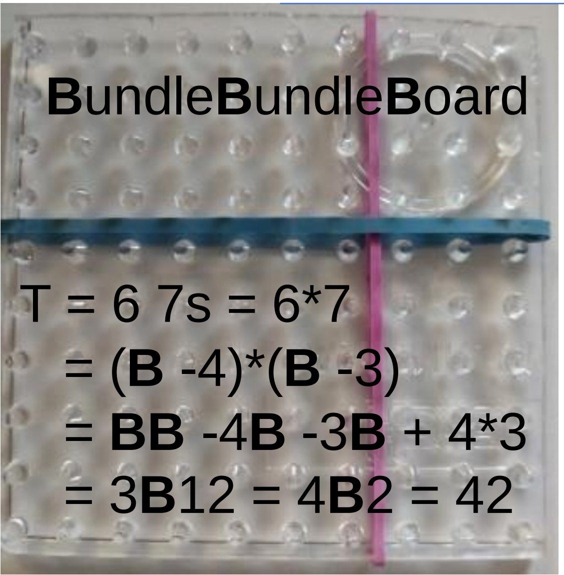
Many before Math!

Mathematics DeColonized by the Child’s own 2D BundleBundle Numbers



Make Math Easy with Many-Math
where Existence precedes Essence

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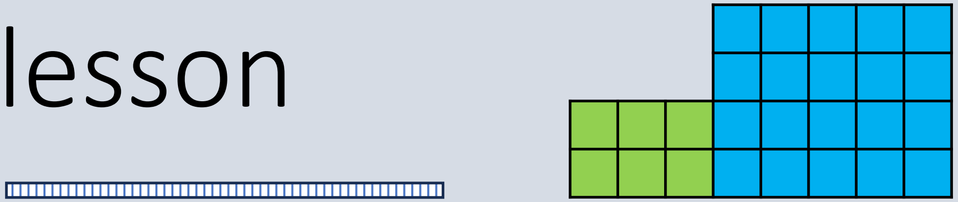
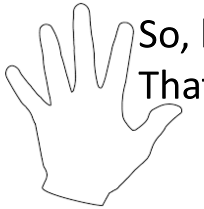
My first math lesson



Hello, my name is Allan. In my first lesson I said "Welcome children, I am your teacher in mathematics, which is about the numbers that you can see on this number line, and that is built upon the fact that one plus one is two as you can see here. So …"

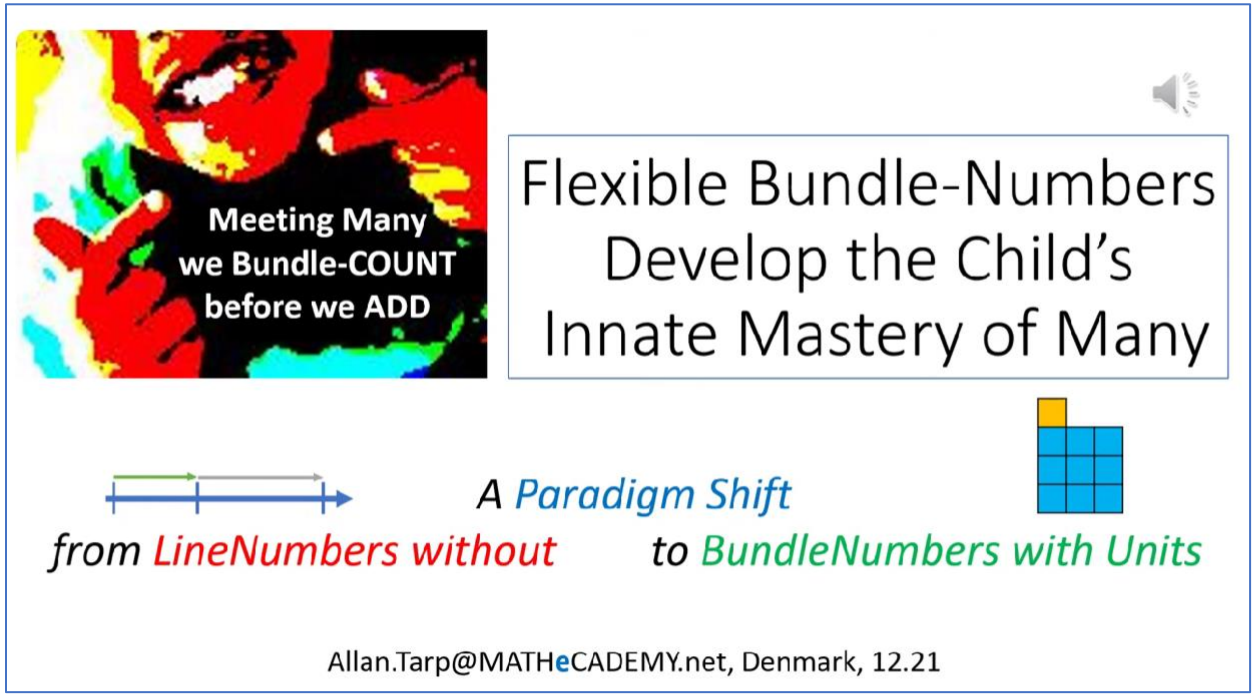
Then a child stopped me, holding four fingers together two by two: "Mister teacher, here is one, and here is one. You say it is two, but we can all see, that it is four". The child separated the fingers, and then held three fingers together on both hands before separating them. "And, here we see that two times three gives six. So, multiplication can be trusted, but not addition. Therefore, please take away your number line addition with its flat-earth-math. Instead, please help us with the numbers we bring to school, multiplication bundle-numbers as 2 3s and 4 5s that we can see on this ten-by-ten peg board, a Bundle-Bundle board, or a BBBoard. And that we would like to add next-to as eights, or on-top as 3s or 5s. If we add them next-to, we add areas, which my uncle calls integral calculus, and if we add them on-top the units must be changed to the same unit, which my uncle calls proportionality. He says it is taught the first year at college, but we need it here to keep and develop the bundle-numbers with units we bring to school instead of having them colonized by your line numbers without units.

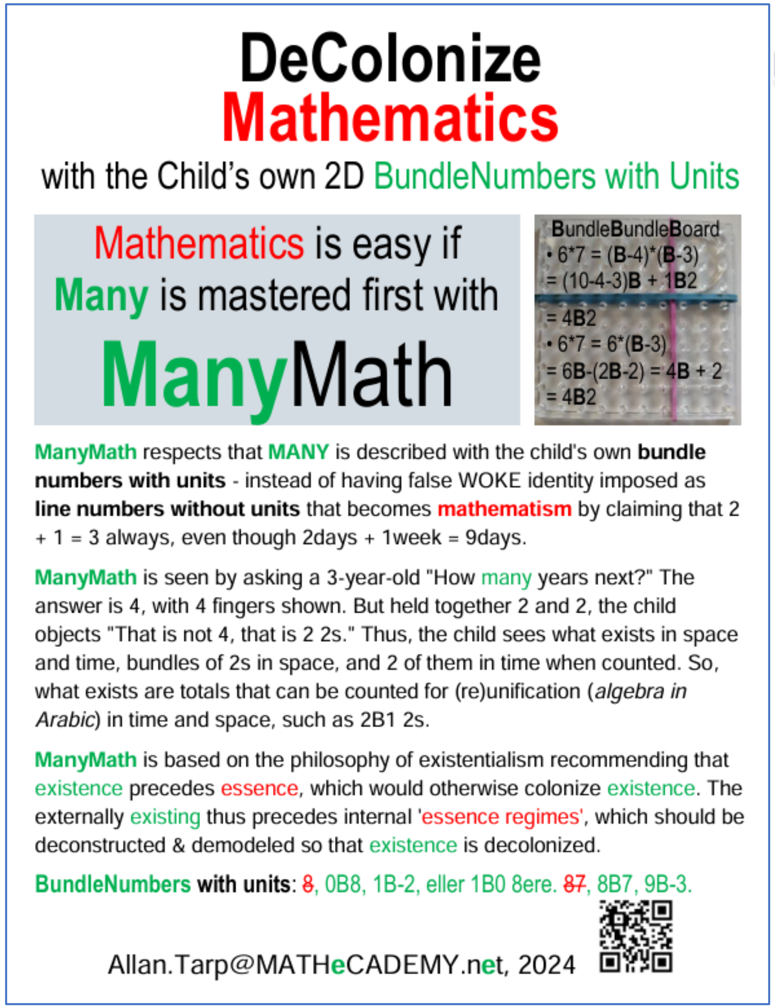
We know that you prefer to bundle in tens, and in ten-tens, and in ten-ten tens, but we like to bundle also in 2s, in 3s, in 4s, in half-tens, etc. And we know that you have not been taught this and that the textbook doesn't teach it, but don't worry, we will teach you. Or better, instead of you colonizing our ways let us find out together what math may grow from our bundle-numbers with units. My uncle is a philosopher, and he calls it existentialism when you treat existence before essence.



So, let us begin with the fingers on a hand. You only see the essence, five, but we see all the ways the fingers may exist as five. That is, as 5 *1s*, as 1 *5s* or 1 **B**undle 0 *5s*, as 1**B** 1 *4s*, as 1**B** 2 *3s* or 2**B** less 1 *3s*, as 1**B** 3 or 2**B**1 or 3**B** less 1 *2s*, or as 1**BB** 0**B** 1 *2s*."

What happened next is seen in this workshop↓ & this folder→





DeColonize Mathematics

with the Child's own 2D BundleNumbers with Units

Mathematics is easy if Many is mastered first with **ManyMath**

BundleBundleBoard
• 6*7 = (B-4)*(B-3) = (10-4-3)B + 1B2 = 4B2
• 6*7 = 6*(B-3) = 6B-(2B-2) = 4B + 2 = 4B2

ManyMath respects that **MANY** is described with the child's own **bundle numbers with units** - instead of having false WOKE identity imposed as **line numbers without units** that becomes **mathematism** by claiming that 2 + 1 = 3 always, even though 2days + 1week = 9days.

ManyMath is seen by asking a 3-year-old "How many years next?" The answer is 4, with 4 fingers shown. But held together 2 and 2, the child objects "That is not 4, that is 2 2s." Thus, the child sees what exists in space and time, bundles of 2s in space, and 2 of them in time when counted. So, what exists are totals that can be counted for (re)unification (*algebra in Arabic*) in time and space, such as 2B1 2s.

ManyMath is based on the philosophy of existentialism recommending that existence precedes essence, which would otherwise colonize existence. The externally existing thus precedes internal 'essence regimes', which should be deconstructed & demodeled so that existence is decolonized.

BundleNumbers with units: ~~8~~, 0B8, 1B-2, eller 1B0 8ere. ~~87~~, 8B7, 9B-3.

