

No, $1+1$ is not 2 but 1 as shown by a collapsing V-sign

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“No, $1+1$ is not 2, but 1 as shown by a Collapsing V-Sign” said the Children in their

Declaration of Independence

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Abstract

The 4th UN Sustainable Development Goal wants to ensure that all youth and most adults achieve literacy and numeracy. This makes education a core institution meant to ‘teach learners something’. But an institution must choose between different views on teaching and learning, and on what to master first, the outside goal or an inside means, Many or math. These choices are discussed by the three grand theories, philosophy and sociology and psychology. When adapting to Many in time and space before school children use their innate number-sense to develop bundle-numbers with units like 2 3s. The educational choice then is: shall existence precede essence, or shall essence be allowed to colonize existence with a ‘no-unit regime’? Will children stay numerate if their own 2D bundle-numbers with units are left uncolonized instead of being colonized by the institutionalized 1D line-numbers without units? The children may think so and formulate their own declaration of independence inspired by the parallel American one.

Keywords: Numeracy, arithmetic, elementary school curriculum, calculus, mathematical applications.

Background, the 4th United Nations Sustainable Development Goal

The fourth of the 17 UN Sustainable Development Goals defines quality education as ‘ensuring inclusive and equitable quality education and promoting lifelong learning opportunities for all.’ Here the subgoal 4.6 wants to “By 2030, ensure that all youth and a substantial proportion of adults, both men and women, achieve literacy and numeracy”.

However, different definitions of ‘numerate’ seem to exist.

Different Views on Numeracy

As to the meaning of the word ‘numerate’, the English Oxford Dictionary defines it as being “competent in the basic principles of mathematics, esp. arithmetic”. In contrast, the American Merriam-Webster dictionary defines it as “having the ability to understand and work with numbers.” The difference in the definitions is interesting.

As to the difference between ‘competent’ and ‘work’, we can ask if assessing competent and work will be equally valid? The word ‘competent’ is a predicate, not an action word, a verb, I cannot competent something, I can only be judged as competent by someone who is already competent. In contrast, ‘work’ is an action word, a verb, since with my body and mind I can work on something.

Also, there is a difference between the words ‘mathematics’ and ‘numbers.’ Again, we can ask if assessing mathematics and numbers will be equally valid? Mathematics is not an action word, I cannot mathematics or even math something. In contrast ‘number’ is both a verb and a noun since I can number something to produce a number for later calculations.

Finally, Many exists in the outside world where humans see and name it differently. In contrast, mathematics does not do so, it is an institutionalized essence that is socially constructed as inside abstractions from outside examples, or as inside examples from inside abstractions.

It thus seems that with the English definition of numeracy, the assessment must be carried out by persons seen as experts on the predicates competent and mathematics and other social constructions. In contrast, the American definition of numeracy allows laymen to judge the actions carried out when working with numbering and numbers.

Furthermore, in their common history, England once colonized America, so we may wonder if the two different views are the views of a former colonizer and a former colonized.

To understand these differences, we now consult the three grand theories, philosophy and sociology and psychology. Philosophy may be able to illuminate the different nature of predicates and verbs. Sociology may be able to illuminate the different inter-human power effects coming from using predicates instead of verbs. And psychology may be able to illuminate the different learning results coming from listening to predicates or practicing verbs.

In this way the three grand theories may help to enlighten and discuss the core of a teacher job formulated as ‘teach learners something’. Here the nature of ‘something’ may be discussed in philosophy. And the power relations in the textbook-teacher-learner interaction may be discussed in sociology. Finally different learning possibilities in working with the subject or hearing about its predicate may be discussed in psychology. Here, discussing may be a better word than enlightening since all three areas contain conflicting theories.

Conflicting Grand Theories

Within philosophy, a discussion between existentialism (Sartre, 2007) and essentialism about the precedence of existence or essence has taken place since philosophy began in ancient Greece. Here the ‘knowing’ sophists argued that to practice democracy we must tell nature from choice to avoid being patronized by choice masked as nature. In other words, we must be able to tell outside existence from its many chosen inside essences and especially the ones that have been institutionalized as the tradition. Against this, the ‘better

knowing' philo-sophists argued that choice is nonexistent since everything physical are but imperfect examples of metaphysical essence as illustrated in Plato's Cave allegory and that is only accessible to philosophers educated at the Plato Academy.

