * 5 = 40" by recounting 40 in 5s:

u * 5 = 40 = (40/5) * 5, so u = 40/5.

Asking "6 8s = ? tens", or "6 * 8 = ?", we inside predict the result by looking at a ten-by-ten square with 6 and 8 as *B*-4 and *B*-2 on the sides. We then see that the 6*8 box is left when from the *B***B* box we pull away a 4**B* and a 2**B* box and add the 4*2 box pulled away twice.

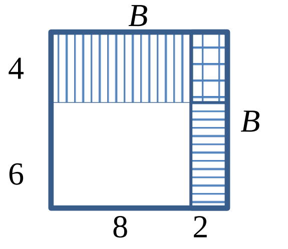	T = 6 * 8 = (B-4) * (B-2) = BB – 2B – 4B + 8 = 4B8 = 48	$T = \begin{pmatrix} 1B-4 \\ 1B-2 \end{pmatrix}$ = 1BB – 2B – 4B + 8 = 10B – 6B + 8 = 4B8 = 48	$T = \begin{pmatrix} 2x-3 \\ 4x-5 \end{pmatrix}$ = $8x^2$ – 10x – 12x + 15 = $8x^2$ – 22x + 15

Figure 2: Multiplying Numbers as Binomials

Inside, multiplying two 'less-numbers' horizontally creates a FOIL-rule: First, Outside, Inside, Last. Multiplying them vertically creates a cross-multiplication rule: First multiply down to get the bundle-of-bundles and the unbundled, then cross-multiply to get the bundles.

Wanting to square a 5-by-4 box, its side is called $\sqrt{20}$, using lines to iconize the square. To find $\sqrt{20}$ we see that removing the 4-square leaves 20 – 4*4 = 4 shared by the two 4**t* boxes in a 4+*t* square, giving *t* = 0.5. A little less since we neglect the *t*-square. Inside, a calculator predicts that $\sqrt{20}$ = 4.472.

Intersection points between lines and circles leads to quadratic equations as x^2+ 6x + 8 = 0, easily solved when rotating the upper of two x by x+3 playing cards to create a square with sides x+3, and with zero area except for the 1 left in the upper 3-by-3 square after 8 is removed.

In total, $(x+3)^2$ = 0 + 1 = 1 = $\sqrt{1}^2$, so x + 3 = ± 1, giving x = - 2 and x = - 4.

MC08. Changing Physical Units with Per-Numbers

The tradition sees shifting physical units as an application of proportionality. Typically finding the unit cost will answer questions as "with 2 kg costing 3$, what does 3 kg cost, and what does 6$ buy?"

A difference is to use 'per-numbers' (Tarp, 2018) coming from double-counting the same quantity in the two units, e.g., *T* = 3$ = (3$/2kg) * 2kg = *p* * 2kg, with the per-number *p* = 3$/2kg, or 3/2 $/kg.

A guiding question can be "How to change physical units?". Outside materials can be colored cubes.

Recounting in the per-number allows shifting units:

T = 6 kg = (6/2)*2 kg = (6/2)*3 $ = 9$; and T = 15$ = (15/3)*3$ = (15/3)*2kg = 10 kg.

Alternatively, we recount the units: $ = ($/kg)*kg = (3/2)*6 = 9; and kg = (kg/$)*$ = (2/3)*15 = 10.

With like units, per-numbers become fractions: 1$/4$ = ¼. The tradition teaches fractions as division: 1/4 of 12 = 12/4. A difference is to see a fraction as a part of a bundle counted in bundles, 1 = (1/4)*4, so 1/4 = 1 part per 4. Finding 3/4 of 12 thus means finding 3parts per 4 of 12 that recounts in 4s as:

T = 12 = (12/4) * 4 = (12/4) * 3parts = 9parts, so 3 per 4 is the same as 9 per 12, or 3/4 = 9/12.

So, finding 3/4 of 100 means finding 3 parts per 4 of 100 = (100/4)*4, giving 75 parts per 100 or 75%.

MC09. Recounting the Sides in a Box Halved by its Diagonal Gives Trigonometry

The tradition teaches trigonometry after plane and coordinate geometry. A difference is to see trigonometry an example of per-numbers, mutually recounting the sides in a box halved by its diagonal.

