The 'KomMod Report', a Counter Report to the Ministry's Competence Report

Allan Tarp, 2002, translated into English in 2017.

The KomMod report provides an alternative response to KOM-project terms of reference, in the expectation that the Science Board of education and the Ministry of education want to respect a common democratic IDC-tradition with Information and Debate between alternatives before a Choice is made. The report replies to the following questions relating to mathematics education:

(a) What is the society's requirements for the education?
(b) To what extent is there a need to renew the existing education?
(c) How can education take into account the new student type?
(d) What content can contemporary mathematics education have?
(e) How can the future of education be organized?
(f) How to secure progression and consistency in education?
(g) What impact will a modified education have for teacher training?
(h) Which competences and qualifications can be acquired at the various stages of the education?
(i) How can competences and qualifications be measured?
(j) How can future teaching materials look like?
(k) How to secure a continuous development of the education?
(l) How can Denmark exchange educational experience with other countries?

Ad a. Our democratic society needs citizens and specialists to have a common number-language to communicate about quantities and calculations. Society needs mathematics as a human right, both as a discursive qualification and as silent competence.

Ad b. There is a need to renew the current mathematics education in order to solve its three main problems: 1. There is a widespread number-languages illiteracy, where many citizens are reluctant to use the number-language. 2. There are major transition issues between primary, secondary and tertiary education. 3. There is a decreasing enrolment to math-based education in science, technology and economy, as well as a large shortage of new secondary school teachers in mathematics.

Ad c. Future mathematics teaching should respect today's democratic, anti-authoritarian youth and its requirements on meaning and authenticity. This can be achieved if the subject respects its historical roots, and re-humanizes itself by presenting abstractions as abstractions and not as examples, i.e. as abstractions from examples (a function is a name for a formula with variable numbers), and not as examples of even more abstract abstractions (a function is an example of a set-relation). In short, the subject should portray itself as mathe-matics, recognizing its outside roots from which it has grown bottom up through abstractions. And the subject must say goodbye to the current 'meta-matics' and its belief that it has meta-physical roots and has grown top down as examples. Finally, the subject should respect the fact that people learn differently. Children learn by touching the world, i.e. by building competences. Young people learn by listening to the world, i.e. by building narratives and skills from the learning question "tell me something I don't know about something I know" (gossip-learning).

Ad d. Mathematics must respect its history as grown through abstractions, and thus also its construction as a number-language grammar, which can only be introduced after the number-language has been developed. The number-language has grown out of the meeting with quantity in time (repetition) and in space (many-ness). This meeting constructed numbers to describe the total, either through counting in pieces, bundles, bundles of bundles, bundles of bundles of bundles etc. Or faster by means of calculations to unite and divide unit-numbers (3$) and per-numbers (3$/day, 3%): Plus and minus unite and divide in variable unit-numbers (3 + 5 =?, 3 +? = 8). Multiplication and division unite and divide in constant unit-numbers (3 * 5 =?, 3 *? = 15). Potency and root & logarithm unite and divide in constant per numbers (3 times 5% =?%, 3 times ?% = 20%, ? times 5% = 20%). Integration and differentiation unite and divide variable per-numbers (5 seconds at 2 m/s growing evenly to 4 m/s =? m, 5 seconds of 2 m/s growing to 4 m/s = 18 m?). In short, the subject must respect the fact that geometry has grown out of what the word means in Greek, earth measurement; and respect that algebra has grown out of what the word means in Arabic, reunion, i.e. uniting and dividing constant and variable unit-numbers and per-numbers. Geometry and algebra must therefore respect their historical roots in an agricultural culture with two main questions: "How to share the Earth, and what it produces?" The number-language has a number of typical applications: Geometry deals with forms and shapes. Formulas deal with number levels. Growth deals with predictable change. Statistics/probability deals with change that is not pre-dictable but post-dictable. It is important to clean teaching of 'killer-Mathematics' (i.e. mathematics, that does not occur outside of the classroom, and that can only be used for one thing, killing students' interest). Addition should only occur within the parentheses, which ensures that the units are equal (T = 2 * 3 + 5 * 3 = (2 + 5) * 3 = 7 * 3 = 21). Fractions should only act together with their totals (1/2 of 2 plus 2/3 of 3 = 3/5 of 5). Equations should be solved by reversed calculation. Since the set concept cannot be well-defined it should be removed, and functions be postponed until it pops up historically after differential calculus.

