Demodeling Calculus: from Deriving Functions to Adding Per-numbers

Chapter CC, Chapter Abstracts and Conclusion of a 20-page article, Allan.Tarp@gmail.com, 10.10.24

Abstract. The fourth United Nations Development Goal wants to ensure that all youth and most adults achieve literacy and numeracy. In mathematics education, numeracy is at the beginning and calculus is at the end, so apparently, calculus has nothing to do with numeracy. This is indeed the case within the present 'essence-based' paradigm built on one-dimensional linear numbers without units. But the opposite is the case within the alternative 'existence-based' paradigm built on the numbers children bring to school, two-dimensional bundle-numbers with units, where 2 3s is short for 2 bundles with 3 per bundle thus containing both a time-based counting number, 2, and a space-based per-number, 3s. Here calculus is needed in grade one for next-to addition of 2 3s and 4 5s as 8s since they add by their areas, and for reverse addition asking "2 3s and how many 5s total 4 8s?". And here, a bundle-bundle square changes to the next when adding 2 bundles and 1 unbundled, which suggests that later 2 bundles will be the result of deriving a bundle-bundle square. So, a communicative turn is now possible also within number-language education if letting existence precede essence as philosophical Existentialism recommends.

CC.1 Background: The 4th UN Development Goal

The fourth of the 17 UN Sustainable Development Goals defines quality education as 'ensuring inclusive and equitable quality education and promoting lifelong learning opportunities for all.' Here the subgoal 4.6 wants to "By 2030, ensure that all youth and a substantial proportion of adults, both men and women, achieve literacy and numeracy".

CC.2 Different Views on Numeracy

As to the meaning of the word 'numerate', the English Oxford Dictionary defines it as being 'competent in the basic principles of mathematics, esp. arithmetic'. In contrast, the American Merriam-Webster dictionary defines it as 'having the ability to understand and work with numbers'. The difference in the definitions is interesting. As to the difference between 'competent' and 'work', we can ask if assessing competence and work will be equally valid? The word 'competent' is a predicate, not an action verb, I cannot competent something, I can only be judged as competent by someone who is already competent. In contrast, ‘work’ is an action verb, since I can work on something with my body and mind.

CC.3 Will Studying Numbers Automatically lead to Mathematics?

In the tradition, number education begins with the number symbols as found on a number-line, 0, 1, 2, etc. Then we learn how to add them, then how to subtract them, then how to multiply them, and finally how to divide them, where subtraction is seen as reversed addition, and multiplication as repeated addition, and division as reversed multiplication. Then reversed calculations called equations show the need for more numbers. First, the equation $3+x=2$ leads on to negative numbers, which makes the whole numbers expand to integer numbers to again be added, subtracted, multiplied and divided.

CC.4 Being Educated by a 3year-old Child

“No, that is not four, that is two twos”. Said a 3year-old child when asked “How many years next time?”; and when seeing four fingers held together two by two. This statement will change mathematics education forever since, as educated, essence is all we see. But as uneducated, the child sees what exists, bundles of twos in space, and two of them when counted in time, i.e., 2 bundles with 2 per bundle. The number ‘two’ thus exists both in and in time but in very different forms.

In space, 2 exists as 2s, a space-number, a per-number, a bundle of 2s, a 2-bundle,

CC.5 What is calculus

The tradition sees calculus as working with functions to find their limits and derivates and primitives (Bressaud et al. 2016). As difficult to learn, it is reserved for committed learners only. But once learned, it is very useful for modeling real-world change problems within the natural and social sciences and economy. So, calculus comes at the end of K-12 education as something that is chosen by a few only. But is calculus so by nature or by choice? Here we quote the ancient Greek sophists warning that unable to tell nature from choice we may be patronized by choices masked as nature. And thus, we may be forced to serve an unnoticed ‘truth-regime’ to use a phrase of the French thinker Foucault (1995).

CC.6 The Role of Calculus in Existence-based Many-math

Existence-based Many-math counts and adds in time and space totals coming from the physical fact Many that exists as duration in time and extension in space. It needs calculus for next-to addition of its bundle-numbers with units since these are two-dimensional areas. But, with existence before essence, counting comes before adding.

