

- In the equation  $u^3 = 8$  we ask "What multiplication number is there 3 of in 8?". The answer is obtained by the factor-finding root,  $u = \sqrt[3]{8}$ . 'Opposite side & sign'.
- In the equation  $2*3 + u*5 = 4*8$ , we ask "2 3s plus how many 5s give 4 8s?" The answer is obtained again by the reverse process, i.e., by pulling-away the 2 3s, and then counting the rest in 5s, also called differentiating where minus precedes division, i.e., the opposite of integrating.

### RECOUNT BETWEEN ICONS AND TENS

The question 'How many 8s in 32' is predicted by the equation  $u*8 = 32$ , with the solution  $u = 32/8$  since 32 recounted in 8s is  $32 = (32/8)*8$ .

The question 'How many tens in 6 7s' is predicted by placing both stacks as underloads on a BundlexBundle board, a **BBBoard**, to learn early algebra:

$6*7 = (B-4) * (B-3) = 10B - 4B - 3B + 4*3$ , as the 4 3s are pulled away twice. Here we see that minus times minus gives plus.

### RECOUNTING GIVES PER-NUMBERS

A quantity of goods can be counted in kg and dollar connected by a per-number, the price, e.g., 4\$ per 5kg, or 4\$/5kg. We then change the unit by recounting in the per-number. This is also known as proportionality.

**Question:** 20kg = ? \$. **Answer:** 20kg = (20/5) \* 5kg = (20/5) \* 4\$ = 16\$.

Nature and STEM are filled with per-numbers. Motion can be counted in meters and seconds, where the per-number meter/second is called speed. Water can be counted in grams and liters with the per-number gram/liter.

On a tile with a bottom, a height, and a diagonal, the height can be recounted in bottoms as

Height = (height/bottom) \* bottom = tangent-angle \* bottom

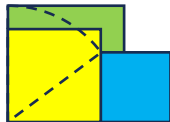
With height 3 and bottom 2,  $3 = (3/2)*2$ , or tangent-angle =  $3/2 = 1.5$ .

Measuring the angle gives 56 degrees. So, at 56 degrees, the height is 1.5 bottoms. Similarly with the other angles up to 90. Thus, we can use a ruler as a protractor. A circle can be divided into many small heights.

So, we find pi as  $\pi = n * \tan(180/n)$  for  $n$  large = 3.14...

A 'diagonal' adds squares as squares:  $\text{bottom}^2 + \text{height}^2 = \text{diagonal}^2$ .

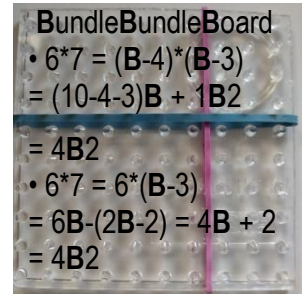
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# DeColonize Mathematics

with the Child's own 2D BundleNumbers with Units

Mathematics is easy if  
Many is mastered first with  
ManyMath



**ManyMath** respects that **MANY** is described with the child's own **bundle numbers with units** - instead of having false WOKE identity imposed as **line numbers without units** that becomes **mathematism** by claiming that  $2 + 1 = 3$  always, even though  $2\text{days} + 1\text{week} = 9\text{days}$ .

**ManyMath** is seen by asking a 3-year-old "How many years next?" The answer is 4, with 4 fingers shown. But held together 2 and 2, the child objects "That is not 4, that is 2 2s." Thus, the child sees what exists in space and time, bundles of 2s in space, and 2 of them in time when counted. So, what exists are totals that can be counted for (re)unification (*algebra in Arabic*) in time and space, such as  $2B1\ 2s$ .

**ManyMath** is based on the philosophy of existentialism recommending that **existence** precedes **essence**, which would otherwise colonize **existence**. The externally **existing** thus precedes internal '**essence regimes**', which should be deconstructed & demodeled so that **existence** is decolonized.