Ad e. Future mathematics lessons can be organized in two main areas: Child math and youth math from respectively grade 1-7 and 8-12. Meeting the roots of mathematics roots, Many in time and space, will develop the learner's two core competences: to count and to add.

Ad f. Progression and consistency in teaching can be ensured by letting the child's math grow out of the local examples of Many, and of agricultural examples from rural and urban areas, and by letting the youth's math grow out of industrial culture and its global diversity. As well as by the child primarily working with unit-numbers, and young people primarily with per-numbers.

Ad g. By dividing education into the child's mathematics and the youth's mathematics, it will also be natural to divide teacher education in primary school teacher and secondary school teacher, as in the rest of the world approximately. This means that all future teacher-training takes place at a university. In the end, this will coincide with the division of the school into a primary school and secondary school that will take place within the next decade in connection with the high school collapse due to increased teacher retirement and decreasing enrollment of new teachers in mathematics and natural science.

Ad h. By meeting Many in time and space, the child develops competences in uniting and dividing constant and variable unit-numbers. In the countryside, bundling and re-bundling leads to multiplication and division. In the city, stacking and re-stacking leads to addition and subtraction. Calculating repetition and diversity develops the skills of young people to unite and divide constant and variable per-numbers. Totaling interest rates leads to power, root and logarithm. Totaling distances leads to integral and differential calculus.

Ad i. Competences are tacit knowledge and can therefore be neither described nor measured, but will evolve automatically through the meeting with meaningful and authentic situations, and grow from the many concrete experiences with Many in time and space, bundling and stacking, uniting and splitting, unit-numbers and per-numbers. Qualifications is measured as now through three types of tasks: Routine tasks, text tasks and projects.

Ad j. Future teaching materials should be short and concise so that time could be dedicated for student learning through self-activity. The material should respect that students have two brains, a reptile's brain for routines and a human brain for conceptual understanding. There should therefore be training tasks with responses, so learners can progress at their own pace and do as many exercises as wanted. As well as textbooks telling how mathematics has grown from practice through layers of abstractions, and accepting different names so concept may be named both bottom-up and top-down, as e.g. growth by adding and linear function etc.

Ad k. A continuous development of education can be ensured by continuously relating mathematics to its roots and not to the current political correctness.

Ad l. Exchange of experience with foreign countries can be done through establishing a Danish development research, in which practitioners can combine being researcher at a University with being attached to a teacher team at a school. This will avoid the current barren ‘ghost research’ performed by researchers without experience background in teaching practice. Development research should be difference-research (Cinderella-research) using practice based and sociological imagination to discover and try out hidden alternatives.

The Difference between the KOM- and KOMMOD Reports

In mathematics education, the two main question are: ‘How do concepts enter into the world and into the student's head - from the outside or from the inside?’ These questions give rise to different answers. Secondary school structuralism says ‘outside-outside’: Concepts exist in the meta-physical world, they are discovered by researchers and mediated by teachers. Primary school constructivism says ‘outside-inside’: Concepts exist in the meta-physical world, but are discovered through experimentation, in which each student construct their own knowledge and abilities (schemata and competences), both being silent and only to be observed through use. Post-structuralism says the ‘inside-outside’: Concepts are created through invention and social construction, and should be presented as such. Apprenticeship says ‘inside-inside’: Concepts are constructed by the apprentice during the participation in the master's practice.

Worldwide, two knowledge wars rage, a math-war between structuralism and constructivism, and a science-war between structuralism and post- structuralism. Instead of acknowledging this diversity, the report is trying to conceal it by taking over the core constructivist concept, competence, but giving it a structuralist content (insight-based action-readiness). The French philosopher Foucault has shown how new words create new clients: ‘Qualification’ creates the unqualified, and ‘competent’ creates the incompetent. But where the unqualified can cure themselves by qualifying themselves, the incompetent cannot cure themselves by ‘competencing’ themselves, and are thus left to be cured by others, the competence-competent. Adoption and modification of the word competence can therefore be interpreted as a structuralist attempt to win the math-war by a coup, instead of using it to a fruitful dialogue with equal partners.

