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MODELING EASED BY DEMODELING AND REROOTING

Modeling should motivate mathematics education, but is not always that easy. Could this be different? Difference research searching for differences making a difference suggests that inside concepts may be de-rooted from the outside world by getting different names and meanings. So de-modeled they may retake their original roles allowing mathematics to again become a number-language communicating about the outside world. The search found children's own flexible bundle-numbers with units, that allow counting and recounting to precede adding. This leads directly to the core of mathematics, using proportionality and calculus to re-unite changing and constant unit-numbers and per-numbers.

DOES MATHEMATICS MODELING HAVE TO BE SO DIFFICULT?

Inspired by the first International Congress on Mathematical Education, ICME-1, I joined the student revolt in 1969 to secure that mathematics would no more be taught without linking it to its use through modeling. I was allowed to write a master thesis on modeling where I chose Game Theory. In 1974 I published my first textbook "Mathematical Growth Models" showing how calculus is modeling predictable non-constant change. And at the ICME-3 I presented a poster "Mathematics, a collection of arbitrary theoretical structures, or model-building of the real world" as well as a short oral address in English and French "Mathematics, an integral part of the real world". From 1975 I worked with Mogens Niss for three years at Roskilde University. We were both interested in modeling the Danish macro-economic cycle. Niss preferred the actual government model where the mathematics was so complicated that it could not be addressed in high school. I saw the model, not as 'since-then' fact model as in physics, but as an 'if-then' fiction model based upon assumptions that could be different. And here Ockham's razor says with two different models to explain the same, you should prefer the simpler one. So, I worked out a simple linear model that could be used in high school (Tarp, 2001). Niss stayed at the university and I returned to the high school and joined a group that succeeded changing the precalculus curriculum so that polynomials of first and second degree were replaced by linear and exponential functions so modeling could enter the classroom. I was allowed to test a special curriculum showing how statistical tables with categories divided into subcategories and changing over time may be modelled by statistics, linear and exponential change (Tarp, 2021), which allowed all students to pass the exam successfully. But the standard textbooks still presented pure mathematics to be learned before it could be applied. Linear and exponential functions were presented as examples of the function concept that was presented as an example of a set-relation. So instead of presenting an abstract concept though its examples, it was presented as an example of a more abstract concept. This was difficult to many students and resulted in so poor exam results, that it was suggested to remove precalculus as a mandatory class at the reform in 2005.

When my students asked for examples, I chose saving money by adding 5 $ per week at home or 5% per year in a bank. "Why can't we call this growth by adding and by multiplying?" the students asked. This made a difference. So, I used deconstruction (Derrida, 1991) to develop a Difference Research searching for differences making a difference (Tarp, 2018). And the government accepted my advice that precalculus should stay as a mandatory subject, but the function concept should be replaced with variables as in in physics and economics, so that we write y = b+a*x instead of f(x) = b+a*x. However, modeling still was difficult to many students. So, I turned to primary school to see if deconstruction by listening to children would make a difference here also. I found that the children see four fingers held together two and two, not as 4, but as two 2s, thus using bundle-numbers with units for what exists, bundles of 2s in space, and 2 of them when counted in time. I described the potentials of deconstruction and bundle-numbers in several MrAlTarp YouTube videos, and in 10+15+16 contributions to the ICME 10-12 (Tarp, 2012). For ICME 13 I wrote 9 papers but was allowed only 1. For the ICMI Study 24 on curriculum I designed several micro curricula where the math core was re-rooted and renamed by the process of counting and recounting before adding (Tarp, 2018, 2020). Also, I had designed a teacher education academy in 'ManyMath', MATHeCADEMY.net, Count & Add in Time & Space. Typically, modeling is eased by demodeling and re-rooting when tested in math labs, libraries and private education. So, the time has come for others to perform a large-scale testing.

Demodeling: from the inside to the outside and back

Modeling goes from the outside to the inside and back. Demodeling does the opposite by going from the inside to the outside and back. So demodeling begins with the core of inside mathematics as seen on a calculator: digits, operations, brackets, multidigit numbers, decimal point, and an equation sign. And then asks the question "What outside things and actions have rooted these inside concepts?"

Demodeling digits and multidigit numbers

Digits and letters may both be seen as symbols. But digits may also be seen as icons with as many sticks as they represent, five sticks in the five-icon, etc., if written 'less sloppy'. A sequence of digits may be seen as one multidigit number obeying a place-value system with ones, tens, hundreds, etc., and seldom with the word 'bundle-of-bundles' used for 'hundred'. But a multidigit number may also be seen as rooted in several numberings of unbundled, bundles, bundles of bundles, etc. (Tarp, 2018).

Recounting a total T of ten in 3s we get T = 3 Bundles 1, or T = 3B 1, or T = 1BB 0B 1 = 1B^2 0B 1 since 3 bundles is 1 bundle-of-bundles. So, bundling bundles roots power, and bundle-counting totals roots polynomials, T = 345 = 3*B^2 + 4*B + 5*1. And it also roots functions as number-language sentences with an outside subject, a verb, and an inside predicate as in word-language sentences.

To bundle-count a total, bundles are pushed away and lifted into a stack to be pulled away to look for unbundles singles. These actions root division as an icon for a push-away broom, multiplication as a lift, subtraction as a pull-away rope, and addition showing two ways to unite stacks, on-top or next-to. Placed on-top of the stack, the unbundled may be seen as a decimal number, or as a fraction when counted in bundles also, or described by what has been pulled away in time from the next bundle, or what is missing in space for another bundle. Recounting 9 in 2s, the end result may thus be written as

T = 9 = 4B1 2s = 4 ½ 2s = 5B-1 2s (an underload), or with an overload, T = 3B3 2s, and T = 2B 5 2s.

As to the process, to recount 8 in 2s we push-away 2s to be stacked as 4 2s, which may be written as

8 = (8/2) x 2 = 8/2 2s, or $T = (T/B) \times B = T/B$ Bs with unspecified numbers.

By changing the unit, this recount-formula roots the proportionality formula $T = a*b$ recounting T in bs. Examples may be meter = meter/sec*sec, recounting a distance in seconds, or \$ = \$/kg*kg, recounting dollars in weight, thus creating 'per-numbers' as meter/sec, \$/kg, etc. Or part = part/whole * whole, recounting a part in wholes and becoming fractions with like units. In time, terminal = terminal/initial * initial recounts the end-value in start-values.

A rectangle has base, b, height, h, and diagonal, d, raising an angle, A. Here, mutual recounting roots trigonometry: $h = (h/b)*b = \tan(A)*b$, $h = (h/d)*d = \sin(A)*d$, and $b = (b/d)*d = \cos(A)*d$. In half a radius 1 circle, splitting the circumference in n parts gives the pi-number $\pi = n*\tan(180/n)$ for n big.