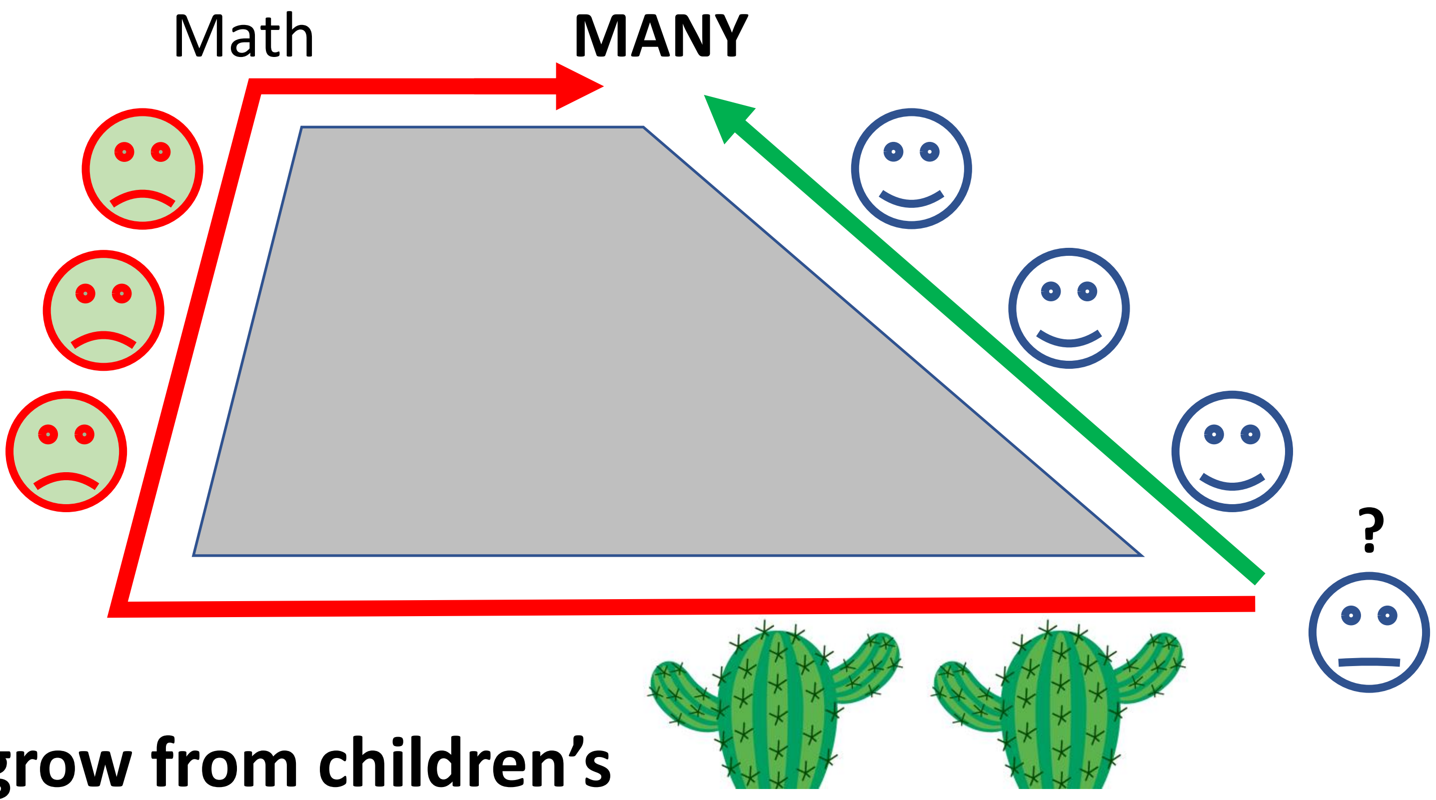
Allan.Tarp@MATHeCADEMY.net, 2024

Workshop: Flexible Bundle Numbers
Develop the Childs Innate Mastery of Many
https://youtu.be/z_FM3Mm5RmE

Mathematics is easy if Many is mastered first with ManyMath
http://mathecademy.net/bundle-bundle-numbers-with-units/

The Goal of Math Education, is that well Defined? What comes first: to master Math - or to master Many?

All say:
The goal is to master Math,
to later master Many.
But Math is hard!
Why not first master Many,
to later master Math?



So, we may ask:

What Mathematics may grow from children's innate mastery of Many, as developed before school?

What to master first, Many or Mathematics? Many, because that makes math just so easy

- Many-math respects that MANY is described by the child's own bundle-numbers with units. Instead of having a false identity imposed with line-numbers without units that becomes ‘Mathema-tism’ by claiming that 2 + 1 = 3 always, despite the fact that 2days + 1week = 9days.
- Many-math is seen by asking a 3-year-old "How many years next?" Typically, the answer is 4, with 4 fingers shown. But held together 2 and 2, the child says “No, that is not 4, that is 2 2s." Thus, the child sees what exists in space and time, bundles of 2s in space, and 2 of them in time when counted. So, what exists are totals that can be counted and united in time and space, like T = 2 2s.
- Many-math is based on the philosophy called existentialism that holds that existence precedes essence.
- That is, what exists outside precedes what is said about it inside.

Aha, with existence before essence counting comes before adding

"No, that is not four, that is two twos". This statement will change mathematics education forever since, as educated, essence is all we see. But as uneducated, the child sees what exists, bundles of twos in space, and two of them when counted in time.

The number 'two' thus exists both in space and in time.

In space, 2 exists as 2s, a space number, a bundle of 2s, a 2-bundle, which can be united with a 3-bundle. Either horizontally to a (2+3) bundle, a 5-bundle, or vertically to a stack of 2B1 2s or to 2B-1 3s with B for bundle.

Here 1 plus 1 does not add up to 2 since the units are not the same, whereas one 2-bundle + one 2-bundle may both be two 2-bundles or one 4-bundle, but not 2 4-bundles.

In time, 2 exists together with the unit that was counted, as 2 units, as a time-number, or a counting-number. So, 2+3 is 5 only with like units. Without units, a counting-number is an operator to be multiplied with a unit to become a total that can be added with another total if the units are the same, or after the units are made the same by recounting the two totals in the same unit.

So, as space-numbers, 2+3 is 5, whereas as time-numbers 2+3 is undecided until their units are known.

The child thus opens our eyes for a different mathematics that, freed from its present essence-bounds, may return to its original identity as a natural science about Many in space and in time. This decolonization allows us to develop a natural number-language to communicate about outside existence instead of about inside essence only.

If existence comes before essence, then counting comes before adding, which is new since normally we just get numbers to add."

Kids use units (2 **3s**), Schools don't. So Math Differs INSIDE & OUTSIDE the 'NoUnitMath' GreenHouse

OUTSIDE

ManyMath

2+1 = ??

depends on the units

2weeks + 1day = 15days

2m + 1m = 3m

2km + 1mm = 2km

20$ + 10% = 22$

INSIDE

NoUnitMath GreenHouse

2+1 = 3 always

MatheMatism

True inside, but seldom outside

2+3 is 5 sometimes. But 2*3 is 6 always. Why? Because here 3 is the unit!

Many-math uses bundle numbers with units

With units, there are only four kinds of numbers in the world: unit-numbers and per-numbers that can be unlike or like, and that may be reunited. The aim of mathematics is therefore not to 'math', because math is not an action word, but to act:

"ReUnite Unlike & Like Unit-Numbers & Per-Numbers"

- 3$ and 2$ are unlike unit-numbers where the calculation 3+2 = 5 predicts the result of uniting them.
- 3 times 2$ are like unit-numbers where the calculation 3*2 = 6 predicts the result of uniting them.
- 3 times 2% are like per-number where the calculation 102%^3 = 106.12% predicts the result of uniting them to 6% and 0.12% extra in 'compound interest'.
- Unlike per-numbers we meet in mixtures, 2kg at 3$/kg plus 4kg at 5$/kg. Here, unit-numbers 2 and 4 unite directly, while the per-numbers 3 and 5 must first be multiplied to unit-numbers before uniting as areas, called integration, where multiplication precedes plus: T = (2+4) kg at (2*3 + 4*5)$, or 6 kg at 26/6 $/kg.

2KG AT 3$/KG
4KG AT 5$/KG
6KG AT 8$/KG

The algebra square shows the 4 types of numbers in the world, unlike & like unit- & per-numbers

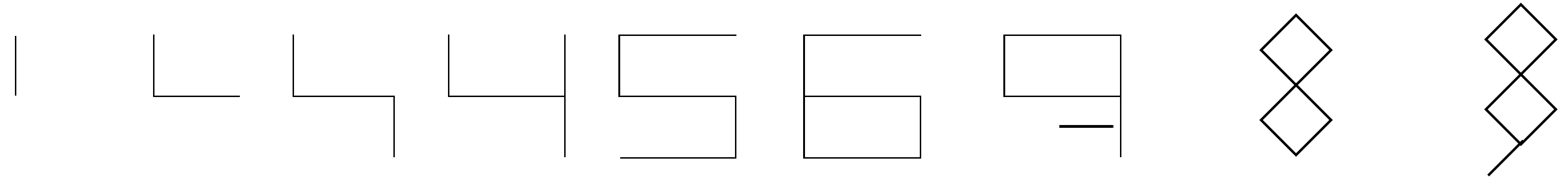
- The Arabic word Algebra means to reunite, that is, to unite and split. Numbers are united in four ways: Addition unites unlike unit-numbers, multiplication unites like unit-numbers, integration unites unlike per-numbers, and power unites like per-number, as you add 5% by multiplying by 105%.
- The opposite is to split into: subtraction splits into unlike unit-numbers, division splits into like unit-numbers, differentiation splits into unlike per-numbers, and the factor-finding root and factor-counting logarithm splits into like per-numbers.