First structuralism tried to solve the math-crisis through the wording ‘responsibility for your own learning’. Students took this seriously and turned their back to ‘meta-matics’ with its meaningless self-reference (a function is an example of a set-relation: bublibub is an example of bablibab).

Now instead the teachers are disciplined and incapacitated by constructing them as incompetent, with a consequent need for competence development through massive in-service training. Omitting the competence ‘experimenting’ shows that the report only respects science as an end-product, and neither the process nor its roots in the outside world. Neither does it respect the way in which young people and especially children acquire knowledge through self-activity and learning.

Defining competence as insight-based, the report assumes that mathematics is already learned, after which the rest of the time can be used to apply mathematics, not on the outside world, but on mathematics itself through eight internal competencies leading to exercising mathematical professionalism. This makes it a report on ‘catholic mathematics’ with eight sacraments, through which the encounter with science can take place. In contrast to this, the counter-report portrays a ‘protestant mathematics’ that emphasizes the importance of a direct meeting between the individual and the knowledge root, Many, through two sacraments, count and add; and emphasizes that linguistic competence precedes grammatical competence. Meaning that also with quantitative competence, the number-language comes before its grammar, mathematics; and as with the word-language, grammar remains a silent competence for most.

Will the math-war end with a KOM-coup? Or will it be settled through a democratic negotiation between opposing views? The choice is yours, and the KomMod report gives you an opportunity to validate the arguments, not from above from political correctness, but from below from the historic roots of mathematics. Best of luck. *Allan.Tarp@MATHeCADEMY.net*

SET-based 'MetaMatics', or Many-based ManyMatics: Learning by Meeting the Sentence or by meeting its Subject

Class 1-2

SETS are united: addition

2 + 3 = 5

47 + 85 = 135

82-65 = 17

PROBLEM:

Addition is a false abstraction:

2 m + 3 cm = 203 cm

2 weeks + 3 days = 17 days

2 C + 3 D = 23 D

3 stones = stone + stone + stone

Country: Bundle & ReBundle

Multiplication is true abstraction:
3 stones = 3 times stone = 3·stone
2·3·days = 6·days
2·m·3·cm = 6·m·cm = 600 cm ^ 2

Bundling and ReBundling:
Total = 6 1s =? 2s
Response: 6·1 = 6 = (6/2)·2 = 3·2

ReBundling-rule: T = (T/b)·b

6/2: Counted in 2s
6·2: Counting 2s

To find the total, count or calculate: ReBundling (division)
Multiplication rebundles in tens:
T = 8·3 = 24 = 2·ten & 4·1
= 2·D & 4·1

Multiplication is division!

Max-height 3:
T = 8 3s = overload
T = 8·3 = 2·3 ^ 2 & 2·3

Unbundled can also bundled in parts, for example in 5s:
T = 8·3 = (24/5)·5 = 4·5 & 4·1
= 4·5 & (4/5)·5 = (4 4/5)·5

Class 3-4

SETS are repeated: multiplication

2·3 = 6

7·85 = 595

372/7 = 53 1/7

City: Stacks & ReStack
T = 653 + 289 =?
653 = 6·C & 5·D & 3·1
279 = 2·C & 7·D & 9·1

T = 8·C & 13·D & 12·1
T = 8·C & (13 + 1)·D & (12-10)·1
T = (8 + 1)·C & (14-10)·D & 2·1
T = 9·C & 4·D & 2·1 = **942**

ReStack rule: T = (T – b) + b

T = 654-278 =?
653 = 6·C & 5·D & 4·1
278 = 2·C & 7·D & 8·1

T = 4·C & -2·D & -4·1
= (4-1)·(C) & (-2 + 10)·D & -4·1
= 3·(C) & (8-1)·(D) & (-4 + 10)·1
= 3·C & 7·D & 6·1 = 376

T = 7·653 =?

T = 7·(6·C & 5·D & 3·1)
= 42·C & 35·D & 21·1
= 42·(C) & (35+2)·(D) & (21-20) ·1
= (42 + 3)·(C) & (37-30)·D & 1·1
= 45·C & 7·D & 1·1 = 4571

T = 653/7 =?