Recounting between icons and tens root equations and early algebra.

Recounting from tens to icons, we ask "How many 2s in 8?" This roots equations solved by recounting 8 in 2s: $u*2 = 8 = (8/2)*2$, so $u = 8/2$ from pushing-away 2s from 8, showing that an equation is solved by reversing the process, i.e., by moving a number to the opposite side with the opposite sign. This follows the formal definition: 8/2 is the number u that multiplied with 2 gives 8, $u*2= 8$. 'To opposite side with opposite sign' may be rooted outside, while the inside balancing method is derived from abstract algebra concepts as group, inverse and neutral elements, associativity and commutativity.

Recounting from icons and tens, we ask "6 7s is how many tens?" This roots early algebra if allowing underloads: T = 6 7s = 6*7 = (B-4)*(B-3) = BB -4B -3B + 4 3s, as seen on a B*B square where the 6 7s is left when from ten bundles we pull-away 4 bundles and 3 bundles, and finally add the 4 3s that was pulled-away twice.

Once counted, stacks may be added, on-top after recounting provides like units, or next-to as areas thus rooting integral calculus, as well as differential calculus when reversing asks "2 3s + ?5s = 4 8s".

Per-numbers are added in mixture problems as "2kg at 3$/kg plus 4kg at 5$/kg give what?". With like units, the unit-numbers 2 and 4 add directly. But per-numbers must be multiplied to unit-numbers before adding as the areas created by the multiplication. So, mixture problems root integral calculus, preceding differential calculus occurring when the problem is reversed.

Inside, outside totals become rectangular stacks as T = 9 5s = 9*5, or squares in the case of bundle-bundles, T = 5 5s = 5*5. So, we may ask "How to square a rectangle?", e.g., T = 9 5s = B*B = B^2 where B is the square root of 45, B = √45, iconized by half a perimeter. Here, half the excessing 4 5s is placed to the right to create a 7*7 square except for the upper 2*2 right corner. Again, half of this is placed to the right to create a 6 5/7-square, since ½*4 = 2 = (2/7)*7. This is close: (6 5/7)^2 = 45.1.

Fact, fiction and fake, the 3 modeling genres

With mathematics as a number-language modeling outside things in space and actions in time, its quantitative literature needs to be divided into fact, fiction or fake, the same genres used in the word-language for qualitative literature. Fact stories are 'since-then' stories that quantify and predict predictable quantities by using factual numbers and formulas; and that need to be checked for units and correctness. Fiction stories are 'if-then' stories that quantify and predict unpredictable quantities by using assumed numbers and formulas; and that need to be supplied with scenarios with alternative assumptions. Fake stories are 'what-then' stories that quantify and predict unpredictable qualities by using fake numbers and formulas; and that need to be replaced by word stories (Tarp, 2001).

Discussion and conclusion

What should name a mathematical concept? Its outside root, or its inside relation to other concepts on a lower or higher abstraction level? Should a function be named a 'sentence' using a verb to link an outside subject to an inside calculation? Or a 'standby calculation' with specified and unspecified numbers? Or an example of a 'many-to-one set relation'? In fact, what we here ask is: The goal of math education, is that to learn to master math to later master Many, or the other way around? Holding that existence precedes essence, existentialist philosophy (Sartre, 2007) prefers the latter. And we see that to master Many to later master mathematics by re-rooting often implies a different name and order of de-rooted concepts. Since, to bring outside Many inside, counting and recounting precedes addition, and 2D flexible bundle-numbers with units replace the traditional 1D line-numbers without, and recounting to change units leads to proportionality, $T = (T/B)\text{x}B$, to per-numbers with different physical units, and to fractions with like units. Likewise, recounting between icons and tens leads to equations solved in the original way by moving to 'opposite side with opposite sign', and to early algebra when 6 7s is rewritten as (B-4)*(B-3). And, recounting rectangular stacks as squares roots the square root, and to quadratic equations. And in a stack halved by its diagonal, mutual recounting between the sides roots trigonometry thus preceding both plane and coordinate geometry. And once counted and recounted, stacks may be added in two ways, on-top, after recounting has made the units like, or next-to as areas thus rooting integral calculus, and differential when the question is reversed. And, since per-numbers must be multiplied to unit-numbers before adding they also add by areas as integral calculus, facilitated by differential calculus trying to rewrite area-strips as differences so the sum of many differences becomes one difference between the end and start values.

So, demodeling and re-rooting inside de-rooted concepts may ease modeling: Now you don't first learn about inside essence, but learn math directly by manipulating and communicating about (Widdowson, 1978) outside existence, e.g., things and actions on a ten-by-ten Bundle-Bundle-Board (Tarp, 2023).

References

Derrida, J. (1991). *A Derrida Reader: Between the Blinds.* Edited P. Kamuf. NY: Columbia University Press.

Sartre, J.P. (2007). *Existentialism is a humanism.* New Haven, CT: Yale University Press.

Tarp, A. (2001). Fact, fiction, fiddle - three types of models, in J. F. Matos & W. Blum & K. Houston & S. P. Carreira (Eds.), Modelling and Mathematics Education: ICTMA 9: Applications in Science and Technology. *Proceedings of the 9th International Conference on the Teaching of Mathematical Modelling and Applications* (pp. 62-71). Chichester UK: Horwood Publishing.

Tarp, A. (2012). *An ICME trilogy.* http://mathecademy.net/icme-papers/.

Tarp, A. (2018). Mastering Many by counting and recounting before adding on-top and next-to. *Journal of Math Education, March 2018, 11*(1), 103-117.

Tarp, A. (2020). De-modeling numbers, operations and equations: from inside-inside to outside-inside understanding. *Ho Chi Minh City Univ. of Education Journal of Science 17*(3), 453-466.

Tarp, A. (2021). *Master Many to later master math, the canceled curriculum chapter in the ICME study 24*. mathecademy.net/appendix-to-curriculum-study-icmi-24/.

Tarp, A. (2023). *With units, mathematism becomes manymath counting and adding in time and space.* www.mathecademy.net/manymath-2030/.

Widdowson, H. G. (1978). *Teaching language as communication.* Oxford, UK: Oxford University Press.

A Text-Free Math Education Found by Difference Research for Protection Against Alien Artificial Intelligence

Artificial Intelligence, AI, friend or foe to math education? Some warn that AI develops into an alien intelligence infiltrating all that is text-bound in a library. So, to protect math education from this, we ask if math may be taught and learned in a text-free form out of reach to AI. Having not yet met math in its text-bound form, 3year old children give the answer by using bundle-numbers with units as 2 2s thus seeing what exists in time and space. This discovery allows difference research to use sociological imagination to design text-free curricula giving priority to outside existence over inside essence, and to use flexible bundle-numbers with units in tales about things and actions on a Bundle-Bundle-Board.