Unite ***Split into***	**Unlike**	**Like**
Unit-numbers (meter, second)	$\mathbf{T = a + b}$ $T - b = a$	$\mathbf{T = a*b}$ $T/b = a$
Per-numbers (m/sec, m/100m = %)	$\mathbf{T = \int f\, dx}$ $dT/dx = f$	$\mathbf{T = a\hat{}b}$ $\sqrt[b]{T} = a \quad \log_a(T) = b$

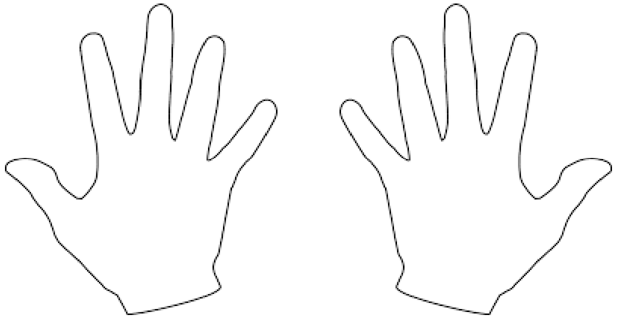
Digits are Icons: I I I I → IIII → 4 →

Children love making number-icons of cars, dolls, spoons, sticks.
Changing four ones to one fours creates a 4-icon with four sticks.
An icon contains as many sticks as it represents, if written less sloppy.
Once created, icons become **UNITS** when counting in bundles, as kids do.

one	two	three	four	five	six	seven	eight	nine
I	II	III	IIII	IIIII	IIIIII	IIIIIII	IIIIIIII	IIIIIIIII
1	2	3	4	5	6	7	8	9



Totals are counted in **Bundles** with units



- 5 fingers are counted as '1 **Bundle** 2' 3s, short as '1**B**2' 3s, or simply '12' 3s.
- Ten fingers are counted as '3**B**1' 3s, or **'1BundleBundle** 0**Bundle** 1' 3s, or '1**BB** 0**B** 1 3s, or simply '101' 3s.
- A total, T, is counted in a unit, for example, T = 3 fours, or T = 3 times 4.
- This is a number-narrative, with a subject, ‘T’, a verb, ‘equal to’, and a predicate, ‘3 times 4’.
- T = 345 has omitted the units, T = 3**BB**4**B**5, where the bundle **B** = ten.
- Counting 5 in 2s is done in 3 ways: **normal**, or '**over-load,**' or '**under-load**'

This eases calculations:

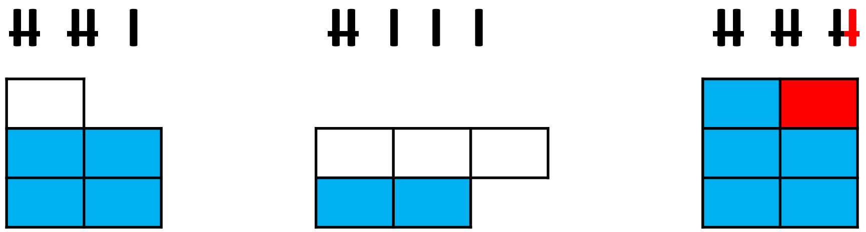
45 + 27 = 4**B**5 + 2**B**7 = 6**B**12 = 7**B**2 = 72

7 * 56 = 7 * 5**B**6 = 35**B**42 = 39**B**2 = 392

392 /7 = 39**B**2 /7 = 35**B** 42 /7 = 5**B**6 = 56

Place values and carrying are unneeded

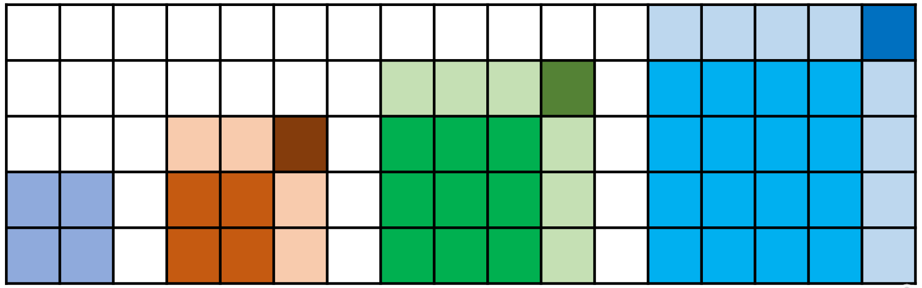
5:



5 = 2**B**1 = 1**B**3 = 3**B**-1 2s

Likewise: T = 47 = 4**B**7 = 3**B**17 = 5**B**-3 tens

Bundle-Bundles are squares



On a **BBB**oard we see, that all the bundle-bundles are squares, 2 2s, 3 3s, 4 4s, 5 5s, etc.

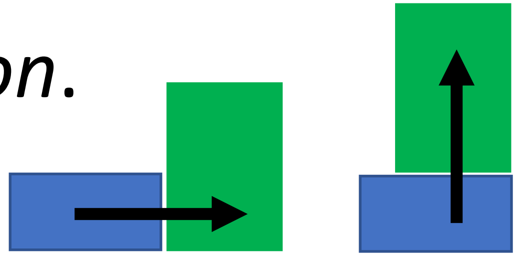
- 2 2s need 2 more 2s, and 1 corner, to be 3 3s.
  - So, 1 **BB** 3s is 1**BB** 2**B** 1 2s, or (2+2)**B** 1 2s or 4**B** 1 2s, or 0**B** 9 tens
- 4 4s need 2 more 4s, and 1 corner, to be 5 5s.
  - So, 1 **BB** 5s is 1**BB** 2**B** 1 4s, or (5+1)**B** 1 4s or 6**B** 1 4s, or 2**B** 5 tens

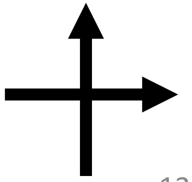
Going the other way, we see that 1 **BB** 4s comes if we from 1 **BB** 5s pull-away 2 bundles, and add the corner that we pull-away twice, which shows that minus times minus must be plus

- 1 **BB** 4s = 1**BB** -2**B** 1 5s = (5-2)**B** 1 5s = 3**B** 1 5s = 1**B** 6 tens
- 1 **BB** 2s = 1**BB** -2**B** 1 3s = (3-2)**B** 1 3s = 1**B** 1 3s = 0**B** 4 tens

Divide & Multiply & Subtract & Add may be 'de-modeled' as Icons also

- From 9 **PUSH** away **4s** we write <u>9/4</u> <u>iconizing</u> a broom, called *division*.
- 2 times **LIFT** the **4s** to a stack we write <u>2x4</u> <u>iconizing</u> a lift called *multiplication*.
- "From 9 **PULL** away 2 **4s** to find un-bundled" we write <u>9 – 2x4</u> <u>iconizing</u> a rope, called *subtraction*.
- **UNITING** next-to or on-top we write **B**+**C** <u>iconizing</u> the two directions, called *addition.*





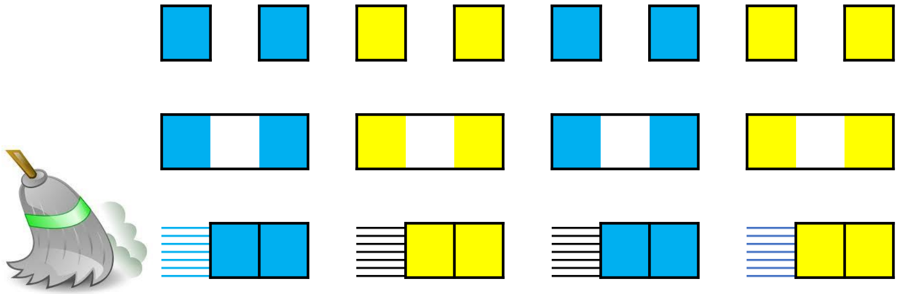
Shifting Units Creates a Recount-Formula

8 = (8/2)x2
T = (T/B)xB

Shift unit from **1s** to **2s**: 8 = ? **2s**

Bundle-counting: 8 = 4 **2s** = 4x2

Predict by a calculation: 8/2 = 4



Recount result: 8 = (8/2)x2

Recount-Formula: **T = (T/B)xB** *" **T** contains **T/B Bs**"*

*u*x2 = 8 = (8/2)x2

u = 8/2

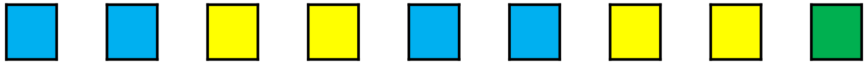
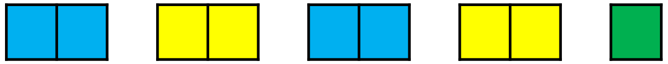
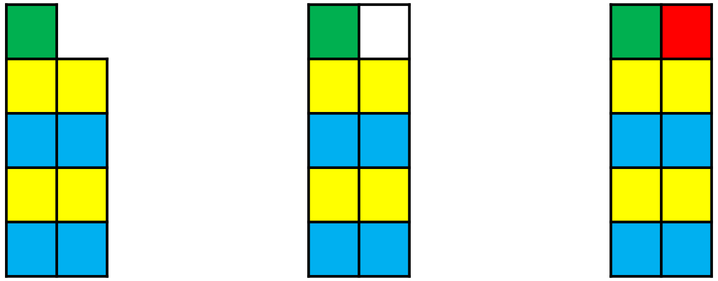
Equations Proportionality

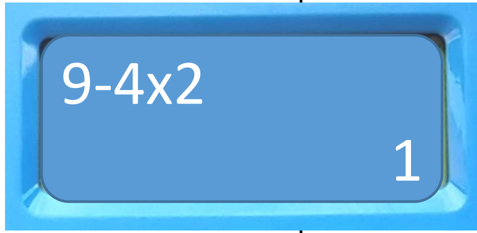
Move: OPPOSITE Side & Sign

Shifting unit	y = k * x
Linearity	Δy = (**Δy/Δx**) * Δx = **m** * Δx
Local linearity	dy = (**dy/dx**) * dx = **y'** * dx
Trigonometry	a = (**a/b**) * b = **tanA** * b
Trade	$ = (**$/kg**) * kg = **price** * kg
STEM	meter = (**meter/sec**) * sec = **speed** * sec

Calculators Predict Counting Results
Bundle-Counting a Total of 9 in **2s**

9/2 **4.some**
9 – 4x2 **1**

Acting	Predicting
 **9**	From 9, 9/2 times, push away **2s**
 *bundled in **2s** with 1 unbundled*	From 9, pull away 4 **2s**, leaves 1 Calculator prediction : T = 9 = 4***B***1 **2s**
 4.1***B*** 4½***B*** 5***B***-1 **2s** *stacked as 4x2 with 1 unbundled placed on-top*	The unbundled are placed on-top • separated by a decimal point, 4.1 **2s** • counted in bundles as 1 = (1/2) x 2 giving 4½***B*** **2s** • counting what is missing in a full bundle, 5***B***-1 **2s** So, inside decimals, fractions, and negatives are rooted in how we outside see the unbundled.



Splitting

$u + 2 = 8$	$u \times 2 = 8$	$2^u = 8$	$u^8 = 2$
$u = 8 – 2$	$u = 8/2$	$u = \log_2(8)$	$u = \sqrt[8]{2}$

The reverse of uniting is splitting, which is predicted by reverse calculations called equations, where we use the letter *u* for the unknown number.

- In the reverse calculation (equation) '*u* + 2 = 5' we ask "What is it that united with 2 gives 5?". The answer, of course, is obtained by the reverse process, by now pulling-away the 2 from 5 by minus, *u* = 5 – 2. The unknown number is thus found by moving the known number to the opposite side with the opposite calculation sign, the "OPPOSITE SIDE & SIGN" method.
- In the equation *u**2 = 6, we ask "How many 2s are in 6?". The answer, of course, is obtained by counting 6 in 2s, 6 = (6/2)*2, so *u* = 6/2. So again, by the reverse process, by pushing 2s away. So, again 'opposite side & sign'.
- In the equation 2^*u* = 8, we ask "How many factors 2 are there in 8?". The answer is obtained by the factor-counting logarithm, *u* = log2(8). So, again 'opposite side & sign'.