**BundleNumbers with units:** 8, 0B8, 1B-2, eller 1B0 8ere. 87, 8B7, 9B-3.



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# 01. MANY-MATH USES BUNDLE-NUMBERS WITH UNITS

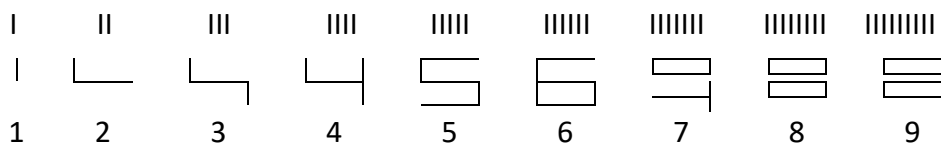
There are two kinds of numbers, unit- and per-numbers, which can be unlike or like that may be reunited. The aim of mathematics is therefore not to 'math', since we can't do that, but to act: "ReUnite Unlike & Like UnitNumbers & PerNumbers"

- 3\$ and 2\$ are unlike unit-numbers where the calculation  $3+2 = 5$  predicts the result of uniting them.
- 3 times 2\$ are like unit-numbers where the calculation  $3*2 = 6$  predicts the result of uniting them.
- 3 times 2% are like per-number where the calculation  $102\%^3 = 106.12\%$  predicts the result of uniting them to 6% and 0.12% extra in 'compound interest'.
- Unlike per-numbers may be mixture, 2kg á 3\$/kg and 4kg á 5\$/kg. Here, unit-numbers 2 and 4 unite directly, while the per-numbers 3 and 5 must first be multiplied to unit-numbers before uniting as areas, called integration, where multiplication precedes plus:  $T = (2+4) \text{ kg to } (2*3+4*5) \text{ kr}$ , i.e. 6 kg á 26/6 kr/kg.

Unite / Split into	Unlike	Like
Unit-numbers (meter, second)	$T = a + b$ $T - b = a$	$T = a * b$ $T / b = a$
Per-numbers (m/sec, m/100m = %)	$T = \int f dx$ $dT/dx = f$	$T = a^b$ $b \sqrt[T]{a} = a \quad \log_a(T) = b$

## DIGITS ARE ICONS

A digit is an icon with the number of strokes that it shows, four in the 4-icon, etc.



## BUNDLE COUNTING

Totals are counted in bundles. 5 fingers are counted as '1 Bundle 2' 3s, short as '1B2' 3s, or simply '12' 3s. And ten fingers are counted as '3B1' 3s, or '1BundleBundle 0Bundle 1' 3s, or '1BB 0B 1 3s, or simply '101' 3s.

$T = 345$  has omitted the units,  $T = 3BB4B5$ , where the bundle B = ten. Counting with unit: 0B1, ..., 0B9, 0Bti or 1B0, 1B1, ..., 9B9, 9Bti, 1BB0B0.

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Counting 5 in 2s can be done in three ways: normal, or with 'over-load' or with 'under-load':  $5 = 2B1 = 1B3 = 3B-1$  2ere. This eases calculations:  $45+27 = 4B5+2B7 = 6B12 = 7B2 = 72$ , and  $7*56 = 7*5B6 = 35B42 = 39B2 = 392$ .

Bundles are squares that grow by 2 sides:  $6^2 = 5^2+2*5 +1$ . The  $T = 6*4$  stack is transformed into a  $\sqrt{T}$ -square by moving half the excess:  $6*4 \approx (6-1)*(4+1) = 5^2$ , so that  $\sqrt{T} \approx 5$  barely before cutting to the corner.

A total T is counted in a unit, e.g.,  $T = 3 \text{ 4s}$ , or  $T = 3*4$ . This is called a number-tale with a subject T, a verb, =, and a predicate,  $3*4$ .

## OPERATIONS ARE ICONS FOR UNITING

Pushing-away 2s by recounting 8 can be iconized by a broom called division,  $8/2$ . 4 times stacking 2s can be iconized by a lift called multiplication,  $4 \times 2$ . Recounting then can be predicted by a **recount formula** used to switch units:  $8 = 4*2 = (8/2)*2$ , or  $T = (T/B)*B$ , with letters for unknown numbers. This proportionality formula states that the number of Bs in T is  $T/B$ .

Pulling-away a stack to find unbundled ones can be iconized by a rope called minus,  $9 - 4*2 = 1$ . Placed on top of the stack, unbundled becomes a decimal, a fraction, underload, or overload:  $9 = 4B1 = 4 \frac{1}{2} = 5B-1 = 3B3 \text{ 2s}$ .