PROTECTING MATH AND ITS EDUCATION FROM BEING INFILTRATED BY AI

It seems only natural that mathematics is a core subject in education because of its many important applications in core societal matters within economy, science, technology, engineering, etc. So, all we need are universities to define and develop mathematics, and to teach teachers how to teach it to students in mathematics classes that it may later be applied in other classes. It is as simple as that. And of course, it goes without saying that first mathematics must be learned to be applied later.

However, a core part of mathematics is geometry, in Greek meaning to measure earth. As well as algebra, in Arabic meaning to reunite changing and constant unit- and per-numbers with addition, multiplication, integration and power. Both thus indicate that inside mathematics has outside roots. So instead of teaching the abstract before its concrete roots, maybe it should be the other way around as suggested by existentialist philosophy (Sartre, 2007) holding that outside existence precedes inside essence that might be power-charged by being not natural but socially constructed (Foucault, 1972)?

Peer-reviewed research may give an answer. But can it be trusted, John Bohannon asked in his 2013 article "Who's Afraid of Peer Review?". And if nobody teaches existence before essence, then peer-review might reject all articles about this arguing they don't discuss or extend established knowledge. So 'difference research' searching for differences making a difference (Tarp, 2018) typically has its papers rejected at conferences' peer-reviews performed by the other contributors. Until now where AI, Artificial Intelligence, with its access to the library may write research articles also in huge numbers. In May 2023, 350 leading scientists and notable figures signed a common statement warning against AI by saying that "Mitigating the risk of extinction from AI should be a global priority alongside other societal-scale risks such as pandemics and nuclear war". A similar warning is found on the YouTube video "AI and the future of humanity" given by Yuval Harari at the Frontiers Forum, May 2023.

To protect math education from infiltration by an alien intelligence seems almost impossible since both mathematics and education are text-bound. So, we may ask: "Can mathematics be taught and learned in a different text-free version?" Let us see what difference research may offer here.

To look for a different version we listen to brains that have not yet been exposed to books, young preschool children. So, we ask a 3year old child "How many years next time?" Typically, the answer is four showing four fingers. But presented by four fingers held together two by two, the child protests: "That is not four, that is two twos". The child thus sees what exists in space and time: Bundles of twos in space, and two of them when counted in time. These rectangular bundle-numbers with units are different from the textbook's linear number-line numbers without units.

Based on this discovery, difference research now uses sociological imagination (Mills, 1959) to design text-free curricula giving priority to outside existence over inside essence, and using bundle-numbers.

Designing micro curricula

Looking at our 5 fingers we observe that bundle-numbers may be flexible when bundle-counting. If we shorten 'Total' to T and 'Bundle' to B we have: T = 0B 5 = 1B 3 = 2B 1 = 3B -1 2s, or T = 1BB 0B 1 = 1B^2 0B 1 = 101 2s if we leave out the units. Here 1B 3 may be called an overload, and 3B -1 may be called an underload. Counting all ten fingers, we get T = 2BB 0B 2 = 1BBB 0BB 1B 0 = 1010 2s. Counting them in 3s, we get T = 3B 1 = 1BB 0B 1 = 101 3s. We notice that with units, the place value system becomes redundant, and that power is the first operation we meet.

Flexible bundle-numbers may also be used with ten as bundle-size: T = 68 = 6B 8 = 5B 18 = 7B -2 tens. This eases standard operations and makes also carrying and borrowing redundant:

T = 23+59 = 2B 3+5B 9 = 7B 12 = 8B 2 = 82; and T = 83 – 59 = 8B 3 – 5B 9 = 3B -6 = 2B 4 = 24

T = 3*59 = 3*5B 9 = 15B 27 = 17B 7 = 177; and T = 84 /3= 8B 4 /3 = 6B 24 / 3 = 2B 8 = 28

Here we met Many in space. In time we also include the unit in the counting sequence: 0B 1, 0B 2, …, 0B 9, 0B ten or 1B 0, 1B 1 etc., enjoying that 'eleven' comes from the Vikings saying '1 left, 2 left'.

We now look at the counting process by asking "How many 2s in 8?". To answer, first we push-away the 2s, which allows division to be iconized as a broom, 8/2. Then 4 times we stack the 2s, which allows multiplication to be iconized as a lift, 4x2. We may now write the result as a 'recount-formula":

8 = 4 2s = 8/2 2s = (8/2) x 2, or $T = (T/B) \times B$ with unspecified numbers.

So, with bundle-counting changing the units from 1s to bundles we get the proportionality formula directly. Also, we meet a formula or function as a number-language sentence with an outside subject, a verb, and an inside predicate as in word language sentences. Also we meet solving equations since our question could be reformulated as *u*x2 = 8 where recounting 8 in 2s gives 8 = (8/2)x2. The equation thus is solved by *u* = 8/2, i.e., by 'moving to opposite side with opposite sign'. Which also follows from the formal definition saying that "8/2 is the number *u* that multiplied with 2 gives 8, *u*x2 = 8".

Likewise, the equation *u*+2 = 5 is solved by moving over as *u* = 5-2 since *u* is a placeholder for a number that with 2 added gives 5, thus found by reversing the action and pulling-away the 2 again.

Solving equations by 'opposite side & sign' is a difference to the traditional balancing method 'do the same to both sides' introduced to motivate teaching teachers the abstract algebra concept 'group'.

When bundle-counting, we also meet decimals, fractions, and negative numbers to account for the unbundled singles: First we pull-away the stack which allows subtraction to be iconized as a rope, e.g., 9 – 4x2 = 1. Then we place the unbundled on-top of the stack, as a decimal number, 9 = 4B1 2s, or as a fraction when counted in 2s also as 1 = (1/2) x 2, 9 = 4 ½ 2s, or as a negative number showing in space what is missing for the next bundle, or what have been pulled away from it in time, 9 = 5B -1.

Above we saw that recounting from tens to icons solves equations: *u**6 = 30 = (30/6)*6, so *u* = 30/6.

Recounting from icons to tens gives multiplication tables that may be seen on a ten-by-ten Bundle-Bundle-Board, a BBBoard where 6*7 may be seen as 6 7s or as '(B-4)*(B-3) which leads to early algebra since the 6 7s are left when from the ten Bs we pull-away 4Bs and 3Bs, and then add the 4 3s we pulled-away twice: T = 6 7s = 6*7 = (B-4)*(B-3) = 10B - 3B - 4B + 4 3s = 3B + 1B 2 = 4B 2 = 42.