Does that mean that calculus first occurs when adding occurs? Of course that depends on how you view calculus. Seeing it as working with areas and per-numbers, calculus occurs from the very start since a bundle-number with units as, e.g., 2 3s exists as 2 bundles with 3 per bundle. This makes the 2 3s a two-dimensional area, a rectangle with 3 as a per-number, with the described total T = 2*3 as its area-number that may or may not be recounted in tens as 2*3 = 6, and with the 2up-by-3out diagonal as a sloping line with the per-number 2/3 as the tangent-number that allows recounting the height in the base as well as finding half the perimeter of the circle as the limit-number n*tan(180/n) for n large. And that later may be added next-to horizontally, e.g., to 4 5s as 8s thus adding the per-numbers as areas, which is what integral calculus does.

CC.6.1 Calculus in primary school

So, seeing it as working with areas and per-numbers, calculus occurs from the very beginning. But before we begin adding the bundle-numbers with units, we want to produce them by counting process where an outside total comes inside as a number-language sentence, e.g., T = 2 3s, connecting the total with a bundle-number with a unit. By using the recount-formula T = (T/B)*B telling that with the bundle B as a unit, T/B is the number of Bs in the total T, and showing the height, T/B, and the base, B, of the rectangle that represents the outside total on a BBBoard. And, before adding the totals we may want to recount a total in another unit.

Numeracy after primary school. At the end of primary school the students' original bundle-numbers with units have developed into counting-numbers in time as unit-numbers, into per-numbers in space as Bundle-numbers, into decimal-numbers and fractions for bundle-parts and negative numbers for what is missing in space or pulled away in time, into square roots for squaring rectangles, into logarithms for recounting from bundles of tens to bundles of 2s, into per-numbers also as bridging physical units with fractions when the units is like, and into tangent-numbers describing the angle and steepness of the diagonal in a stack. They have seen digits and operations as icons. And seen place-values and carrying as unneeded if including units to show numbers' polynomial nature as a union of unbundled singles, bundles, bundle-bundles, etc.

CC.6.2 Middle school

In middle school the BBBoard is replaced by a coordinate system, and the pocket calculator is replaced by a simple graphical CAS calculator. To coordinate geometry and algebra in coordinate geometry, a coordinate system now has two number-line containing also zero and negative numbers. And in algebra total-formulas now are replaced by formulas predicting y from a calculation containing numbers and an x-variable called a function where an unspecified function is written as $f(x)$. And here change-steps is repelled by difference equations allowing change calculation to split in two parts, predictable change and unpredictable change leading to and statistics and probability.

CC.6.3 High school

Geometry expands geometry from two to three dimensions by introducing linear algebra and vectors.Algebra introduces complex numbersIntegral calculus introduces areas under a locally constant curve where the formal definitions of piecewise and locally constancy (continuous) -as well as local linearity- occur by interchanging epsilon and delta.

CC.7 Consulting the three Grand Theories

To understand the differences between essence-based mathematism and existence-based Many-math we now consult the three grand theories, philosophy and sociology and psychology. Philosophy may be able to illuminate the different nature of predicates and verbs. Sociology may be able to illuminate the different inter-human power effects coming from using predicates instead of verbs. And psychology may be able to illuminate the different learning results coming from listening to predicates or practicing verbs. In this way the three grand theories may help to enlighten and discuss the core of a teacher job formulated as 'teach learners something'. Here the nature of 'something' may be discussed in philosophy. And the power relations in the textbook-teacher-learner interaction may be discussed in sociology. Finally different learning possibilities in working with the subject or hearing about its predicate may be discussed in psychology. And, discussing may be a better word than enlightening since alle three areas contain conflicting theories.

CC.8 Discussion and Conclusion

The UN Sustainable Development Goals wants to "By 2030, ensure that all youth and a substantial proportion of adults, both men and women, achieve literacy and numeracy". However, it turned out that two different views exist on numeracy, one pointing to knowing basic mathematics, and the other to using numbers.

Seeing numbers as one-dimensional line-numbers without units obeying the assumption that 1+1=2, these two views do not conflict since they both have addition using place value systems and carrying in the beginning, and calculus in the end to find derivatives and primitives. Therefore, calculus cannot be a part of numeracy.