Uniting stacks can be iconized by a cross called addition, +, showing the two directions that 2 3s and 4 5s can be united, horizontally, and vertically.

## SPLITTING

The reverse of uniting is splitting, which is predicted by reverse calculations called equations, where we use the letter u for the unknown number.

- In the reverse calculation (equation) ' $u + 2 = 5$ ' we ask "What is it that united with 2 gives 5?". The answer, of course, is obtained by the reverse process, by now pulling-away the 2 from 5 by minus,  $u = 5 - 2$ . The unknown number is thus found by moving the known number to opposite side with the opposite calculation sign.
- In the equation  $u*2 = 6$ , we ask "How many 2s are in 6?". The answer, of course, is obtained by counting 6 in 2s,  $6 = (6/2)*2$ , so  $u = 6/2$ . So again, by the reverse process, by pushing 2s away. So, again 'opposite side & sign'.
- In the equation  $2^u = 8$ , we ask "How many factors 2 are there in 8?". The answer is obtained by the factor-counting logarithm,  $u = \log_2(8)$ . 'Opposite side & sign'.

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Modeling real world is difficult for mathematism that also fails to distinguish between the three genres fact, fiction, and fake ('Since-then/If-then/What-then, or 'room/rate/risk' models). All models are said to be approximations. By using formulas from the start, Many-math avoids modeling problems, as it sees itself as a number-language parallel to the word-language, both of which have a meta-language (a grammar) and three genres: fact, fiction, and fake. Fake models are, for example, mathematism adding numbers without units, as well as averages of numbers that could not have been equal.

## DECOLONIZING BY DEMODELING & DECONSTRUCTION

2 colonizations: BundleNumbers by matematism LineNumbers, by metamatism sets

	Mathematism, ESSENCE	ManyMath, EXISTENCE
Digits	Symbols	Icons
345	Place value system	$T = 3BB \ 4B \ 5, BB = B^2, BBB = B^3$
Operations	Functions, order $+ - \times / \wedge$	Icons, order $\wedge / \times - +$
$3 + 4$	$3 + 4 = 7$	Meaningless without units
$3 * 4$	$3 * 4 = 12$	$3 * 4 = 3 \ 4s$
$9 = ? \ 2s$	Meaningless, only tens	$9 = 3B3 = 5B-2 = 4B1 = 4\frac{1}{2} \ 2s$
$8 = ? \ 2s$	Meaningless, only tens	$8 = (8/2)*2, T = (T/B)*B, \text{prop.}$
$2*u = 8$	$(2*u)*\frac{1}{2} = 8*\frac{1}{2}, u = 4$	$2*u = 8 = (8/2)*2, \text{s\aa } u = 8/2$
$6*7 = ?$	eh 44, eh 52, eh 42? OK	$(B-4)*(B-3) = (10-4-3)B \ 12 = 4B2$
$4kg = 5\$, 6kg=?$	$1kg = 5/4\$, 6kg = 5/4*6\$\$	$6kg = (6/4)*4kg = (6/4)*5\$\$
$1/2 + 2/3 = ?$	$1/2 + 2/3 = 3/6 + 4/6 = 7/6$	$1/2*2 + 2/3*3 = 3/5*5$
$2 \ 3s + 4 \ 5s$	$2*3+4*5 = 10*5 = 6+20= 26$	$2*3 + 4*5 = 3B2 \ 8s, \text{integration}$
$6 + 9 = ?$	$6 + 9 = 15$	$2B3 \ 6s = 2B-3 \ 9s = 1B \ (1+4) = 15$
Tangent = ?	Tan = sine/cosine	raise = (raise/run)*run, tan = raise/run

### References (Existence before essence)

Tarp, A. (2018). *Mastering Many by counting, re-counting and double-counting before adding on-top and next-to*. Journal of Math. Education, 11(1), 103-117.

Tarp, A. (2019). *A decolonized curriculum*. Mathecademy.net/a-decolonized-curriculum/

Tarp, A. (2020). *De-modeling numbers & operations: From inside-inside to outside-inside understanding*. Ho Chi Minh City Univ. of Educ. Journal of Science 17(3), 453-466.