Inside, outside totals become rectangular stacks as T = 8 4s = 8*4, or squares in the case of bundle-bundles T = 4 4s = 4*4. So, we may ask "How to square a rectangle?", e.g., T = 8 4s = B*B = B^2 where B is called the square root of 32, B = √32, iconized by half a perimeter. We begin by adding half of the excess, ½*(8-4) 4s= 2 4s to both sides of the 4x4 square, which gives a 6x6 square with a total of 36. This is too much since also the upper right corner must be included. So instead we add a number *t* determined by (4+*t*)^2 = 32. On a drawing we see that the square (4+*t*)^2 has four parts, 4^2 and *t*^2 and 2*4**t*, so (4+*t*)^2 = 4^2 + *t*^2 + 8**t* = 32, or *t*^2 + 8**t* + 16 = 32 , or *t*^2 + 8**t* – 16 = 0, called a quadratic now rooted in transforming a 32-rectangle into a '4+*t* square'. Likewise, we may rewrite the quadratic *t*^2 + *b***t* + *c* = 0 as *t*^2 + 2**b*/2**t* + (*b*/2)^2 = (*b*/2)^2 – *c* = *D*/4, or as (*t*+ *b*/2)^2 = *D*/4 where *D* is called a discriminant. This shows that a *D*/4 rectangle may be transformed into a *b*/2+*t* square thus providing the solution to the quadratic as *t* = -*b*/2 ±√(*D*/4).

Changing the unit, the recount-formula roots the proportionality formula *T* = *a***b* recounting *T* in *b*s. Examples may be meter = meter/sec*sec, recounting a distance in seconds, or $ = $/kg*kg, recounting dollars in weight, thus creating 'per-numbers' as meter/sec, $/kg, etc. Or part = part/whole * whole, recounting a part in wholes and becoming fractions with like units. In time, the end value may be recounted in the start-value: end = end/start * start, where end/start is the change-factor, e.g., 105%.

Finally, in a rectangle with a base, *b*, a height, *h*, and a diagonal, *d*, mutual recounting roots trigonometry as per-numbers: *h* = (*h*/*b*)**b* = tangent(Angle)**b* where tan(*A*) is the per-number *h*/*b*. Splitting the circumference of half a unit-circle in *n* parts gives the number *π* = *n**tan(180/*n*) for *n* big.

Once counted, stacks may be added, on-top after recounting has provided like units, or next-to as areas thus rooting integral calculus, as well as differential calculus when reversing asks "2 3s + ?5s = 4 8s".

Per-numbers add in mixture problems as "2kg at 3$/kg plus 4kg at 5$/kg gives what?". With like units, the unit-numbers 2 and 4 add directly. But per-numbers must be multiplied to unit-numbers before adding as the areas created by the multiplication. So, mixture problems root integral calculus, becoming differential calculus when the problem is reversed. So, integral calculus should be introduced before its inverse differential calculus show that many differences add as one difference.

Fact, fiction & fake, the 3 modeling genres

With mathematics as a number-language for outside things in space and actions in time, its quantitative literature or models needs to be divided into fact, fiction or fake, the same genres used in the word-language for qualitative literature. Fact stories are 'since-then' stories that quantify and predict predictable quantities by using factual numbers and formulas; and that need to be checked for units and correctness. Fiction stories are 'if-then' stories that quantify and predict unpredictable quantities by using assumed numbers and formulas; and that need to be supplied with scenarios with alternative assumptions. Fake stories are 'what-then' stories that quantify and predict unpredictable qualities by using fake numbers and formulas; and that need to be replaced by word stories (Tarp, 2001).

Conclusion, yes we can

So, the answer to our question is: Yes, mathematics may be taught and learned in a version free from text and AI if we follow the advice of existentialist philosophy and let outside text-free existence precede inside text-bound essence; and use children's own flexible bundle-numbers with units instead of the textbook's line-numbers without units (Tarp, 2020-2023). This allows a communicative turn in number-language as the one that took place in the word-language education in the 1970s (Widdowson, 1978). Core mathematics will then be learned automatically as tales about Many on a Bundle-Bundle-Board. As to teacher education and research, the MATHeCADEMY.net offers a corresponding free online teacher education using Bundle-numbers with units where learning takes place through guided activities that allow questions to be answered by the subject in the laboratory instead of by an instructor in a library. The academy also contains many articles showing the possibilities of leaning to master Many before mathematics. They propose that studies of the discovered differences are carried out as design research (Bakker, 2018). And studies on the ability to bring back brains from special education is especially needed in a subject that has deprived children of their own bundle-numbers with units.

References

Bakker, A. (2018). *Design research in education.* Oxon, UK: Routledge.

Foucault, M. (1995). *Discipline & punish.* New York, NY: Vintage Books.

Mills, C. W. (1959). *The sociological imagination.* Oxford, UK: Oxford University Press.

Sartre, J.P. (2007). *Existentialism is a humanism.* New Haven, CT: Yale University Press.

Statement on AI Risk. (2023). https://www.safe.ai/statement-on-ai-risk.

Tarp, A. (2001). Fact, fiction, fiddle - three types of models, in J. F. Matos & W. Blum & K. Houston & S. P. Carreira (Eds.), Modelling and Mathematics Education: ICTMA 9: Applications in Science and Technology. *Proceedings of the 9th International Conference on the Teaching of Mathematical Modelling and Applications* (pp. 62-71). Chichester UK: Horwood Publishing.

Tarp, A. (2018). Mastering Many by counting and recounting before adding on-top and next-to. *Journal of Math Education, March 2018, 11*(1), 103-117.

Tarp, A. (2020). De-modeling numbers, operations and equations: from inside-inside to outside-inside understanding. *Ho Chi Minh City Univ. of Education Journal of Science 17*(3), 453-466.

Tarp, A. (2021a). *Flexible bundle-numbers develop the child's innate mastery of Many.* Workshop. YouTube video. youtu.be/z_FM3Mm5RmE.

Tarp, A. (2021b). *Master Many to later master math, the canceled curriculum chapter in the ICME study 24*. mathecademy.net/ appendix-to-curriculum-study-icmi-24/.

Tarp, A. (2022). *Woke Math never forces fixed forms upon flexible Totals*. www.mathecademy.net/wokemath/.

Tarp, A. (2023). *With units, mathematism becomes manymath counting and adding in time and space.* www.mathecademy.net/manymath-2030/.

Widdowson, H. G. (1978). *Teaching language as communication.* Oxford, UK: Oxford University Press.

Decolonizing mathematics when demodeling it by using the child's uncolonized 2D bundle-numbers with units

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The names of any other people who will assist with running the Workshop None

The name of the Workshop as it should appear in the ICME-15 program:

Decolonizing mathematics when demodeling it by using the child's natural 2D bundle-numbers with units.

The aim of the Workshop

The workshop aims at experiencing how 'existence precedes essence', the slogan of existentialist philosophy, may create a different 'counting precedes adding' mathematics that uses the child's natural bundle-numbers with units occurring when a 3year-old reacts to four fingers held together two by two by saying "That is not four, that is two twos". The child thus sees what exists, bundles of twos in space that serve as units when later counted in time. An 'existence-based unit-number math' may perhaps serve as a decolonization of the traditional 'essence-based non-unit math'. Furthermore, it is outside the reach of AI by being text-free since, with units, numbers as 2 3s become physical rectangles on a ten-by-ten Bundle-Bundle-Board, a 'BBBoard'.