This disagreement between sophists and philosophers about nature versus choice, and existence versus essence runs through history. Medieval times saw a controversy between the realists and the nominalists as to whether a name is naming something or a mere sound. In the late Renaissance, a controversy occurred between Hobbes arguing that their destructive nature forces humans to accept patronization, and Locke arguing, like the sophists, that enlightenment enables humans to practice democracy without any physical or metaphysical patronization. In the counter-enlightenment, Hegel reinstalled a patronizing Spirit expressing itself through art and through the history of different people. This created the foundation of Europe's line-organized office-preparing Bildung schools; and for Marxism and socialism, and for the critical thinking of the Frankfurter School, reviving the ancient sophist-philosopher debate by fiercely debating across the Rhine with the French Enlightenment republic's post-structuralism inspired by Heidegger (1962) who argued that "P is Q" is a statement judging an outside existence, P, with an inside constructed essence-predicate that may be a preconceived prejudice, e.g., gossip (Gerede), which therefore should be met with skepticism and possibly be deconstructed. With 'essence' coming from Latin 'esse' meaning 'being', Heidegger gave four answers to the question "What is is?" pointing down, up, over, or nowhere: is for example, is an example of, is like, or is period. A function thus may be an example of a subset in a set-product where first-component identity implies second-component identity, or an expression with both specified and unspecified numbers, or a number-language sentence including a subject, a verb, and a predicate as in a word-language sentence, or simply what it is, a stand-by calculation ' $3*x$ ' in contrast to ' $3*5$ '.

Later, the two extreme clashes in 20th century Europe between highly institutionalized countries, both terminated by the low institutionalized American Enlightenment republics, made thinkers as Baumann (1990) and Arendt (1963) warn against a built-in tendency in institutions that, despite created to reach a goal, quickly become tempted to perform a ‘goal displacement’ by using not reaching the goal as a means to more funding and more research. And, reluctant to follow an order, you can find another job in the private sector, but not in an institution. Here the necessity of keeping a job may force you to carry out also evil orders, called the banality of evil. As an example of an institutionalized knowledge regime (Foucault, 1995), this also is an issue in mathematics education.

Psychology has a conflict within constructivism where Vygotsky sees education as adapting to the institutionalized essence-regime by building ladders from it to the learners’ proximal learning zones. Piaget replaces this top-down view with a bottom-up view inspired by American Grounded Theory allowing categories to emerge from concrete experiences. This conflict has sociological consequences. Presented top-down from above as examples of inside abstractions, concepts become hard to learn, which forces many learners to stop learning what is meaningless to them and to accept patronization by those who accept such meanings. In contrast, bottom-up concepts grounded from below in the outside world are easier to learn for children through the concrete examples that root the concepts; and for teenagers since knowing the subject in a sentence makes learning gossip-like.

Within Sociology, mathematics education and its research may be discussed by asking the basic question: in a social space, do we need patronization, or can we find mutual solutions by using the threefold information-debate-choice method of a democracy? As pointed out, the debate on patronization began in ancient Greece between the philosophers and the sophists; and the debate is still with us today between socialist top-down critical theorists and skeptical bottom-up existentialist theorists. The power exercised by the social

institution education can be patronizing; or emancipating if providing self-supporting resilience ('Mündigkeit' in German). Europe maximizes the power-component by using line-organized office-preparing education to force learners to stay at the line as long as possible, and to accept that their difficulties are caused by their inferiority to the children of the public officeholders helping their children inherit their offices created to support the non-self-supported. Since they cannot use the alphabet as in ancient mandarin China, they use concepts instead. Whereas North America from secondary school minimizes the power-component by using daily lessons in self-chosen half-year blocks to uncover and develop the individual talent of the learner.

Mathematics may also serve both purposes. Presented from above as a top-down self-referring essence regime, mathematics becomes so hard to learn that it forces many learners to stop learning it. This is a minor problem with half-year blocks since leaving math does not mean leaving school, but it is a big problem at line-organized schools where leaving the line means leaving school for good. Presented bottom-up from below grounded in the existing fact Many, mathematics becomes easier to learn; and the learner can keep on choosing more blocks until the interest may disappear. The need to keep the job forces teachers to follow the orders of their specific institution. But when trained, teachers should as potential change agents be informed about the choices in an educational institution and within Mathematics, so the individual teacher knows the difference between choice and nature, i.e., what can be changed and what cannot, to prevent being forced to become an agent practicing the tradition's banality of evil by preventing the children from developing their own number-language.