• *Matematik er bare s\aa let*, <https://youtu.be/zUlaXnSBJ4Y>.

• *Flexible Bundle Numbers Develop the Childs Innate Mastery of Many*, [youtu.be/z\\_FM3Mm5RmE](https://youtu.be/z_FM3Mm5RmE).

<http://mathecademy.net/calculus-adds-pernumbers/>

<http://mathecademy.net/trigonometry-before-geometry/>

<http://mathecademy.net/two-competences-or-eight/>

<http://mathecademy.net/math-modeling-models/>

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## ADDITION, BUT HORIZONTAL OR VERTICAL?

When totals are counted and recounted, they can be added, but horizontally or vertically? For example, horizontally we ask ' $T = 2 \ 3s + 4 \ 5s = ? \ 8s$ '.

The recounting formula predicts that ' $T/8 = 3.\text{more}$ ', and ' $\text{more} = T - 3*8 = 2$ ', so ' $T = 3B2$ '  $8s$ . This is also called integration as we add areas. To add vertically, recounting must first make the units like, e.g.,  $3s, 5s$ , or tens.

Then the recount formula will predict, e.g., that ' $T/3 = 8.\text{more}$ ', and ' $\text{more} = T - 3*8 = 2$ ', so ' $T = 8B2$ '  $3s$ . This is called proportionality.

Single digits:  $6+9 = 2B3 \ 6s = 2B-3 \ 9s = 1B \ (1+4) \ \text{tens} = 1B5 = 15$ .

$9-6 = 1B0 - 1B-3 \ 9s = 3$ . So,  $0 - -3 = 0 + 3$ .  $6-9 = 1B-3 - 1B0 \ 9s = -3$ .

Before adding, per-numbers must be multiplied to unit-numbers thus becoming areas. Per-numbers therefore add as the area under their curve, i.e., as integration.

## TOTALS IN TIME AND SPACE, GROWTH AND STATISTICS

**In time**, a total grows by being added or multiplied by a number, called addition-growth and multiplying-growth, or linear and exponential growth.

Addition-growth: Final number = Initial number + growth-number \* growth times, or shortly,  $T = B + a*n$ . The number  $a$  is also called the slope. Multiplying-growth: Final number = Initial number \* growth-factor ^ growth times, or shortly,  $T = B * a^n$ , since  $200\$ + 5\% = (200*105\%) \ \$$ , so here the number  $a$  is  $1 + \text{interest rate}$ .

Combined growth (savings in a bank): Here we have that  $A/a = R/r$ , where  $A$  is end-dollars,  $a$  is the period-dollars,  $R$  is the end-rate,  $r$  is the period-rate, and  $1+R = (1+r)^n$ , where  $n$  is the number of periods.

100% split in  $n$  parts will give the Euler number  $e = (1+1/n)^n$  for  $n$  large.

Changing the growth-number constantly will give a quadratic growth with a parabola curving upwards or downwards if the change increases or decreases.

Changing the curvature constantly will give cubic growth with a double parabola with curvature and counter-curvature. Decreasing the growth-factor constantly will give logistic saturation growth with a hill-curve in infections. Confusion between exponential and saturation growth can cause unnecessary damage.

**In space**, a total can be divided into several subtotals that could be as large as their average, but where the deviation then tells how far away from the average they are. However, averaging numbers only makes sense if they could be equal. Students in the 1st and 9th grades do not attend the 5th grade on average.

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## 02. MATHE-MATISM USES LINE NUMBERS WITHOUT UNITS

Many-math with units is based on the concrete existence 'Many', and uses bundle-numbers with units, and distinguishes between unit-numbers and per-numbers.

Set-mathematics without units is based on the abstract essence 'set', and does not accept per-numbers, but is based upon line-numbers without units that become 'mathe-matism', always true inside but rarely outside class, by claiming that  $2 + 1 = 3$  even though  $2 \text{ pairs} + 1 = 5$ . And by claiming that digits and fractions are numbers when instead they are operators needing a number to become a number.

That sets lead to a self-reference paradox is neglected: The set of sets that do not belong to themselves, does it belong to itself or not? This is equivalent to asking, "This phrase is untrue", is it true or untrue?