The activities that will run

The child's natural 2D bundle-numbers with units makes linearity and calculus enter at once in grade one. Linearity enters when asking "2 3s is how many 5s?". This recounting will change the form but not the area of the total. Calculus enters when next-to addition of totals may ask "2 3s and 4 5s total how many 8s?"

In both cases the answer may be predicted by a 'recount-formula', Total = (Total/Bundle)xBundle, or T = (T/B)xB, exemplified by 8 = (8/2)x2 saying that "the number of 2s in 8 is 8/2". Digits have already entered as icons with as many strokes as they represent, and now also division and multiplication enter as icons for a broom and a lift pushing-away and stacking bundles. Likewise, subtraction enters as an icon for a rope to pull-away the stack to find unbundled that are placed on-top of the stack as decimals, fractions, negatives ('underloads'), or 'overloads', e.g., 9 = 4B1 = 4½ = 5B-1 = 3B3 2s; and 48 = 4B8 = 5B-2 = 3B18 tens.

Recounting from tens to digits may ask "How many 7s in 42?". This leads to the equation "*u*x7 = 42" where recounting 42 in 7s as 42 = (42/7)x7 gives the answer *u* = 42/7, found by moving 'to opposite side with opposite sign'. As when restacking T = (T–B)+B says that "pulled-away from T, B is placed next-to T–B".

Recounting from digits to tens may ask "6 7s is how many tens?". This leads to early algebra when shown on the BBBoard as (B-4)x(B-3) which is left after pulling-away 3 right- and 4 top-bundles, and adding the upper right 4 3s removed twice, so 6x7 = (B-4)x(B-3) = BB-3B-4B+4x3 (FOIL). Here, minus x minus gives plus.

Recounting ten fingers in 3s leads to bundle-bundles with 9 as 3 bundles, or 1 bundle-bundle, which makes ten to 1BB0B1 or 1(B^2)0B1 3s. Or to 1(B^3)0(B^2)1B0 2s. Bundle-bundles thus leads to squares growing by adding an extra top and side and a corner so that 6^2 is 5^2+2x5+1. And to square roots almost found by moving half the extra top to the side, e.g., 6 4s = (6-1)x(4+1) = 5x5 almost. And to quadratics where a (*u*+3) square contains two squares, *u*^2 and 3^2, and two stacks, 2x(3x*u*). This means that with *u*^2+6*u*+8 = 0, only 9-8 = 1 is left for the (*u*+3) square, which gives pull-away 2 and pull-away 4, or -2 and -4, as solutions.

Recounting meters in seconds gives a 'per-number' as 4m/5sec or 4/5 m/sec that bridges the units by recounting in the per-number: 20m = (20/4)x4m = (20/4)x5sec; and 20sec = (20/5)x5sec = (20/5)x4m. With the same units, per-numbers become fractions: 3m/4m = ¾; and 3m/100m = 3/100 = 3%.

The 3 4s stack has a base B, and a height H, and a diagonal D. Mutually recounted, the three leads to trigonometry, e.g., H = (H/B)xB = tangent (Angle) x B, where tan(A) = H/B. In a low stack the height is almost the same as the perimeter in a circle with its center at the other end of the base. So pi, the full circle's half perimeter counted in the base, is almost *n* x tan(180/*n*) with *n* as a very high number.

There is a 4x4 square inside a 6x4 stack where a new unit, *k*, makes the upper right corner shrink along the diagonal to become a corner in a 6*k* x 4*k* stack. When it meets the square, 6x*k* is 4 = (4/6)x6, so *k* = 4/6, which changes 4*k* into 4x4/6 or 16/6. So now it is a corner in a 4x16/6 stack. Later it meets the square's falling diagonal where the height 6*k* now is 4 – 4*k*, so that 4 is 10*k*, or *k* = 4/10. This changes 6*k* and 4*k* into 24/10 and 16/10. It is a corner in a 2.4x1.6 stack. A BBBoard or a squared paper confirms these predictions.

Once counted and recounted totals may add, but on-top or next-to? Adding 2 3s and 4 5s on-top, recounting first must make the units like. Adding 2 3s and 4 5s next-to as 8s means adding areas as in integral calculus that becomes differential calculus if reversing the question to "2 3s and how many 5s gives 3 8s?"

Mixture problems exemplify adding per-numbers or fractions: To add 2kg at 3$/kg and 4kg at 5$/kg, the unit numbers 2 and 4 add directly whereas, before adding as $, the per-numbers 3 and 5 must be multiplied which creates areas. So, per-numbers add by the area under the per-number curve, i.e., as integral calculus.

Adding 5% to 200$ can take place by adding 5% to the 200 or to the unit to create a new unit, 105%x$. Adding 5% 7 times thus changes the unit from $ to 105%^7$. Constantly adding the same percentage thus leads to power, or if reversed, to the factor-finding root, or to the factor-counting logarithm.

In Arabic, algebra means to reunite numbers. So, we now have an 'Algebra square' as a number-language to reunite the world's four different number-types: multiplication and addition unite like and unlike unit-numbers, and power and integration unite like and unlike per-numbers. And splitting is performed by the reverse operations: division and subtraction, and root or log and differentiation.

Now it is time to use the number-language to create quantitative tales, reports, and literature. The first two tales are about space and time. In space, the Greek word geometry means to measure earth. In time, pre-calculus is about change where the change-number and -percent may be constant or constantly changing, or predictable with a change formula, or unpredictable without thus leading to statistics and probability.

With a number-language describing outside things in space and actions in time, its literature or models needs to be separated in fact, fiction and fake, the same genres that exist when using the word-language to produce qualitative literature. Fact stories are 'since-then' stories that quantify and predict predictable quantities by using factual numbers and formulas. They need to be checked for correctness and units. Fiction stories are 'if-then' stories that quantify and predict unpredictable quantities by using assumed numbers and formulas. They need to be supplied with scenarios building on alternative assumptions. Fake or fiddle stories are 'what-then' stories that quantify and predict unpredictable qualities by using fake numbers and formulas. They need to be replaced by word stories.

Any other specific requirements i.e. multimedia facilities needed to run the activities.

Participants could bring matches, snap cubes, and a pegboard with rubber bands.

Workshops will have a professional focus, not a commercial one.

There is no commercial focus. Professionally it connects to our discussion group proposal "Decolonizing mathematics, can that secure numeracy for all, and be protected from AI?" as well as to my two papers/ posters in Topic Study Groups 3.4 and 5.10, "Modeling Eased by Demodeling and Rerooting", and "A Text-Free Math Education Found by Difference Research for Protection Against Alien Artificial Intelligence."