T = 6/7·C & 5/5·D & 3/7
= 65/7·D & 3/7
= (65-2)/7·(D) & (20 + 3)/7
= 9·D & 23/7
= 9·D & 3 2/7 = 93 2/7
(double book-keeping)

Class 5-6

SETS are divided: fractions

$\frac{1}{2} + \frac{2}{3} = ?$

$\frac{1}{2} + \frac{2}{3} = \frac{3}{6} + \frac{4}{6} = \frac{7}{6}$

PROBLEM:

$\frac{1}{2} + \frac{2}{3} = \frac{1+2}{2+3} = \frac{3}{5}$ if

1 coke of 2 bottles plus 2 cokes of 3 bottles is (1 + 2) cokes of (2 + 3) bottles.

City: weighted average

$T = \frac{1}{2}\cdot 2 + \frac{2}{3}\cdot 3 = 3 = \frac{3}{5}\cdot 5$, or

$T = \frac{1}{2}\cdot 4 + \frac{2}{3}\cdot 3 = 4 = \frac{4}{7}\cdot 7$

So there are many different answers to the question

$\frac{1}{2} + \frac{2}{3} = ?$

But NEVER more than 1!

Trade calculation

5 kg cost 60 \$, 3 kg cost ? \$

ReBundle $	ReBundle kg
$ = ($/kg)·kg	3 kg = (3/5)·5kg
$ = (60/5)·3	3 kg = (3/5)·60$
$ = 36	3 kg = 36$

Percentages part 1

• 8 has 2, so 100 has ?
100 = (100/8)·8 has (100/8)·2 = 25

• 100 has 25, so 8 has ?
8 = (8/100)·100 has (8/100)·25 = 2

• 100 has 25,so ? has 2
2 = (2/25)·25 had by (2/25)·100 = 8

Class 6-7

Solution-**SETS**: open statements (equations)

2 + 3·x = 8	3·x = 6
(2+3·x)-2 = 8-2	(3·x)/3 = 6/3
(3·x + 2)-2 = 6	(x·3)/3 = 2
3·x + (2-2) = 6	x·(3/3) = 2
3·x + 0 = 6	x·1 = 2

L = {x ∈ R | 2+3·x = 8} = {2}

PROBLEM:

The weight-metaphor hides the count process, and creates many error possibilities as e.g. If 2 + 3·x = 8, then 5·x = 8

Castle & Monastery: Coding
2 + (3·5) = 17 → 2 + (3·x) = T
DeCoding (solving an equation):
ReStacking 8 in two stacks:
2 + (3·x) = 8 = (8-2) + 2
3·x = 8-2 = 6
ReBundling from 1s to 3s:
3·x = 6 = (6/3)·3
x = 6/3 = 2
Forward- & back calculations :
To opposite side with opp. sign

Forward		Back
2 + 3·x	=	8
+ 2	↑↓	*-2*
3·x	=	8-2 = 6
·3	↑↓	*/3*
x	=	6/3 = 2

Percentage part 2

• 25% of 8 is ?
0.25·8 = x

• 25% of ? is 2
0.25·x = 2, så x = 2/0.25 = 8

• ? % of 8 is 2
x·8 = 2, so x = 2/8 = 0.25 = 25%

Class 8-9

SETS are connected: functions

Function: an example of a many-one set-relation

E.g. f (x) = 2 + 3·x

A function's value and graph

PROBLEM:

The function came after calculus! A syntax error to confuse the language and meta-language: the function's value corresponds to the verb's tie.

City: Trade and Tax
Per-numbers: Tax, custom, exchange and interest rates, profit, loss, bonds, assurance.
Adding per-numbers:
3 kg at 4$/kg + 5 kg at 6$/kg gives 8 kg at ? $/kg
Geometry: area and volume of plane and spatial forms. Right-angled triangles: Pythagoras, sine, cosine & tangent.
Linear funct.: growth by adding:
T = b + a + a + a + … = b + a·n
A function is a name for a calculation with variable numbers, such as. T = 2 + 3 x. (Euler 1748)
Calculations give fixed and functions give variable number.
The change of a function can be shown in tables or on curves.
The Inn: Redistribution by games
Winn on pools, lotto, roulette.
Statistics counts number of wins.
Risk = Consequence·propability.