How Numerate are Children?

Looking at four fingers held together two by two, we see four fingers, the essence. But, before school, children see what exists, bundles of twos in space, and two of them when

counted in time. So, we may ask how mathematics may be taught to children if using their own two-dimensional bundle-numbers with units instead of the school's one-dimensional line-numbers without units. In other words, we may ask how children may learn mathematics by working with existence instead of listening to essence. Here we follow existentialism by holding that existence precedes essence. This will mean that counting precedes adding since outside totals must first be counted to later be recounted and added inside. In this 'Many-math' approach, inside math concepts are 'de-modeled' as outside existing examples instead of being defined as inside examples (Tarp, 2018, 2020). Here one-dimensional lines on a ruler are replaced by two-dimensional rectangles on a ten-by-ten Bundle-Bundle Board, a BBBoard. Here, the name-sequence 'one, two, three' is used to pull-away 1s in time to create the bundles '1s, 2s, 3s' in space until we reach the unit that becomes 1B0 followed by 1B1, etc. Here, a total of 5 1s may be recounted in 2s as 2B1, or with an overload as 1B3, or with an underload as 3B-1, or as 1BB0B1 since 2 2s is 1 bundle-of-bundles. So here, tens, hundreds, and thousands become bundles, bundle-bundles, and bundle-bundle-bundles, as does 2, 4 and 8 when counting in 2s instead of tens. And here outside existing subjects are linked to inside essence-predicates in a number-language sentence as in a word-language sentence where $T = 2B1\ 3s$ tells that the total contains 2 bundles with 3 per bundle and 1 unbundled. This flexibility together with the unit makes both place values and carrying unneeded since $36 + 47 = 3B6 + 4B7 = 7B13 = 8B3 = 83$ when counting in tens.

It thus seems that children will stay numerate and learn core mathematics on the way if allowed to keep and develop their own bundle-numbers with units. So, they may want to slightly reformulate the American Declaration of Independence.

The Children's Declaration of Independence

We, the children, declare unanimously that when it becomes necessary for one group of people to dissolve the educational bands which have connected them with another, and to

assume among the separate and equal station to which their human nature entitle them, a decent respect requires that they should declare the causes which impel them to the separation of an education that prohibits this.

We hold these truths to be self-evident, that all humans are created equal, that they are endowed by their Creator with certain unalienable Rights, that among these are Life, Liberty and the pursuit of realizing their individual talents.--That to secure these rights, education is instituted among humans, deriving their just powers from the consent of the educated, --That whenever any Form of education becomes destructive of these ends, it is the Right of the learners to alter or to abolish it, and to institute new education, laying its foundation on such principles and organizing its powers in such form, as to them shall seem most likely to allow them to realize their individual potentials.

Prudence, indeed, will dictate that education long established should not be changed for light and transient causes; and accordingly, all experience hath shewn, that learners are more disposed to suffer, while evils are sufferable, than to right themselves by abolishing the forms to which they are accustomed. But when a long train of abuses evinces a design to reduce them under absolute Despotism, it is their right, it is their duty, to throw off such education, and to provide new guidance for their future learning. --Such has been the patient sufferance of we the children; and such is now the necessity which constrains us to alter their former Systems of education. The history of the present mathematics education is a history of repeated injuries and usurpations, all having in direct object the establishment of an absolute Tyranny over the learners. To prove this, let Facts be submitted to a candid world about the Tyranny of mathematics education.

It deprives us of our own number-language that is developed to count and add in time and space, i.e., to adapt us to outside quantities through appropriate actions. It has done so by forcing upon us a belief that to master the outside fact Many as it occurs as multiplicity in

space and repetition in time that we can access directly with our senses, hands and thoughts, first we must master inside mathematics that is not an action word, a verb, that allows us to ourselves judge whether our actions are appropriate. Instead, mathematics is a judging word, a predicate, needing mathematicians to judge if what we do is mathematics. And that use rootless self-referring definitions all being examples of the mother concept set despite Russell proving that self-reference is meaningless with his set-paradox saying that the set of sets not belonging to itself will do so only if it does not.