References

Armstrong, J. & Jackman, I. (2023). *The decolonisation of mathematics.* arXiv:2310.13594.

Sartre, J.P. (2007). *Existentialism is a humanism.* Yale University Press.

Tarp, A. (2001). *Fact, fiction, fiddle - three types of models,* in J. F. Matos & W. Blum & K. Houston & S. P. Carreira (Eds.), Modelling and Mathematics Education: ICTMA 9: Applications in Science and Technology. Proceedings of the 9th International Conference on the Teaching of Mathematical Modelling and Applications (pp. 62-71). Chichester UK: Horwood Publishing.

Tarp, A. (2012). *ICME 10-15 papers.* Mathecademy.net/ICME-papers/

Tarp, A. (2018). Mastering Many by counting, re-counting and double-counting before adding on-top and next-to. *Journal of Mathematics Education, 11*(1), 103-117.

Tarp, A. (2020). De-modeling numbers, operations and equations: From inside-inside to outside-inside understanding. *Ho Chi Minh City Univ. of Education Journal of Science 17*(3), 453-466.

Tarp, A. (2021). *Flexible bundle numbers develop the child's innate mastery of Many*. https://youtu.be/z_FM3Mm5RmE. And other 'MrAlTarp' YouTube videos on deconstruction.

Tarp, A. (2022). *A decolonized curriculum.* Mathecademy.net/a-decolonized-curriculum/

Widdowson, H. G. (1978). *Teaching language as communication.* Oxford University Press.

DECOLONIZING MATHEMATICS, CAN THAT SECURE NUMERACY FOR ALL, AND BE PROTECTED FROM AI?

A discussion group proposed by Shuhua An at California State University Long Beach, and Le Trung at Ho Chi Minh City University of Education, and Allan Tarp at MATHeCADEMY.net, Denmark

The purpose

This group will discuss the question: 'Quality Education', the fourth of the 17 UN Sustainable Development Goals, has as a goal target to "By 2030, ensure that all youth and a substantial proportion of adults, both men and women, achieve literacy and numeracy". If decolonizing mathematics is relevant in this connection, how could it take place, and how to protect it from AI?"

Introduction

"That is not four. That is two twos." Said a 3year-old when asked "How many years next time?" and seeing 4 fingers 2 by 2. Which indicates that children have their own number-language before they are asked to shift to the school's version. The child sees what exists, bundles of twos in space serving as units when counted in time. And as in the word-language, the child's number-language also uses a full sentence with an outside existing subject, a linking verb, and an inside predicate.

The school thus could help children to further develop their own number-language that uses two-dimensional bundle-numbers with units where multiplication always holds by simply changing the unit, e.g., from 4s to tens where 3 x 4 = 12 states that 3 4s may be recounted in tens as 1.2 tens.

So, by what right and how ethical is it when the school imposes upon the children its own one-dimensional non-unit numbers where addition only holds without units inside the school but seldom with units outside the school where 2+1 = 3 is often falsified, e.g., by 2 days + 1week = 9 days?

To separate reliable 'multiplication-math' from unreliable 'addition-math' the latter should maybe be called 'mathematism', true inside but seldom outside school. But then, why teach addition of non-unit numbers inside when students outside need to add numbers with units?

We therefore could ask: To impose unreliable addition of one-dimensional non-unit numbers upon students that use multiplication in their two-dimensional unit numbers, isn't that an example of "a colonization of the life world by the system", the key concept in the sociology of Jürgen Habermas?

If so, then we should ask if demodeling could be used to bring inside concepts back once more to its outside roots in order to decolonize mathematics and its education to meet the fourth of the 17 UN Sustainable Development Goals that is called 'Quality Education' aiming at "ensuring inclusive and equitable quality education and promoting lifelong learning opportunities for all". And having as a goal target to "By 2030, ensure that all youth and a substantial proportion of adults, both men and women, achieve literacy and numeracy". Decolonization will not be easy as seen by different definitions of 'numerate'. The English Oxford Dictionary defines it as being "competent in the basic principles of mathematics, esp. arithmetic". In contrast, the American Merriam-Webster dictionary defines it as "having the ability to understand and work with numbers."

In their common history, England once colonized America. So, the difference in the definitions is interesting. The former uses the passive term 'being' where the latter uses the active term 'having'. The former only includes the inside institutionalized essence that has already colonized numeracy, mathematics, and arithmetic. While the latter connects the definition directly to the outside existence of numbers. So, a search for ways to decolonize should use only the methods coming from the philosophy, sociology, and psychology of the colonized, i.e., from American pragmatism and symbolic interactionism, from Piagetian psychology, and from existentialist philosophy holding that existence should precede essence to prevent the latter from colonizing the former.

One example of a decolonized mathematics education that respects the children's bundle-numbers with units may be found in the article "Mastering Many by counting, re-counting and double-counting before adding on-top and next-to." Using postmodern deconstruction, this article shows that a 'counting before adding' approach leads to the same concepts as a traditional approach but with different identities, and in a different order. Also, working with 2D multiplication-numbers on a ten-by-ten 'Bundle-Bundle-Board' will allow learning to be text-free out of the reach of AI.

Counting in 3s leads to 9 as a bundle-bundle, a B^2, which leads on to squares, square roots, and quadratics. Counting transforms the operations into icons where division and multiplication become a broom and a lift that pushes-away bundles to be stacked later as shown when recounting 8 in 2s as 8 = (8/2) x 2, or with T and B for Total and Bundle, T = (T/B) x B, that creates per-numbers when recounting in physical units, $ = ($/kg) x kg. Subtraction becomes a rope that pulls-away the stack to find the unbundled that placed on-top of the stack as part of an extra bundle become decimals, fractions, or negatives, e.g., 9 = 4B1 = 4½ = 5B-1 2s.

Finally, addition becomes a cross showing the two ways to add stacks, on-top using the linearity of recounting to make the units like, or next-to creating integral calculus by adding areas, that is also used when adding per-numbers needing to be multiplied to areas before adding. All this provides an 'Algebra Square' showing how to unite the four types of existing numbers: multiplication and addition unite like and unlike unit-numbers, and power and integration unite like and unlike per-numbers. And how to split totals with the opposite operations: division and subtraction, together with root or logarithm and differentiation.

Reuniting like and unlike unit- and per-numbers is "ability to understand and work with numbers" to produce quantitative tales, reports, and literature; and to discuss to which genre they belong, fact or fiction or fake. Which will allow a communicative turn in the number-language as the one that took place in foreign language education in the 1970s allowing all to use the English language without first knowing its abstract grammar. Which again may create a world where numeracy is no longer a privilege of an elite colonizing the number-language with unreliable mathematism.