A guiding question can be "How to recount the sides in a box halved by its diagonal?". Outside materials can be tiles, cards, peg boards, and books.

Recounting the height in the base, height = (height/base) * base = tangent A * base, shortened to

$h = (h / b) * b = \tan A * b = \tan A$ bs, thus giving the formula tangent A = height / base, or $\tan A = h/b$.

Using the words 'run' and 'rise' for 'base' and 'height', we get the formula: tan A = rise/run, giving the steepness or slope of the diagonal. The word 'tangent' is used since the height will be a tangent in a circle with centre in A with the base as its radius.

This gives a formula for the circumference since a circle contains many right triangles: In an h-by-r half-box, h recounts in r as $h = (h/r) * r = \tan A * r$.

A half circle is 180 degrees that split in 100 small parts as 180 = (180/100)*100 = 1.8 100s = 100 1.8s. With A as 1.8 degrees, the circle and the tangent, h, are almost identical. So, half the circumference is

$\frac{1}{2}C = 100 * h = 100 * \tan 1.8 * r = 100 * \tan (180/100) * r = 3.1426 * r$

Calling the circumference for $2 * \pi * r$, we get a formula for the number π:

$\pi = \tan (180/n) * n$, for n sufficiently large.

MC10. Adding Next-To and On-Top

The tradition sees numbers as one-dimensional cardinality with addition defined as counting on. A difference is to accept childrens' bundle-numbers as 2-dimensional boxes that add next-to and on-top. A guiding question can be "How to add 2 3s and 4 5s on-top and next-to?". Material: cubes and abacus.

To add 2 3s and 4 5s on-top, the units must be made the same, outside by squeezing one or both; inside recounting changes units. Or to use the recount formula to predict the result by entering (2*3+4*5)/B, where B can be 3 or 5 or 8. Added next-to by areas is called integral calculus.

Adding 20% to 30$, we have two units with the per-number 30$ per 100%. Adding 20% to 100% gives 120%, recounting in 100s as 120% = (120/100)*100% = (120/100)*30 $ = 120%*30$. So, we add 20% by multiplying with 120%, also called to multiply with the index-number 120.

Reversing adding next-to and on-top, a guiding question can be "How many 3s to add to 4 5s to get a total of 6 5s or 5 8s?". Outside materials can be cubes and an abacus.

To find the answer outside, we pull way the 4 5s from the total *T* before recounting in 3s, which is predicted inside by asking the calculator: (6*5 – 4*5)/3, or (5*8 – 4*5)/3, i.e., as Δ*T*/3. Using a difference to calculate the change in the total, Δ*T*, before recounting, this is called differential calculus.

MC11. Adding Per-Numbers and Fractions

Seeing fractions as numbers that add without units, the tradition teaches 'mathematism', true inside but seldom outside classrooms where, e.g., 2m + 3cm = 203cm. A difference is respecting that fractions and per-numbers are not numbers, but operators needing numbers to become numbers before adding.

A guiding question can be "What is 2kg at 3$/kg plus 4kg at 5$/kg?" Outside materials can be a peg board with rubber bands, vertically placed in the distances 2 and 6, and horizontally in 3 and 5.

Inside we see that unit-numbers add directly. Whereas, per-numbers first must be multiplied to become unit-numbers. And since multiplication creates a box with an area, per-numbers add by their areas, i.e., as the area under the per-number curve. And again, adding areas is called integral calculus.

And again, the opposite is called differential calculus asking "2kg at 3$/kg plus 4kg at how many $/kg total 6 kg at 5$/kg." The two connect by the fact that adding serial differences, the middle terms disappear leaving only the difference between the end and initial numbers: $(b\text{-}a) + (c\text{-}b) + (d\text{-}c) = d\text{-}a$.

MC12. Change by Adding or by Multiplying

The tradition taches adding arithmetic and geometric sequences as series. A difference is adding constant unit-numbers and per-numbers. A guiding question can be "What is 2$ plus 3$/day?" and "What is 2$ plus 3%/day?" Outside materials can be a peg board and an abacus.