Equations are solved by moving the numbers to **OPPOSITE** side with **OPPOSITE** calculation sign

In the equation *u*^3 = 8 we ask: "What multiplication number is there 3 of in 8?". The answer is obtained by the factor-finding root, *u* = 3√8. So, again 'opposite side & sign'.

- In the equation 2*3 + *u**5 = 4*8, we ask "2 3s plus how many 5s give 4 8s?" The answer is obtained again by the reverse process, i.e., by pulling-away the 2 3s, and then counting the rest in 5s, also called differentiating where minus precedes division, i.e., the opposite of integrating.

$u + 2 = 8$	$u \times 2 = 8$	u^8 = 2	2^u = 8
$u = 8 - 2$	$u = 8/2$	$u = \sqrt[8]{2}$	$u = \log_2(8)$

A hymn to equations

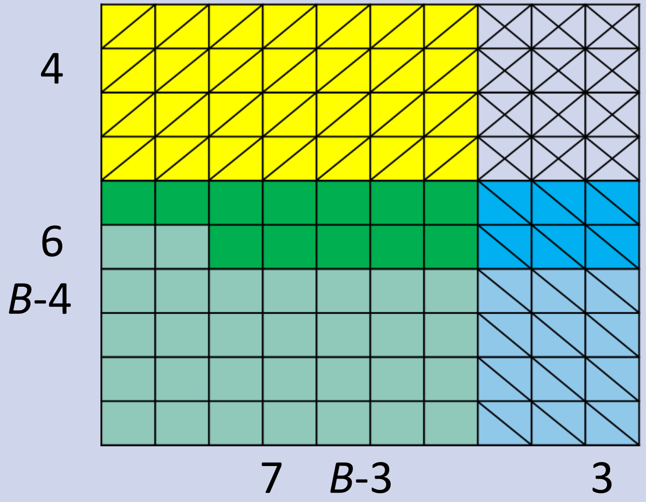
Equations are the best we know, they're solved by isolation. But first the brackets must be placed around multiplication. We change the sign, and take away, so only u itself will stay. We just keep on moving, we never give up. So feed us equations, we don't want to stop.	$3*u + 2 = 14$ $(3*u) + 2 = 14$ $3*u = 14 - 2$ $u = (14 - 2)/3$ $u = 4$

<u>Walking the equation 3*u + 2 = 14, in time:</u>

	**3*		*+2*	
u	→	**3****u***	→	**3****u* + 2**
4 = (14-2)/3	←	14-2	←	**14**
	/3		*-2*	

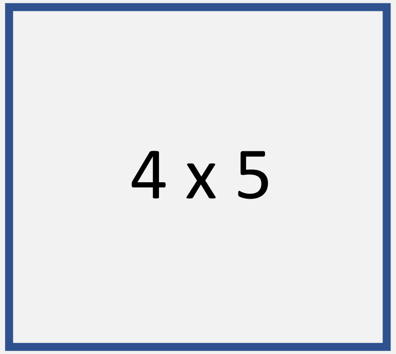
Recount between icons and 10s

- The question 'How many 8s in 32' is predicted by the equation *u**8 = 32, with the solution *u* = 32/8 since 32 recounted in 8s is 32 = (32/8)*8.
- The question 'How many tens in 6 7s' is predicted by placing them both as stacks with underload on a BundlexBundle board, a BBBoard, to learn early algebra:
- 6*7 = (**B**-4) * (**B**-3) = 10**B** - 4**B** - 3**B** + 4*3, as the 4 3s are pulled away twice.

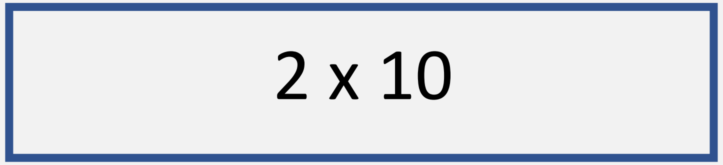
Act	Predict
	<u>T = 6 **7s** = ? **tens**</u> = 6 x 7 = (*B*-4) x (*B*-3) = $\left(\begin{matrix}B-4\\B-3\end{matrix}\right)$ = *BB* - 3*B* - 4*B* - - 4 x 3 = 3*B* + 1*B*2 = 4*B*2 = 42, so 6 **7s** is 4*B*2 **tens** *- - is + since it is pulled away twice*

Increasing the base will decrease the height. A Total may be STACK'ed, TEN'ed or SQUARE'd

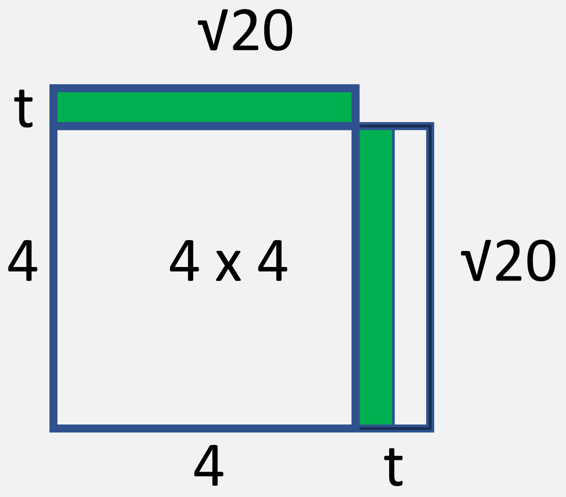
STACK'ed



TEN'ed



SQUARE'd



The surplus 4 1s is shared by the two 4xt stacks:

$8*t = 4$, or $t = 4/8 = 0.5$

So $\sqrt{20} \approx 4.5$

Calculator: $\sqrt{20} = 4.472$

Recounting gives **per-numbers** and fractions

A quantity of goods can be counted in kg and dollar connected by a **per-number** as 4kg per 5$, or 4kg/5$. We then change the unit by recounting in the per-number. This is also known as proportionality.

- Question: 20kg = ? $.
- Answer: 20kg = (20/4) * 4kg = (20/4) * 5$ = 25$.

Nature and STEM are filled with per-numbers.

- Motion can be counted in meters and seconds, where the per-number meter/second is called the speed.
- Water can be counted in grams and liters, with the per-number gram per liter.

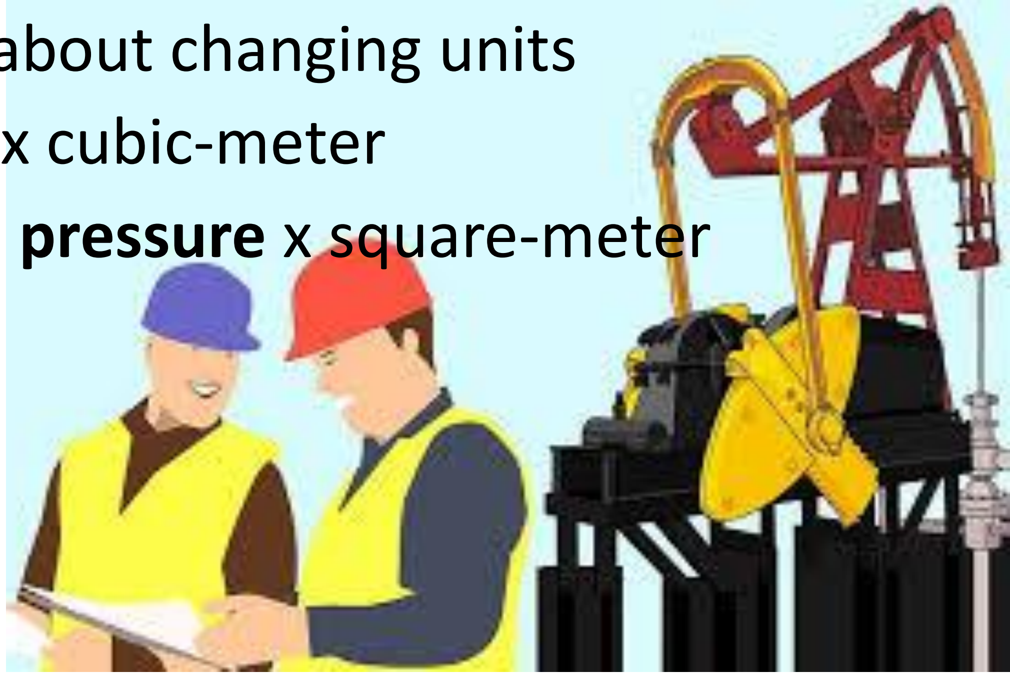
With like units, per-numbers become fractions, 4$/5$ = 4/5, and 40$/100$ = 40%

- Question: with 40$ = 100%, 8$ = ?%
- Answer: 8$ = (8/40)*40$ = (8/40)*100% = 20%
- Question: with 40$ = 100%, 80% = ?$
- Answer: 80% = (80/100)*100% = (80/100)*40$ = 32$

The ReCount formula is the core of STEM

STEM typically contains multiplication formulas about changing units

- kg = (**kg/cubic-meter**) x cubic-meter = **density** x cubic-meter
- force = (**force/square-meter**) x square-meter = **pressure** x square-meter
- meter = (**meter/sec**) x sec = **speed** x sec
- energy = (**energy/sec**) x sec = **Watt** x sec
- energy = (**energy/kg**) x kg = **heat** x kg
- gram = (**gram/mole**) x mole = **molar mass** x mole
- Δ momentum = (**Δ momentum/sec**) x sec = **force** x sec
- Δ energy = (**Δ energy/ meter**) x meter = **force** x meter = work
- **energy/sec** = **(energy/charge)** x **(charge/sec)** or **Watt** = **Volt** x **Amp**



Recounting sides in a stack halved by its diagonal gives trigonometry before geometry, and Pi