It deprives us of our innate number sense that allows us to see the outside total 2 3s as 2 bundles with 3 per bundle by enforcing upon us a place value system that leaves out units by simply writing 47 instead of what exists, 4B7 tens, i.e., 4 bundles with ten per bundle plus 3 unbundled ones. And it never presents ten, hundred and thousand as what exists, a bundle B, a bundle-bundle BB, and a bundle-bundle-bundle BBB, easily seen when counting five fingers as 1BB0B1 2s, and ten fingers as 1BBB0BB1B0 2s, or 1BB0B1 3s.

It prevents us from using number language sentences with an outside subject, a bridging verb, and an inside predicate, e.g., “The total is 5 7s”, which abbreviates to a formula “ $T = 5*7$ ”. Instead, it just wants the number of tens in 5 7s by simply asking “ $5*7 = ?$ ” without telling what is calculated outside or why it is relevant to change unit from 7s to tens. Also, it calls a calculating sentence a function defined by self-reference as a subset in a set-product where first-component identity implies second-component identity.

It never presents Bundle-Bundles as squares where the formulas $T = 1BB2B1$ and $T = 1BB-2B1$ produce the following and previous square.

Figure 1.

And it never shows that rectangular stacks almost are recounted in squares with the square root as the side when moving half the surplus from the top to the side. A method that leads directly to solving quadratic equations on a ten-by-ten Bundle-Bundle-Board, a

BBBoard. Instead, quadratic equations are postponed to late middle school and solved by factoring.

Figure 2.

It forces upon us its own one-dimensional line-numbers without units together with the claim that $1+1 = 2$ always despite the fact that collapsing a V-sign clearly shows that one 1s plus one 1s gives one 2s and not two 2s. Likewise adding 2 1s and 1 2s cannot give 3 3s as taught, but 1 4s since the per-numbers 1s and 2s must be multiplied to unit-numbers before adding, $2*1 + 1*2 = 4$, thus added or integrated by their areas as in integral calculus.

Figure 3.

It prevents us from seeing the digits as icons with the number of strokes they represent, four strokes in the four-icon, etc., thus making us see both digits and letters as mere symbols. And it prevents us from understanding that ten has no digit since it is counted, not as ten ones but as one bundle and none unbundled, 1B0, or 10 without unit.

Figure 4.

It also prevents us from seeing operations as icons and as they occur naturally in the counting process. First, we meet the power 2 and 3 in bundles-of-bundles, and in bundles-of-bundles-of-bundles when recounting ten fingers in 2s or 3s. And when 5 times folding a paper in 2s to see it with 1, 2, 4, 8, 16, and 32 edges, which may be written as 2^0 , 2^1 , 2^2 , 2^3 , 2^4 , and 2^5 if using the symbol $^$ for folded. Here the logarithm counts the folding in time, and the root counts the folding in space. Then we meet division as a broom to push-away bundles to push-back and stack by a multiplication lift before being pulled-away by a subtraction rope to find the unbundled that are included on-top the stack as decimals, fractions or negative numbers, e.g., $7 = 3B1 = 3 \frac{1}{2} = 4B-1 \text{ 2s}$. Finally, we pull-back the stacks using a cross to show they may add horizontally next-to or vertically on-top.

It prevents us from using flexible bundle-numbers to ease standard calculations by using overloads and underloads. Instead, it enforces an unneeded place value and carrying system, so we are not allowed to write.

Figure 5.

It forces us to learn to add without units despite this is meaningless. It does so to define subtraction as reversed addition, and multiplication as repeated addition, and division as reversed multiplication. $4*5$ is presented as 20 even if it is 4 times 5s that may or may not be recounted in tens. And $8/2$ is presented as 4 without saying that this has two different meanings, $8s/2times = 4s$ in space, and $8s/2s = 4times$ in time.

It prevents us from using our own two-dimensional flexible Bundle-Bundle numbers with units to count and recount outside totals such that 8 recounted in 2s gives the recount formula, $8 = (8/2)*2$, or $T = (T/B)*B$ with T and B for Total and Bundle, which is used all over science and technology and economics to change units.

It deprives us of the joy of recounting, both from tens to icons by solving equations, and from icons to tens with the multiplication tables.