Inside we see that adding 3$/day to 2$ gives a total of $T = 2 + 3*n$ after n days. This is called change by adding, or linear change with the general formula $T = b + a*n$. And that adding 3%/day to 2$ gives a total of $T = 2*103\%\text{^}n$ after n days since adding 3% means multiplying with 103%. This is called change by multiplying or exponential change having the general formula $T = b * a\text{^}n = b*(1+r)\text{^}n$.

Reversing change by adding means facing an equation as $100 = 20 + 5*u$, easily solved by restacking and recounting: 100 = (100-20) + 20 = 80 + 20, so $u*5 = 80 = (80/5)*5$, so $u = 80/5 = 16$.

Reversing change by multiplying gives two equations. In the equation $20 = u\text{^}5$, we want to find the factor u of which 5 gives 20, predicted by the factor-finding root $\sqrt[5]{20} = 1.82$. In $20 = 5\text{^}u$ we want to find the number u of 5-factors that give 20, predicted by the factor-counting logarithm $\log_5(20) = 1.86$. We now know all the ways to unite parts into a total, and to split a total in parts, the 'Algebra-square':

Operations **unite**/split Totals in	Changing	Constant
Unit-numbers m, s, kg, $	$\boldsymbol{T = a + n}$ $T - n = a$	$\boldsymbol{T = a*n}$ $\frac{T}{n} = a$
Per-numbers m/s, $/kg, $/100$ = %	$\boldsymbol{T = \int a*\mathrm{d}n}$ $\frac{\mathrm{d}T}{\mathrm{d}n} = a$	$\boldsymbol{T = a\text{^}n}$ $\sqrt[n]{T} = a \qquad n = \log_a T$

Figure 3: The 4 Ways to Unite Parts into a Total, and the 5 Ways to Split a Total into Parts.

OBSERVATIONS

The special education students were asked to write or phone short messages to a friend about how they experienced the twelve micro curricula. Typical answers expressed positive attitudes towards learning that digits are icons with sticks, that hundred is a bundle-of-bundles, that with bundles you don't need the place value system, that over- and underloads are allowed, that recounting is predicted by a recount-formula that also solves equations, that negative numbers and decimals and fractions simply tell how to see the unbundled, that the multiplication tables come when recounting from icons to tens, that boxes can be squeezed to change units or to become squares, that physical units are changed by recounting in the per-numbers, that recounting a box halved by its diagonal introduces trigonometry and a formula for pi, that number-boxes can be added on-top, but also next-to as integration adding areas, also occurring when adding per-numbers. And they proudly talk about returning to general education and becoming stars when teaching fellow students and the teacher new ways to do math.

CONCLUSION

The ancient Greek civilization talked about common ethos, individual ethics and collective moral. As to a school, we would expect a civilized ethos of this institution to be its original meaning, a timeout to reflect ongoing activity, as when called by a coach in a match. Foucault (1972) thus warns against a school becoming a 'pris-pital' mixing the power techniques of a prison and a hospital: the 'pati-mates' must return to their cells daily, and accept the diagnose 'un-educated' to be cured by, of course, education as defined by a self-referring 'truth regime'. To avoid this, mathematics education should be a timeout where the ethics of civilized educators would be that of foster-parents guiding their foster-children to better master and communicate about Many. And as to moral, we would expect a civilized mathematics education curriculum to support the foster-parents when helping children improve their mastery of outside existence instead of forcing them to first master an inside constructed essence. That, as an institutionalized means, may be tempted by a goal displacement (Bauman, 1990, p. 84): By making itself so difficult that only few will arrive at the outside end-goal of mastering Many, inside mathematics could increase its power and funding in order to finally, if ever, reach its outside goal.

Thus, to the benefit of all students, a moral mathematics education should use guidance to develop the mastery and communication that children build up when adapting to Many before school. And it would be immoral and unethical to force upon them the necessity of a construct called one dimensional place-value line-numbers adding without units in order to exclude some to special education just repeating the same at a slower pace. This paper shows that this immoral mathematics education is not there by necessity, but by choice. And that a moral version comes from not teaching but learning from children.

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