Class 10	Class 11	Class 12
Set theory Function theory: Domains & values. Algebraic functions: Polynomials and polynomial fractions. First- & second-degree polynomials. Trigonometry. Analytical geometry.	Function theory: reverse and composite function. Non-algebraic functions: trigonometric functions. Logarithm- & exponential functions as homomorphisms: $f(x*y) = f(x) \# f(y)$ Stochastic functions. Core calculus.	Vector spaces. Main calculus. Simple differential equations.
The Renaissance: Constant per-numbers Numbers as many-bundles (polynomials): $T = 2345 = 2\cdot B^3 + 3\cdot B^2 + 4\cdot B + 5\cdot 1$ Reversed calculations with powers: $B^4 = 81$ $4^n = 1024$ $B = \sqrt[4]{81}$ $n = \log1024/\log4$ Interest rates: Single r, total R, compound RR $(1 + r)^n - 1 = R = n\cdot r + RR$ Change with constant per-number and percentage: $x: +1 \rightarrow T: +a\$$ linear change $T = b + a\cdot x$ $x: +1 \rightarrow T: +r\%$ exponential $T = b\cdot(1+r)^x$ $x: +1\% \rightarrow T: +r\%$ power change $T = b\cdot x^r$ $x: +1 \rightarrow T: +r\% + a\$$ savings $T = a\cdot R/r$ Change with unpredictable (random) variation $\Delta T = ?$ $T = MID \pm 2\cdot SPR$ Adding percentages by their areas (integration): $300\$$ at 4% and $500\$$ at 6% is $800\$$ at ? %. Change percentage: $T = a\cdot b$: $\Delta T/T \approx \Delta a/a + \Delta b/b$ $T = a/b$: $\Delta T/T \approx \Delta a/a - \Delta b/b$ Trigonometry: SIN & COS: short sides in percent of the long. TAN: the one short side in percent of the other.	**Industry: Variable per-numbers** Coordinate geometry: Geometry & algebra, always together, never apart. Curve fitting with polynomials: $T = A + B\cdot x + C\cdot x^2 + D\cdot x^3$ (or $y = A + B\cdot x + C\cdot x^2$) A: level, B: rise, C: curvature, D: counter-curvature Variable, predictable change: Differential calculus: $dT = (dT/dx)\cdot dx = T'\cdot dx$ The non-linear is locally linear: $(1+r)^n \approx 1 + n\cdot r$ ($= 1 + n\cdot r + RR$: with a small interest, the compound-interest can be neglected) $T = x^n$: $dT/T = n\cdot dx/x$, $dT/dx = n\cdot T/x = n\cdot x^{(n-1)}$ Optimization tasks in engineering and economics. Integral calculus: $\Delta T = T2 - T1 = \int dT = \int f\cdot dx$, Total change = terminal – start = the sum of single changes, regardless of their number or size. Integration is done by rewriting to change form: Since $6\cdot x^2 + 8\cdot x = d/dx\,(2\cdot x^3 + 4\cdot x^2) = d/dx(T)$ then $\int(6\cdot x^2 + 8\cdot x)\,dx = \int d(2\cdot x^3 + 4\cdot x^2)$ $= \int dT = \Delta T = T2 - T1$ Accumulation tasks in engineering and economics.	**Major works in the Quantitative Literature:** Geometry, Trade, Economics, Physics, Biology. The three genres for quantitative literature: - *Fact or since-then calculations* quantifies the quantifiable, and calculates the calculable: since the price is $4\$/kg$, then the cost of 6 kg is $6\cdot 4\$ = 24\$$. - *Fiction or if-then calculations* quantifies the quantifiable, and calculate the incalculable: if my income is $4m\$/year$, then 6 years of income will be $6\cdot 4m\$ = 24$ million \$. - *Fiddle or so-what calculations* quantify the non-quantifiable: If the consequence 'broken leg' C is taken to be 2 million \$, and if the probability p is taken to 30%, then the risk R will be $R = C\cdot p = 2m\$\cdot 0.3 = 0.6$ million \$. The three courses of action: fact models are controlled especially for the units; fiction models are supplemented with alternative scenarios; fiddle models are referred to a qualitative treatment. Change equations solved by numerical integration. Functions of two variables. Differentiation and integration. Optimization and accumulation. Vectors used in trade and in the movement on a surface and in space.