Unless numbers are the two-dimensional bundle-numbers with units that children use before school, and that may become areas on a ten-by-ten BundleBundleBoard. Here adding the bundle-stacks 2 3s and 4 5s as 8s means adding areas thus rooting a primary school integral calculus that becomes differential calculus when asking the reverse question: "2 3s and how many 5s total 4 8s".

Here digits have different roles in space and time: 2 3s describes bundles of 3s existing in space, and 2 of them when counted in time. So, with 2 3s as 2 bundles with 3 per bundle, 2 may be called a counting-number or a unit-number since it only has meaning as attached to a unit describing what was counted, 3s, or meters, or dollars, etc. And 3 may be called a bundle-number or a per-number also without meaning unless connected with a counting-number telling how many there are. Furthermore, neither obey the assumption that 1+1 = 2 as seen when four fingers arranged as an open and a closed V-sign show that 2 1s and 1 2s total 1 4s and not 3 3s.

These two views on numbers may be called ‘essence-based’ and ‘existence-based’ mathematics when using the two core concepts of philosophical Existentialism holding that existence should precede essence which entails that counting and recounting outside totals should precede adding them inside.

Essence-math has calculus in the end. In contrast, existence-math has calculus in the beginning as the core of numeracy. It is needed in space for horizontal direct or reversed next-to addition of stacks. And for recounting rectangular stacks as quadratic bundle-bundles with the square root as the side. And when adding squares to the square formed by their bottom-top line. And to ease multiplication tables by seeing 6*7 as (B-4)*(B-3) on a BBBoard where the 6 7s are found by subtracting and adding areas. Also, it is helpful for finding the area of a circle.

Furthermore, in time differential calculus shows how one bundle-bundle square changes to the next by adding two bundles and one unbundled thus introducing how to derive quadratics later. And, it occurs as change-formulas creating orbits as straight or bended lines on a BBBoard or in a coordinate system. And used to show a simple prey-predator simulation.

Where essence-based mathematics is textbook-bound with a lot of written exercises, existence-math is physical with totals occurring both as extension in space and duration in time, and with counting preceding adding to bring outside totals inside as bundle-numbers with units on a BBBoard.

Consequently, it will be possible to realize the UN numeracy goal, but only if the United Nations also declare that a number-language is a human right to prevent a colonization of the concept ‘numerate’ by the essence-based tradition claiming that numerate means ‘being competent in the basic principles of mathematics’ with its one-dimensional line-numbers built on the assumption that 1+2 = 3 always, despite the fact that 1 week + 2 days = 9 days, and that 1 ones + 2 ones = 1 threes, whereas 1*2 = 2 always, which shows that “Outside, addition folds but multiplication holds”.

Instead, a number-language will become a human right only if its education accepts and develops the children’s own two-dimensional bundle-numbers with units with its numerate concept expressed by the alternative existence-based view as ‘having the ability to understand and work with numbers.’ But only if the numbers are the bundle-numbers that children bring to school. And that has calculus in the sense of ‘adding per-numbers’ in their very nature since a bundle-number with units as 2 3s is 2 bundles with 3 per-bundle that add with 4 5s as areas when adding the two per-numbers to 8s. And since bundle-bundle numbers are squares needing two bundles to change to the following square, except for a small corner.

Before making the choice between the essence- and existence-based views on numeracy it may be a good idea to listen to the three grand theories, philosophy and sociology and psychology. Here philosophical Existentialism discusses the difference between essence and existence. And here sociology warns that despite institutions are created as goal-directed means, a goal displacement will occur if the institutionalized essence-based mathematic will use its monopoly to use ‘not reaching the goal’ as a means to become the goal itself. And here psychology points out that meeting concrete existence and listening to institutionalized essence are two different ways of being educated.

In the 1970s a communicative turn brought general literacy to word-language education (Widdowson 1978). Calculus now has the chance to let a communicative turn bring general numeracy to number-language education also by helping children to add the per-numbers they use to communicate about what exists in space and what happens in time on a BundleBundleBoard.

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