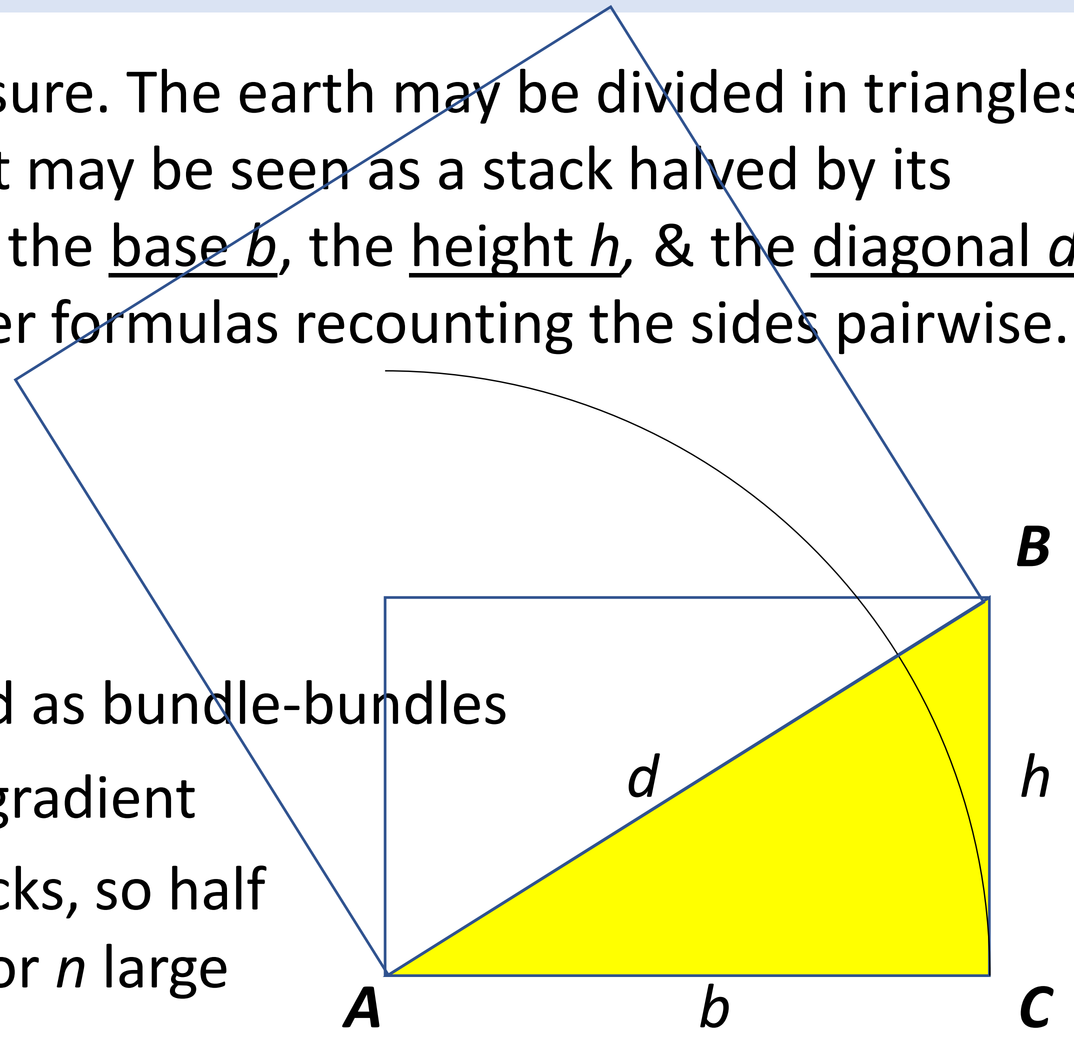
In Greek, geo-metry means to earth-measure. The earth may be divided in triangles; that may be divided in right triangles; that may be seen as a stack halved by its diagonal. This ‘half-stack’ has three sides: the <u>base *b*</u>, the <u>height *h*</u>, & the <u>diagonal *d*</u>, connected with the <u>angle *A*</u> by per-number formulas recounting the sides pairwise.

$h = (h/b) \times b = \tan\boldsymbol{A} \times b$
$h = (h/d) \times d = \sin\boldsymbol{A} \times d$
$b = (b/d) \times d = \cos\boldsymbol{A} \times d$

$\underline{h \times h + b \times b = d \times d}$, so the sides add as bundle-bundles

$\tan\boldsymbol{A} = h/b = \Delta y/\Delta x$ = rise/run = diagonal gradient

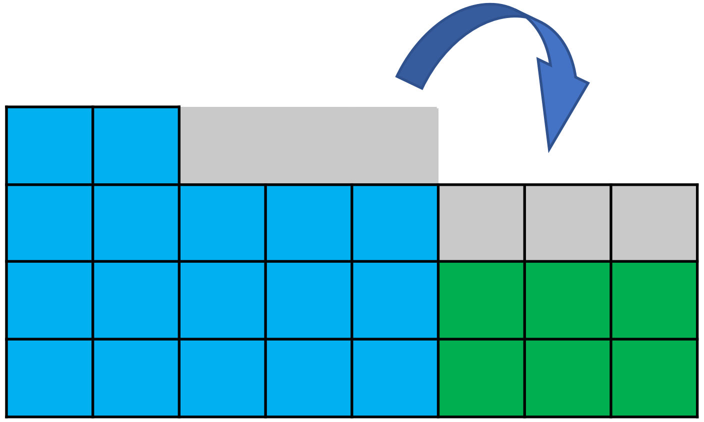
A circle contains very many small half-stacks, so half the circumference is: $\boldsymbol{\pi = n \times \tan(180/n)}$ for n large

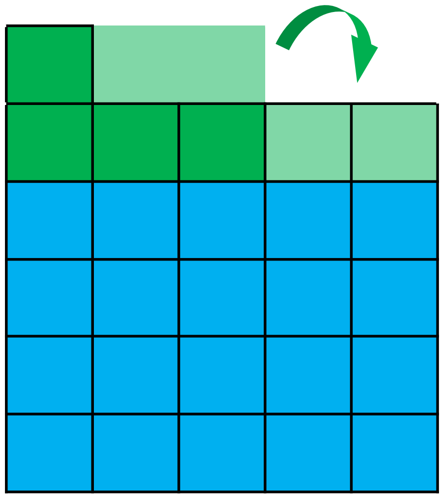


MATHeCADEMY.net : Math as MANYmath - a Natural Science about MANY

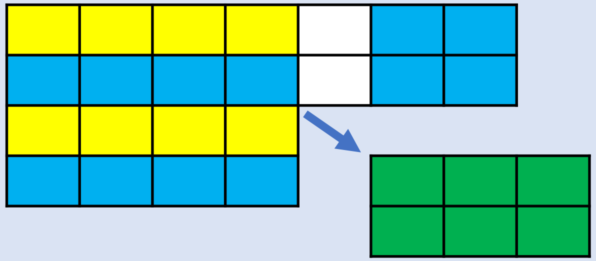
Once Counted & Recounted, Totals may Add

BUT: **NextTo** →	**or** **OnTop** ↑
4 **5s** + 2 **3s** = 3**B**2 **8s**	4 **5s** + 2 **3s** = 5**B**1 **5s**
The areas are integrated *Adding areas = Integration*	The units are changed to be the same *Change unit = ReCounting = Proportionality*





Reversing next-to addition



"If T1 = 2 **3s** and T2 add next-to as 4 **7s**, what is T2?"

We pull away the initial block T1 before recounting the rest in **4s**.

The recount formula predicts the result:

T2 = (T2/**B**) x **B**

= ((4x7 – 2x3)/4) x 4 = 5.2 **4s**

(4x7 – 2x3)/4	**5.some**
(4x7 – 2x3) – 5x4	**2**

Since reversed next-to addition finds area-differences, it is called differential calculus. Here subtraction precedes division; which is natural as reversed integration.

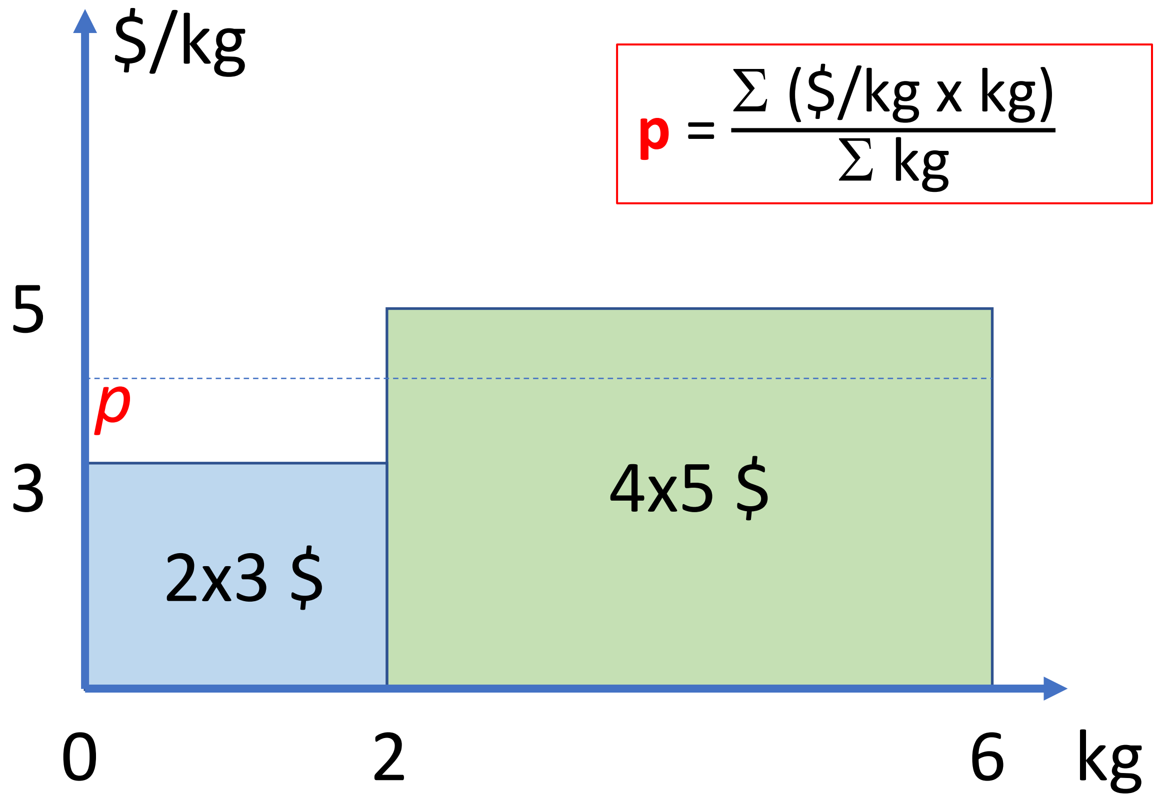
Per-numbers add as Areas (Integral Calculus)

Here, the per-number *p* is **piecewise constant**, which gives the sum Σ ($p*\Delta x$) that becomes ∫$p*dx$, if it is **locally constant**, by interchanging epsilon and delta

"2kg at **3$/kg** + 4kg at **5$/kg** = 6kg at **?** **$/kg**?"

2 kg	at	**3 $/kg**
+ 4 kg	at	**5 $/kg**
(2+4) kg	at	***p*** **$/kg**

- Unit-numbers add directly.
- Per-numbers must be multiplied to unit-numbers, thus adding as **areas** under the per-number curve.
- Here, multiplication before addition
- So, per-numbers and fractions are not numbers, but operators needing numbers to be numbers.



Subtracting PerNumbers (Differentiation)

"2kg at **3$/kg** + 4kg at **what** = 6kg at **5$/kg**?"