Mathematics considers digits and operations as symbols just like letters. Multi-digit numbers are said to follow a place-value system, but ten is not called 'bundle', hundred is not called 'bundle-bundle', which would allow power as the first operation. Negative numbers are not allowed at a place. Reuniting occurs with the same operations; however, they are presented not simultaneously, but in the opposite order: addition, subtraction, multiplication, division, power.

$3+1 = 4$  is presented as different number-names for the same total. And not as a tale about a total,  $T = 3+1 = 4$ . That is, both the subject and the verb are omitted. Only an equivalence between number-names is included. Over- and under-load are not accepted, instead carrying and borrowing are used.

$2+3*4$ , is that 20 or 14? This is determined by the definition known as the PEMDAS math hierarchy. Even though  $T = 2+3*4 = 2 \text{ 1s} + 3 \text{ 4s}$ , which can only be recounted as 1B4 tens or 14.

$6*7$  is presented as another number-name for 42, even though  $6*7$  is 6 7s, which may or may not be recounted to tens as 4B2 tens or  $4.2*10$  or 42 if leaving out the unit and the decimal point; and here increasing the width from 7 to ten will decrease the height from 6 to 4.2.

$8/2$  is 8 divided into 2 4-bundles, instead of 8 counted in 4 2-bundles.

The tables are memorized,  $6*7 = 42$  instead of saying  $6*7 = (B-4)*(B-3)$ , or  $6*7 = 6*(B-3) = 6B - 18 = 6B - (2B-2) = 4B2 = 4.2*10$ , with less height. Here we see that minis times minus gives plus.

Letter calculations such as  $2ab + 3bc = (2a+3c)*b$  are presented as applying the distributive law, where numbers can be moved in or out of parentheses. And not by finding the common unity,  $b$ 's:

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Number of  $b$ 's is  $2a + 3c$ , so  $T = (2a + 3c) b$ 's  $= (2a+3c)*b$ .

Division leads on to fractions, decimals, and percent. Fractions are presented without units:  $1/2 + 2/3 = 7/6$ , even though  $1/2$  of 2 apples +  $2/3$  of 3 apples is  $3/5$  of 5 apples, and, of course, not 7 apples of 6.

Proportionality tasks are solved by going over the unit.

Negative numbers are introduced as independent numbers, where minus times minus is defined to be plus.

Splitting numbers is called solving equations with two numbers-names, whose equivalence is expressed in a statement that retains its truth value if the same operation is performed on both numbers-names. When transforming a number-name, three laws are used, a communicative, an associative and a distributive law.

$2*x = 8$ ;  $(2*x)*\frac{1}{2} = 8*\frac{1}{2}$ ;  $(x*2)*\frac{1}{2} = 4$ ;  $x*(2*\frac{1}{2}) = 4$ ;  $x*1 = 4$ ;  $x = 4$

The quadratic equation in the 10th class omits drawing  $x^2 + 6x - 8=0$  as  $(x+3)^2$  if 4 parts that disappear except  $3^2 - 8 = 1$ . So  $(x+3)^2$  is 1 or -1, i.e.,  $x = -2$  or  $x = -4$ .

Formulas from geometry lead to the function concept. Euler defined a function as a calculation containing both numbers and letters. In set-mathematics, a function is defined as a subset of a set product, where first-component identity implies second-component identity.

Where  $x$  represents an unspecified number,  $f(x)$  represents an unspecified formula with  $x$  as a variable. The term  $f(2)$  is therefore meaningless, since 2 is not a variable but a constant.

Linear and exponential functions are then defined as examples of homomorphisms:  $f(x) = a*x$ , and  $f(x) = a^x$ , i.e., without initial number  $b$ .

In geometry, plane geometry and coordinate geometry are presented before the trigonometry.

Calculus is presented last with differentiation before integration, although mixture calculations means adding piecewise constant per-numbers, which later become locally constant per-numbers that are rewritten as increments,  $p*dx = dy$ , which add as one difference between end-y and start-y, as all intermediaries disappear.

In addition, mathematism introduces 8 so-called mathematics competences, where many-math have only 2: count and add in time and space, the CATS approach.

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