Some questions

With a 'decolonized' and a 'colonized' version of math education we ask: What are advantages and shortcomings of the two approaches? Do both "ensure inclusive and equitable quality education and promote lifelong learning opportunities for all"? Can both be protected from Artificial Intelligence?

Call for papers

The discussion group invites participants to submit a four-page paper or essay before May 15 using the conference template. The papers will all be accessible on the MATHeCADEMY.net website. Some will be chosen for a five-minute presentation followed by a ten-minute discussion.

References

Armstrong, J. & Jackman, I. (2023). *The decolonisation of mathematics.* arXiv:2310.13594.

Habermas, J. (1981). *Theory of communicative action.* Beacon Press.

Tarp, A. (2012). *ICME 10-15 papers.* Mathecademy.net/icme-papers/

Tarp, A. (2018). Mastering Many by counting, re-counting and double-counting before adding on-top and next-to. *Journal of Mathematics Education, 11*(1), 103-117.

Tarp, A. (2020). De-modeling numbers, operations and equations: From inside-inside to outside-inside understanding. *Ho Chi Minh City Univ. of Education Journal of Science 17*(3), 453-466.

Tarp, A. (2022). *A decolonized curriculum.* Mathecademy.net/a-decolonized-curriculum/

Widdowson, H. G. (1978). *Teaching language as communication.* Oxford University Press.

ABSTRACTS ON THE ICME15 WEBSITE

Discussion group

This group will discuss the question: ‘Quality Education’, the fourth of the 17 UN Sustainable Development Goals, has as a goal target to “By 2030, ensure that all youth and a substantial proportion of adults, both men and women, achieve literacy and numeracy”. If decolonizing mathematics is relevant for this, how could it take place, and how to protect it from AI?”

“That is not four! That is two twos.” Said a 3year-old when asked “How many years next time?” and seeing 4 fingers held together 2 by 2. This indicates that children have a different uncolonized number-language before it is colonized by the school’s version. The child sees what exists, bundles of twos in space serving as units when counted in time. The school thus could help children to further develop their original number-language that uses two-dimensional bundle-numbers with units where multiplication always holds by simply changing the unit, e.g., from 4s to tens where 3 x 4 = 12 states that 3 4s may be recounted in tens as 1.2 tens.

So, by what right and how ethical is it when the system imposes upon the children its essence with one-dimensional non-unit numbers where addition without units holds only inside the school but seldom outside where 2+1 = 3 may be falsified with units: 2 days + 1week = 9 days? In sociology, Habermas calls this “a colonization of the life world by the system”. May 'existence precedes essence' be a text-free decolonization of mathematics protected from AI?

This discussion group is coordinated with the workshop "Decolonizing mathematics when demodeling it by using the child’s uncolonized 2D bundle-numbers with units".

Workshop

The workshop aims at experiencing how ‘existence precedes essence’, the slogan of existentialist philosophy, may create a different ‘counting precedes adding’ mathematics built from the child’s uncolonized number-language using the bundle-numbers with units that occur when a 3year-old reacts to four fingers held together two by two by saying “That is not four, that is two twos!” The child thus sees what exists, bundles of twos in space that serve as units when later counted in time. An ‘existence-based unit-number mathematics’ may perhaps serve as a decolonization of the traditional ‘essence-based non-unit mathematics’. Furthermore, it is outside the reach of AI by being text-free since, with units, numbers as 2 3s become physical rectangles on a ten-by-ten Bundle-Bundle-Board.

In a ‘essence after existence’ de-modeling, linearity and calculus enters immediately when asking “2 3s is how many 5s?”, and “2 3s and 4 5s total how many 8s?” Both answers is predicted by a ‘recount-formula’, Total = (Total/Bundle)xBundle, or T = (T/B)xB, exemplified by 8 = (8/2)x2 saying that “the number of 2s in 8 is 8 push-away 2”. Digits have already entered as icons with as many strokes as they represent, and now division and multiplication enter as icons for a broom and a lift pushing-away and stacking bundles. Likewise, subtraction enters as an icon for a rope to pull-away the stack to find unbundled that are placed on-top of the stack as decimals, fractions, negatives (‘underloads’), or ‘overloads’, e.g., 9 = 4B1 = 4½ = 5B-1 = 3B3 2s.

This workshop is coordinated with the discussion group " Decolonizing mathematics, can that secure numeracy for all, and be protected from AI?".

DE-COLONIZING MATHEMATICS BY DE-MODELING & RE-ROOTING

A digital poster

ICME-15
7-14 July 2024 • ICC Sydney, Australia
Come and be counted

TSG 3.4: Mathematical applications and modelling in mathematics education

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De-colonizing mathematics by de-modeling & re-rooting

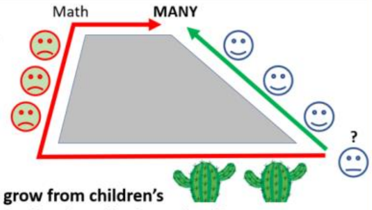
De-modeled & re-rooted, de-rooted math may finally change from 'mathematism' and its no-unit regime to Many-math, a natural science about Many in time and space, where counting precedes adding, to let existence precede essence as philosophical existentialism holds.

So, let mastery of Many precede mastery of math

The Goal of Math Education, is it well Defined?
What comes first: to master Math - or to master Many?

All say:
The goal is to master Math,
to later master Many.
But Math is hard!
Why not first master Many,
to later master Math?

So, we may ask:



What Mathematics may grow from children's innate mastery of Many, as developed before school?

MATHeCADEMY.net - Math as MANYmath - a Natural Science about MANY

Findings:

"That is not four, that is two twos". Said a 3year old child when asked
"How many years next time?" And seeing 4 fingers held together 2 by 2.

So, un-colonized by the 1D number **no-unit essence-regime**, children describes existence by 2D **bundle-numbers with units**. A **curriculum** built on **existence before essence** and **counting before adding**, will lead directly to the core of mathematics:

- digits & operations as **icons** • **re-counting** to shift units and to solve equations • fractions and **trigonometry** as **per-numbers**
- **add on-top** after recounting gives like units • **add next-to** by **calculus**, also adding **piecewise & locally constant per-numbers.**

There are two numbers-types in the world: UNIT-numbers & PER-numbers
which may be unlike or like, & which may be united or split

The aim of math education therefore is not to 'learn to math', because math is not an action verb, but to actively act to:	**Re-unite unlike & like UNIT-numbers & PER-numbers**

- 3\$ and 2\$ are unlike unit-numbers where the calculation 3+2 = 5 predicts the result of uniting them.
- 3 times 2\$ are like unit-numbers where the calculation 3*2 = 6 predicts the result of uniting them.
- 3 times 2% are like per-number where the calculation 102%^3 = 106.12% predicts the result of uniting to 6% and 0.12% extra.
- Unlike per-numbers as mixture: 2kg at 3\$/kg and 4kg at 5\$/kg. Here, the unit-numbers 2 and 4 add directly while the per-numbers 3 and 5 must first be multiplied to unit-numbers before adding as areas, called integration, where multiplication precedes plus: T = (2+4)kg to (2*3+4*5)\$, i.e., 6kg á 26/6\$/kg. In Aarabic, 'algebra' means to re-unite.