2 kg	at	**3**	**$/kg**
+ 4 kg	at	**?**	**$/kg**
6 kg	at	**5**	**$/kg**

We remove the initial 2x3 block, and recount the rest in 4s to get the per-number.

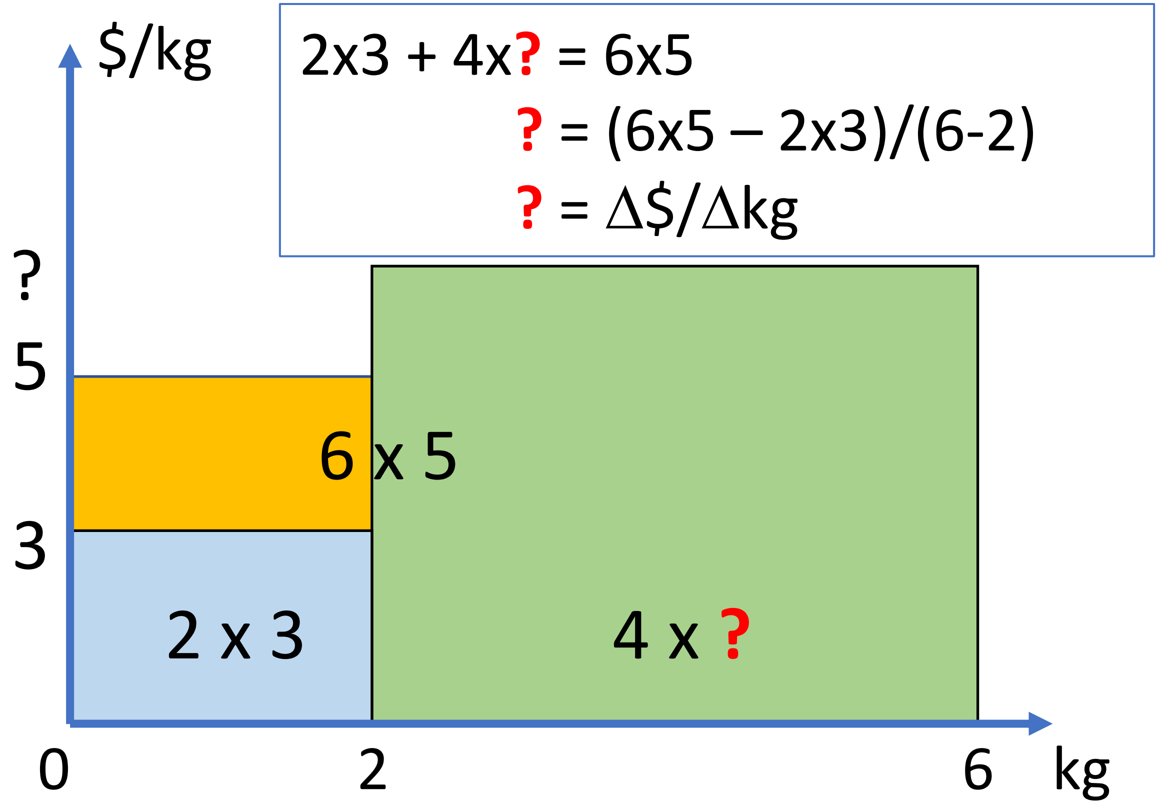
So, reversed per-number addition means subtracting areas to be reshaped, called differential calculus.

Here subtraction (giving a change, Δ) comes before division, the reverse of multiplication before addition in integral calculus.

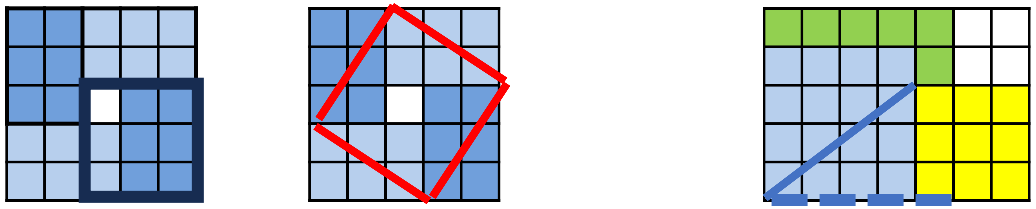
2x3 + 4x**?** = 6x5

? = (6x5 – 2x3)/(6-2)

? = Δ$/Δkg



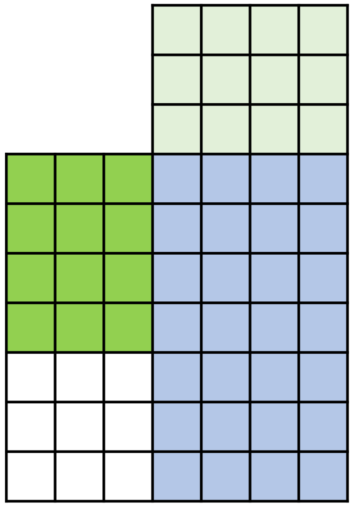
Adding Bundle-Bundle squares



- On a BBBoard we place four 2-by-3 stacks so they form a 5-by-5 square that inside contains two squares, 2 2s and 3 3s as well as two stacks. But it also contains one square formed by the diagonals in the stacks as well as four half stacks.
- So, in this stack, adding the height and the bundle as squares gives the square of the diagonal. This rule is named by the ancient Greek thinker, Pythagoras.
- The two squares 4 4s and 3 3s thus add as the square formed by the mutual Bottom-Top BT line having the length as the square-root of the sum, so, $\sqrt{(4^2 + 3^2)} = 5$.

Subtracting Bundle-Bundle squares

From a 7-by-7 square,
we pull-away a 3-by-3 square
This leaves a 7-by-(7-3) stack, and a (7-3)-by-3 stack …
… that may be turned to 3-by-(7-3) stack,
totaling a (7+3)-by-(7-3) stack.



So, 7^2 – 3^2 = (7 – 3)*(7 + 3).

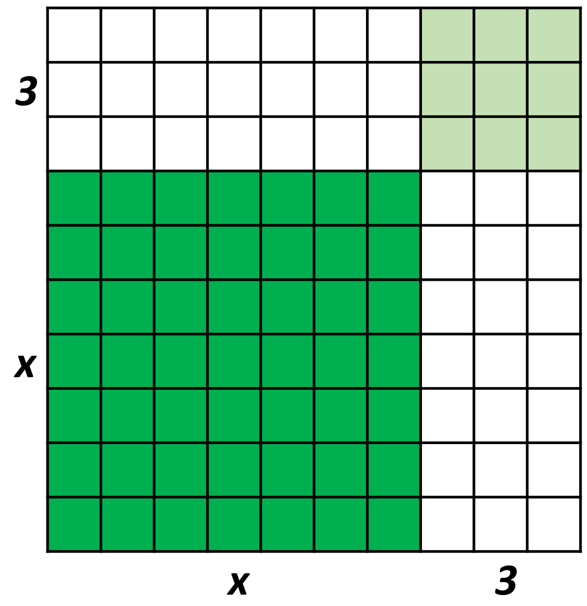
Adding squares also solve quadratic equations

On a BBBoard we see that T = (*x*+3)*(*x*+3) is a square with four parts, two squares *x*^2 and 3^2, and two stacks 2*3**x*, so that T = *x*^2 + 6**x* + 9.

The quadratic equation *x*^2 + 6**x* + 8 = 0 then makes the whole square go away except for 9-8 = 1.

So (*x*+3)^2 = 1, which gives two solutions , *x* = -2 and *x* = -4

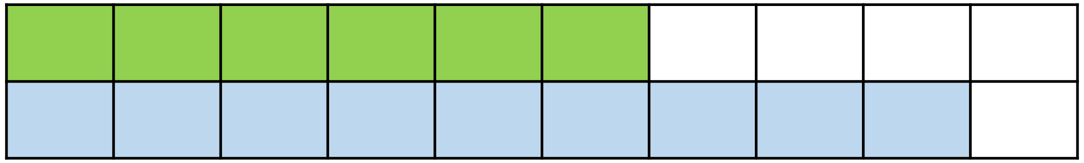
The quadratic equation *x*^2 + 6**x* + 10 = 0 has no solutions since here (*x* + 3)^2 = -1:
(*x* + 3)^2 = *x*^2 + 6**x* + 9 = *x*^2 + 6**x* + 10 – 1 = 0 – 1 = -1



Subtracting Numbers with Like Units, 9-6

Inside the ‘no-unit-math’ paradigm, numbers subtract **serial** next-to on the number line. We find the result by counting backwards.

Outside, in the ‘unit-math’ paradigm, numbers subtract **parallel** on-top.
We see that $T = 9 – 6 = 3$; and that $T = 6 – 9 =$ less3, since 6 = 9less3



Subtracted directly as less-numbers:

$T = 9 – 6 = B\text{-}1 – (B\text{-}4) = 0 \text{ -}1 \text{ - - } 4 = \text{-}1 + 4 = 3$,

$T = 6 – 9 = B\text{-}4 – (B\text{-}1) = 0 \text{ -}4 \text{ - - } 1 = \text{-}4 + 1 = \text{-}3$, both showing that - - is +

Adding Numbers with Like Units, 7+9

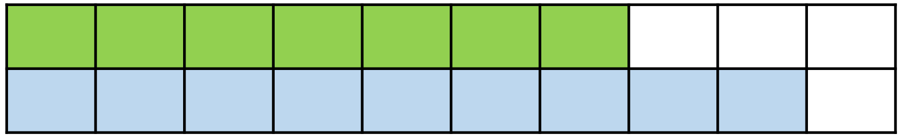
Inside the ‘no-unit-math’ paradigm, numbers add **serial** next-to on the number line. We find the result by counting on from 7 or 9.

Outside, in the ‘unit-math’ paradigm, numbers add **parallel** on-top.