Unite / *split into*	Unlike	Like
UNIT-numbers (meter, second)	**T = a + b** *T – b = a*	**T = a*b** *T/b = a*
PER-numbers (m/sec, m/100m = %)	**T = ∫ f dx** *dT/dx = f*	**T = a^b** *b√T = a* *loga(T) = b*

- Tarp, A. (2018). *Mastering Many by counting, re-counting and double-counting before adding on-top and next-to*. Journal of Math. Education, 11(1), 103-117.
- Tarp, A. (2019). *A decolonized curriculum*. Mathecademy.net/a-decolonized-curriculum/
- Tarp, A. (2020). *De-modeling numbers & operations: From inside-inside to outside-inside understanding*. Ho Chi Minh City Univ. of Educ. Journal of Science 17(3), 453-466.
- Tarp, A. (2021). *Flexible Bundle Numbers Develop the Childs Innate Mastery of Many*, youtu.be/z_FM3Mm5RmE.

TSG 3.4: Mathematical applications and modelling in mathematics education

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What should name a mathematical concept, *what exemplifies it OUTSIDE - or from what it exemplifies INSIDE?*

The goal of math education, is that to learn to master math to later master Many, or the other way around?

- Traditionally, the goal of math education is seen as learning to master math to later master Many. So, a difference could be to see the goal of math education as learning to master Many directly to indirectly learning math on the way, at least the math core as displayed on a calculator: digits, operations, and equations.
- Traditionally, these all occur as **products** in space, so a difference could be to see them as **processes** in time by letting **outside-Many precede inside-math**. And the math core is different as **tales about Many** existing as rectangular totals of bundle-stacks on a plastic ten-by-ten bundle-bundle board, a **BBBoard**.
- To see if a 'process-based' 'Many-first' education will make a difference to the traditional 'product-based' 'Math-first' education, **micro-curricula** are designed using **flexible bundle-counting** to bring outside totals inside as **flexible bundle-numbers with units**, that are rectangular where the bundle-bundles are squares.
- Here both **digits and operations are icons**. Digits when uniting sticks. And operations with division to **push-away** bundles that multiplication **lifts** into a stack. Subtraction **pulls-away** stacks so unbundled are included as decimals, fractions, or negatives. The addition-cross shows the **two ways to add**, next-to & on-top.
- Once counted, **changing unit** may be predicted on a calculator by **the recount formula T = (T/B) x B**, saying that the total T contains T/B Bundles. Here recounting from tens to icons and vice versa leads to **equations**, and to **multiplication tables** existing as the stack left when removing the 2 surplus stacks from the full bundle-bundle on a BBBoard. And here recounting a rectangle as a square introduces its side as the **square root**, and a way to solve quadratics.
- Here recounting in two physical units leads to **per-numbers** as 4$/5kg bridging the two units; and becoming fractions with like units, 4$/5$ = 4/5, 4$/100$ = 4%.
- Here mutual recounting the sides and the diagonal in a stack leads to **trigonometry before geometry**.
- Once counted, totals may **add on-top** after recounting makes the units like or **add next-to** as areas as integral calculus becoming differential calculus if reversed.
- **Per-numbers and fractions are operators** needing numbers to become numbers, so also adding by their areas after being multiplied to unit-numbers to add.
- So, outside totals inside appear in an '**Algebra Square**' where unlike and like unit-numbers and per-numbers are united by addition and multiplication, and by integration and power. And later again split by the reverse operations: subtraction and division, and by differentiation and factor-finding root or -counting logarithm.
- Once process-based Many-first Many-math micro curricula have been designed, they are **tested** in online education, as well as **in special education** to see if **BBBoards 'Bring Back Brains'** excluded from the 'Math-first' education.

Details: A. Tarp. "*ManyMath 2030: Decolonizing 1D Mathematism into 2D Many-math*", http://mathecademy.net/manymath-2030/

TSG 3.4: Mathematical applications and modelling in mathematics education Allan.Tarp@gmail.com , MATHeCADEMY.net

2 Colonizations: Bundle-numbers by Line-numbers, then by abstract SETS

De-colonization by de-modeling & re-rooting & de-construction will change

	ESSENCE-math, Mathematism into	EXISTENCE-math, ManyMath
Digits	Symbols	Icons
345	Place value system	T = 3**BB** 4**B** 5, **BB** = **B**^2, **BBB** = **B**^3
Operations	Functions, order: **+ - x / ^**	Icons, opposite order: **^ / x - +**
3 + 4	3 + 4 = 7	Meaningless with no units
3 * 4	3 * 4 = 12	3 * 4 = 3 **4s** *that may be recounted to 1.2 tens*
9 = ? 2s	Meaningless, only ten-counting	9 = 3**B**3 = 5**B**-2 = 4**B**1 = 4½ **2s**
8 = ? 2s	Meaningless, only ten-counting	8 = (8/2)*2, **T = (T/B)*B**, proportionality
2*u = 8	(2*u)*½ = 8*½, (u*2)*½ = 4, u*(2*½) = 4, u*1 = 4, u = 4	2*u = 8 = (8/2)*2, så u = 8/2
6*7 = ?	eh, 44? eh, 52? eh, 42? OK	(**B**-4)*(**B**-3) = (10-4-3)**B**+12 = 3**B**12 = 4**B**2
4kg=5$, 6kg=?	1kg = 5/4$, 6kg = 5/4*6$	6kg = (6/4)*4kg = (6/4)*5$
1/2 + 2/3 = ?	1/2 + 2/3 = 3/6 + 4/6 = 7/6	1/2*2 + 2/3*3 = 3/5*5
2 3s + 4 5s	2*3+4*5 is 10*5 = 50, or 6+20= 26, decided by definition	2**B**0 **3s** + 4**B**0 **5s** = 3**B**2 **8s**, integration
6 + 9 = ?	6 + 9 = 15	1**B**0 **6s** + 1**B**3 **6s** = 1**B**-3 **9s** + 1**B**0 **9s** = 2**B**3 **6s** = 2**B**-3 **9s** = 1**B**5 **tens** = 15
Tangent = ?	Tangent = sine/cosine	raise = (raise/run)*run, tan = raise/run

2+3 = 5, seldom, since 2weeks + 3days = 17days. **Mathematism adds without units, true inside & maybe outside.**
2x3 = 6, always, since 2 3s = 6 1s. **ManyMath adds with units, true inside & outside.**

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