We see that *T* = 7 + 9 = 2*B*2 7s = 2*B*-2 9s = 2*B*-4 tens = 1*B*6 tens = 16

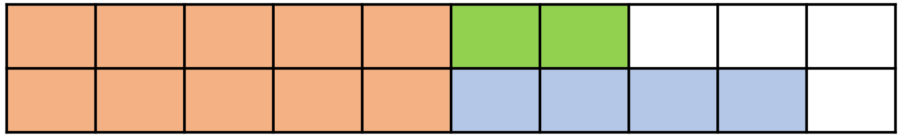
Added directly as less-numbers:

T = 7 + 9 = *B*-3 + *B*-1 = 2*B*-4 = 16



Added directly as half-bundles

T = 7 + 9 = ½B 2 + ½B 4 = 1B 6 = 16



Decolonizing by demodeling & deconstruction

2 colonizations: BundleNumbers, first by MateMatism & LineNumbers, then by MetaMatism & sets

	Mathematism, ESSENCE	ManyMath, EXISTENCE
Digits	Symbols	Icons
345	The place value system tells it	T = 3BB 4B 5, BB = B^2, BBB = B^3
Operations	Functions, order + - x / ^	Icons, order ^ / x - +
3 + 4	3 + 4 = 7	Meaningless without units
3 * 4	3 * 4 = 12	3 * 4 = 3 4s
9 = ? 2s	Meaningless, only ten counting	9 = 3B3 = 5B-2 = 4B1 = 4½ 2s
8 = ? 2s	Meaningless, only ten counting	8 = (8/2)*2, T = (T/B)*B, proportionality
2*u = 8	(2*u)*½ = 8*½, so (u*2)*½ = 4, so u*(2*½) = 4, so u*1 = 4, so u = 4	2*u = 8 = (8/2)*2, so u = 8/2
6*7 = ?	eh 44, eh 52, eh 42? OK	(B-4)*(B-3) = (10-4-3)*B+12 = 3B12 = 4B2 tens
4kg = 5$, 6kg = ?	1kg = 5/4$, 6kg = 5/4*6$	6kg = (6/4)*4kg = (6/4)*5$
1/2 + 2/3 = ?	1/2 + 2/3 = 3/6 + 4/6 = 7/6	1/2*2 + 2/3*3 = 3/5*5
2*3 + 4*5	2*3+4*5 = 10*5, sorry, 6+20 = 26	2*3 + 4*5 = 2 3s + 4 5s = 3B2 8s, by integration
7+ 9 = ?	7 + 9 = 16	2B2 7s = 2B-2 9s = ½B2 + ½B4 = 1B6 tens = 16
Tangent = ?	Tan = sine/cosine	raise = (raise/run)*run, tan = raise/run

Totals in time & space, growth & statistics I

In time, a total grows by being added or multiplied by a number, called addition-growth and multiplying-growth, or linear and exponential growth.

- Addition-growth: Final number = Initial number + growth-number * growth times, or shortly, $T = B + a*n$. The number a is also called the slope.
- Multiplying-growth: Final number = Initial number * growth-factor ^ growth times, or shortly, $T = B * a\^n$, since 200$ + 5% = (200*105%) $, so here a is 1 + interest rate.
- Combined growth (savings in a bank): Here we have that $A/a = R/r$, where A is end-dollars, a is the period-dollars, R is the end-rate, r is the period-rate, and $1+R = (1+r)\^n$, where n is the number of periods.

100% split in n parts will give the Euler number $e = (1+1/n)\^n$ for n large.

Totals in time & space, growth & statistics II

- Changing the growth-number constantly will give a quadratic growth with a parabola curving upwards or downwards if the change increases or decreases.
- Changing the curvature constantly will give cubic growth with a double parabola with curvature and counter-curvature.
- Decreasing the growth-factor constantly will give logistic saturation growth with a hill-curve in infections. Confusion between exponential and saturation growth can cause unnecessary damage.

In space, a total can be divided into several subtotals that could be as large as their average, but where the deviation then tells how far away from the average they are. However, averaging numbers only makes sense if they could be equal. Students in the 1st and 9th grades do not attend the 5th grade on average.

Mathe-matism uses line numbers without units

- Many-math with units is based on the concrete existence 'Many', and uses bundle-numbers with units, and distinguishes between unit-numbers and per-numbers.
- Set-mathematics without units is based on the abstract essence 'set', and does not accept per-numbers, but is based upon line-numbers without units that become 'mathe-matism', always true inside but rarely outside class, by claiming that 2 + 1 = 3 even though 2 pairs + 1 = 5. And by claiming that digits and fractions are numbers when instead they are operators needing a number to become a number.

That sets lead to a self-reference paradox is neglected:

'The set of sets that do not belong to themselves', does it belong to itself or not?

Well, it does, if it does not. And it does not, if it does.

This is equivalent to asking:

"This sentence is false", is it true or false?

Well, it is true, if it is false. And it is false, if it is true.

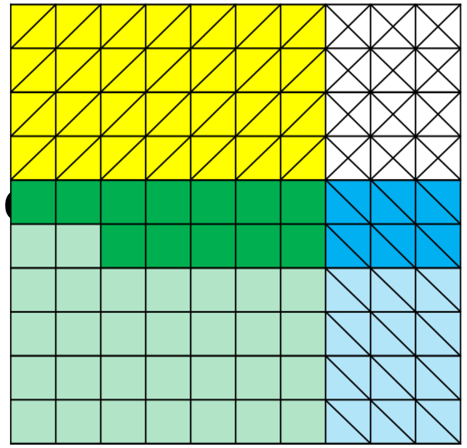
In short, self-reference is meaningless.

More sides of school mathematism

- Mathematics considers digits and operations as symbols just like letters. Multi-digit numbers are said to follow a place-value system, but ten is not called ‘bundle’, hundred is not called ‘bundle-bundle’, which would allow power as the first operation. Negative numbers are not allowed at any place.
- Reuniting occurs with the same operations; however, they are presented, not simultaneously, but in the opposite order: addition, subtraction, multiplication, division, power.
- 3+1 = 4 is presented as different number-names for the same total. And not as a tale about a total, T = 3+1 = 4. That is, both the subject and the verb are omitted. Only an equivalence between number-names is included. Over- and under-load are not accepted, carrying and borrowing are used instead.
- 2+3*4, is that 20 or14? This is determined by the definition known as the PEMDAS math hierarchy. Even though T = 2+3*4 = 2 **1s** + 3 **4s**, which can only be recounted as 1**B**4 **tens** or 14.

More sides of school mathematism

6*7 is presented as another number-name for 42, even though 6*7 is 6 7s, which may or may not be recounted to tens as 4**B**2 tens or 4.2*10 or 42 if leaving out the unit and the decimal point; and here increasing the width from 7 to 10 will decrease the height from 6 to 4.2.



8/2 is 8 divided into 2 4-bundles, instead of 8 counted in 4 2-bundles.

The tables are memorized, 6*7 = 42 instead of saying 6*7 = (**B**-4)*(**B**-3), or 6*7 = 6*(**B**-3) = 6**B** -18 = 6**B** – (2**B**-2) = 4**B**2 = 4.2*10, with less height.

Here we see that minus times minus gives plus.

Letter calculations such as 2ab + 3bc = (2a+3c)***b** are presented as applying the distributive law, where numbers can be moved in or out of parentheses.

And not by finding the common unit, **b**’s:

Number of **b**'s is 2a + 3c, so T = (2a + 3c) **b**'s = (2a+3c)***b**.

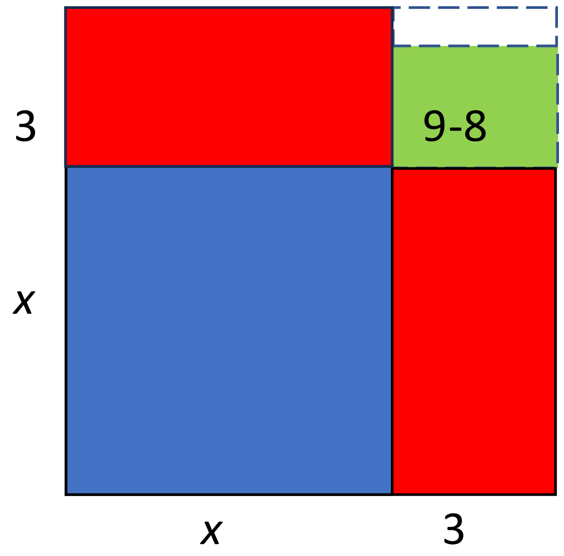
More sides of school mathematism

- Division leads on to fractions, decimals, and percent. Fractions are presented without units: 1/2 + 2/3 = 7/6, even though 1/2 of 2apples + 2/3 of 3 apples is 3/5 of 5 apples, and, of course, not 7 apples of 6.
- Proportionality tasks are solved by going over the unit.
- Negative numbers are introduced as independent numbers, where minus times minus is defined to be plus.
- Splitting numbers is called solving equations with two numbers-names, whose equivalence is expressed in a statement that retains its truth value if the same operation is performed on both number-names. When transforming a number-name, three laws are used, a commutative, an associative and a distributive law. And two abstract concepts, 1 over 2 as the inverse element to 2, as well as the neutral element 1. Instead of simply moving to opposite side with opposite sign.

$$
\begin{aligned}
2*x &= 8\\
(2*x)*½ &= 8*½\\
(x*2)*½ &= 4\\
x*(2*½) &= 4\\
x*1 &= 4\\
x &= 4
\end{aligned}
$$

$$
\begin{aligned}
2*x &= 8\\
x &= 8/2\\
x &= 4
\end{aligned}
$$

More sides of school mathematism



- **Quadratics** omit drawing x^2 + 6x + 8 = 0 as the square (x+3)^2 with 4 parts, x^2, and 3x, and 3x, and 3^2, that disappear except for 3^2 - 8 = 1.
- If the school presented inner essence as stemming from external existence, then functions could be presented as number-language sentences that connect an outer subject with an inner predicate, as in the word-language.
- Instead, the school says: A function is a rule that assigns to each element in one set one and only one element in another set.
- And in teacher training: A function is a subset of a set-product in which first-component identity implies second-component identity.

More sides of school mathematism

- Where x represents an unspecified number, $f(x)$ represents an unspecified formula with x as a variable. The term $f(2)$ is therefore meaningless, since 2 is not a variable but a constant number.
- Linear and exponential functions are then defined as examples of homomorphisms:
- $f(x) = a*x$, and $f(x) = a$^x, i.e., without initial number b.
- In geometry, plane geometry and coordinate geometry are presented before trigonometry.
- Calculus is presented last with differentiation before integration, although mixture calculations mean adding piecewise constant per-numbers, which later become locally constant per-numbers that are rewritten as increments, $p*dx = dy$, which add as one difference between end-y and start-y, as all intermediaries disappear.
- In addition, mathematism introduces eight so-called 'mathematics competences', where many-math have only two: **C**ount & **A**dd in **T**ime & **S**pace, the **CATS** approach.

8 Competences in MatheMatism, only 2 in ManyMath: Count & Add in Time & Space, the CATS approach

ManyMath
OUTSIDE

To master MANY, first we COUNT then we ADD

MatheMatism
INSIDE
NoUnit GreenHouse

8 competences needed to be applied outside

Fact & fiction & fake, the 3 genres of both the word-language and the number-language

- Once we know how to count and recount totals, and how to unite and split the four number-types, we can now actively use this number-language to produce tales about numbering and numbers, and about totaling and totals in space and time. This is called modeling.
- As in the word-language, number-language tales also come in three genres: fact, fiction, and fake models that are also called since-then, if-then, and what-then models, or room, rate, and risk models.
- Fact models talk about the past and present, and they only need to have the units checked.
- Fiction models talk about the future, and they need to be supplemented with alternative models built upon alternative assumptions.
- And fake models typically add without units, e.g., when claiming that ‘2+3 = 5’ always despite 2weeks + 3days = 17days, thus transforming mathematics to ‘mathematism’.

The 'Unit-Math' Paradigm allows Finally to have a Communicative Turn in Mathematics Education

- Modeling real world problems is difficult for mathematism that also fails to distinguish between the three genres fact, fiction, and fake ('Since-then/If-then/What-then, or 'room/rate/risk' models). All models are said to be approximations.
- By using formulas from the start, Many-math avoids modeling problems, as it sees itself as a number-language parallel to the word-language, both of which have a meta-language (a grammar) and three genres: fact, fiction, and fake.

> *Fake models are, e.g., mathematism adding numbers without units, as well as averages of numbers that could never be equal.*

	WORD language	NUMBER language
Meta-language	*This is a sentence*	*This is a formula*
Inside language	This is a chair	T = 3 4s
Outside world		

A Final Question

SUSTAINABLE DEVELOPMENT GOALS

4 QUALITY EDUCATION

Should Ethical Quality Education force children inside a ‘no-unit-math’ greenhouse that slowly strangles their innate number-language by using line-numbers to learn no-unit addition that folds outside?

Where children’s innate mastery of Many just waits to be developed by flexible bundle-numbers available at their fingertips.

Sociology explains mathematism

- Sociology sees people as individuals working towards goals. Both individually, and jointly through institutions with employees who should work for the common goal to be achieved, but who are tempted to make a so-called goal-displacement by instead working to ensure that the goal is not achieved, as this will ensure continued employment, and more resources for extra hours, and for more employees.
- The school's goal is to make children and young people more self-reliant than they already are. This is also true in terms of their mastery of Many as it appears in time and space.
- To achieve this goal, the school has hired mathematism, which unfortunately has fallen into the temptation to make a goal-displacement so that it has become the goal itself, and which therefore constantly wants more resources for extra hours, and for more employees to teach and research the 'difficult' mathematism.
- By instead firing mathematism, the school can avoid this goal-displacement and allow mastery of many to precede mastery of mathematics, which is then achieved automatically anyhow.

Theoretical Background

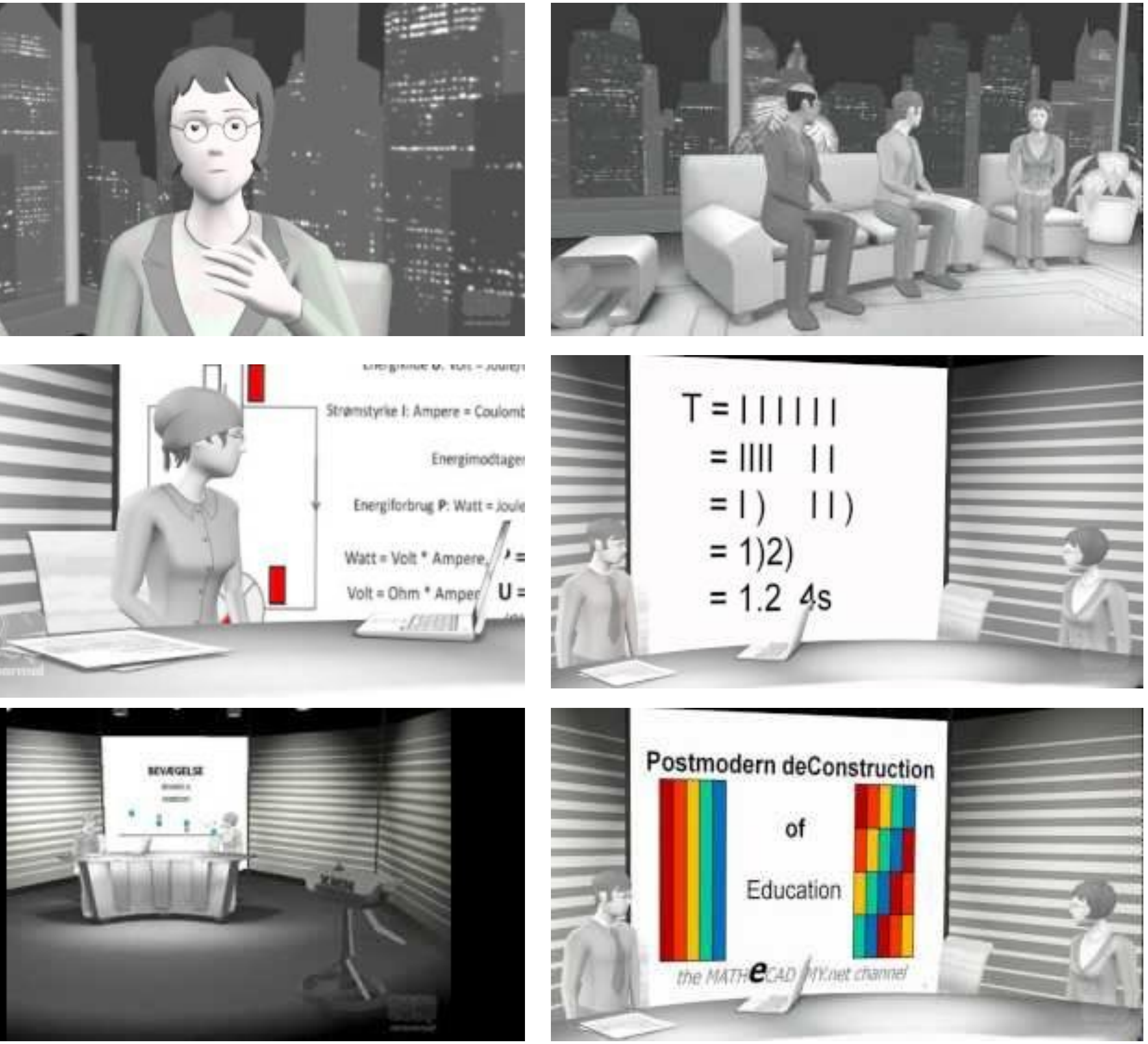
Tarp, A. (2018). Mastering Many by counting and recounting before adding on-top and next-to. *Journal of Math Education, March 2018, 11*(1), 103-117.

Tarp, A. (2020). De-modeling numbers, operations and equations: from inside-inside to outside-inside understanding. *Ho Chi Minh City Univ. of Education Journal of Science 17*(3), 453-466.

References to MATHeCADEMY.net material

- MATHeCADEMY.net, e.g.,
  http://mathecademy.net/math-with-playing-cards/
  http://mathecademy.net/calculus-adds-pernumbers/
  http://mathecademy.net/refugee-camp-math/
  http://mathecademy.net/trigonometry-before-geometry/
  http://mathecademy.net/dk/math-modeling-models/
  http://mathecademy.net/dk/two-competences-or-eight/

Some of MrAlTarp's 25 YouTube videoes



Many before Math! Math DeColonized by the Child's own 2D BundleBundle Numbers
Online math opens for a communicative turn in number language education.
AI and Difference Research in Math Education
Continuous means locally constant
From STEAM to STEEM part II

Flexible Bundle Numbers Develop the Childs Innate Mastery of Many
Children's innate Mastery of Many developed by flexible bundle-numbers
To master Many Recount before Adding
Bring Back Brains from Special Education in Mathematics
From STEAM to STEEM

Trigonometry Before Geometry Probably Makes Every Other Boy an Excited Engineer
Introducing the MATHeCADEMY dot net
Mathematics language or grammar
The two infection formulas, part 1
The two infection formulas, part 2

CupCount and ReCount before you Add
Preschoolers learn Linearity & Integration by Icon-Counting & NextTo-Addition
Deconstructing Calculus
Deconstructing PreSchool Mathematics
Deconstructing PreCalculus Mathematics

Deconstructing Fractions
A Postmodern Deconstruction of World History
8 Missing Links of Mandarin Math I
8 Missing Links of Mandarin Math II
A Postmodern Mathematics Education

ManyMath solving a facebook puzzle
Question Answer

Without units	With units
1 + 4 = 5	1 **1s** + 4 **1s** = 5
2 + 5 = 12	2 **1s** + 5 **2s** = 12
3 + 6 = 21	3 **1s** + 6 **3s** = 21
8 + 11 = ?	8 **1s** + 11 **4s** = **52**

Conclusion

We began by observing the difference between ‘mathematism’ adding without units, which is true inside but seldom outside, and ‘Many-math’ instead using bundle-numbers with units inspired by how the uneducated child sees the outside existing fact Many. We then explored the consequences of letting existence come before essence by letting counting and recounting come before adding. We saw how Many-math may be learned by working with things and actions on a 2D Bundle-Bundle-Board.

But, will this allow the learner to learn mathematics or even to be numerate as wished by the UN Sustainable Development Goals?

Apparently, different definitions of ‘numerate’ exist where existence and essence have different order. The English Oxford Dictionary defines it as being “competent in the basic principles of mathematics, esp. arithmetic”. In contrast, the American Merriam-Webster dictionary defines it as “having the ability to understand and work with numbers.”

In their common history, England once colonized America. So, the difference in the definitions is interesting. The former uses the passive term ‘being’ where the latter uses the active term ‘having’. The former connects the definition to the inside essence of mathematics while the latter connects it directly to the outside existence of numbers.

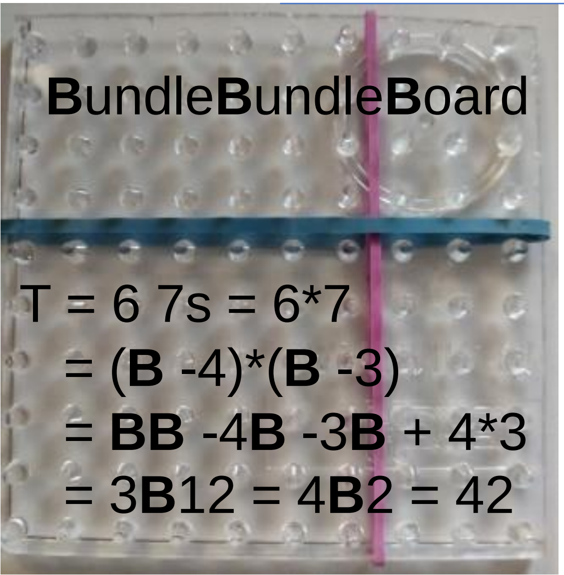
The choice thus is: shall existence precede essence as philosophical Existentialism holds, or shall essence be allowed to colonize existence with a ‘no-unit regime’ to use a Foucault-phrase? Maybe it is time to see if children stay numerate if their own 2D bundle-numbers with units are not colonized by 1D line-numbers without units.

Maybe it is finally time for a Kuhnian paradigm shift in number-language education.

Therefore, think things. Or, in the Viking version: “Derfor, tænk ting”.

Many before Math!

Mathematics DeColonized by the Child's own 2D BundleBundle Numbers



Make Math Easy with Many-Math where Existence precedes Essence

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