It thus deprives us of solving equations as $u*2 = 8$ asking ‘how many 2s in 8?’ by recounting 8 in 2s as $8 = (8/2)*2$ showing that, of course, $u = 8/2$, which is found by moving ‘to opposite side with opposite sign’. This is in accordance with the formal definition of division: $8/2$ is the number u that multiplied with 2 gives 8, $u*2 = 8$. Instead, it forces our teachers to learn the unneeded concepts from abstract algebra: commutativity, associativity, inverse elements, and neutral elements, which allows introducing a ‘do the same to both side’ method:

If $2*u = 8$, then $(2*u)^{1/2} = 8^{1/2}$, then $(u*2)^{1/2} = 4$, then $u*(2^{1/2}) = 4$, then $u*1 = 4$, so $u = 4$.

Figure 6.

And, it also deprives us of a ten-by-ten BundleBoard, a BBBoard to learn the multiplication tables by seeing and feeling and counting the 2s, and the 3s, and the 4s, and the 5s. And to see 6 as $\frac{1}{2}B1$, and 7 as $\frac{1}{2}B2$, etc. So, we never enjoy that $4*5$ is $4*\frac{1}{2}B$ or $2B$ or 20. Or that $4*7$ is $4*\frac{1}{2}B2$ or $2B8$ or 28. Or that $6*7 = (B-4)*(B-3) = 1BB - (4+3)B + (4*3) = 3B12 = 4B2 = 42$. And we never enjoy learning early algebra when finding the answer by pulling-away $3B$ and $4B$ from the total BB to get $3B$, and then by adding the $4*3$ s that was pulled away twice. So again, $6*7 = 3B12 = 4B2 = 42$. Written in a short way this shows the FOIL method: $6*7 = (B-4)*(B-3) = BB - 3B - 4B + 4*3$, as well as showing that minus times minus must be plus. Instead, it enforces rote learning of the tables, which creates widespread math inability and dislike.

Figure 7.

It prevents us from the joy of finding per-numbers when recounting in physical units where recounting 5 kg as 3\$ creates the per-numbers $3\$/5\text{kg}$ to bridge the units by simply recounting:

$$20\text{kg} = (20/5)*5\text{kg} = (20/5)*3\$ = 12\$; \text{ and } 15\$ = (15/3)*3\$ = (15/3)*5\text{kg} = 25\text{kg}.$$

Instead, it enforces a ‘pass the unit’ method or cross multiplication. And it never talks of the medieval ‘Rule of 3’ method that also includes the units by saying: first alter the units, then multiply before you divide.

$$? \$ = 20\text{kg} * 3\$ / 5\text{kg}, \quad \text{and} \quad ? \text{kg} = 15\$ * 5\text{kg} / 3\$.$$

It prevents us from seeing fractions as per-numbers with like units as $3\$/5\$ = 3/5$. And from realizing that both digits, per-numbers and fractions are not numbers but operators needing a number to become a number. Instead, it adds fractions without units by claiming that $1/2 + 2/3 = 7/6$ even though 1 red of 2 apples plus 2 red of 3 apples gives 3 red of 5 apples and of course never 7 red of 6 apples.

It prevents us from meeting trigonometry when mutually recounting the sides and diagonal in a stack: $\text{rise} = (\text{rise}/\text{run}) * \text{run} = \text{tangent}(\text{Angle}) * \text{run}$.

And we never split a circle in many small pieces to see π calculated as $n * \tan(180/n)$ for n large. Instead, it postpones trigonometry to after plane geometry.

It prevents us from adding our numbers as 2 3s and 4 5s on-top after recounting has changed the units to the same, or next-to as areas leading to integral calculus and differential calculus when reversed. Instead, it changes the names, ‘recounting to change units’ is called ‘proportionality’ or ‘linearity’; ‘area-addition’ is called ‘integral calculus’; ‘change by adding and multiplying’ is called ‘linear and exponential functions’; ‘local constancy and linearity’ is called ‘continuity and differentiability’; etc. And, even if we need it at once in grade one, it postpones proportionality to middle school, and calculus to late high school where it is made accessible for only few by letting its reverse ‘differential calculus’ precede it with its over-formalization of limits and derivatives.

It prevents us from seeing the beauty of our ‘Algebra Square (Tarp, 2018) showing how to unite or split the world’s four different number-types. Here addition and multiplication unite unlike and like unit-numbers while integration and power unite unlike and like per-numbers. And here subtraction and division splits into unlike and like unit-numbers while differentiation and log or root splits into unlike and like per-numbers. This is in accordance with that in Arabic, algebra means to reunite.

Table 1.

Finally, it prevents us from using our number-language to produce the same genres that exist in word-language tales (Tarp, 2018), fact and fiction and fake, where a factual since-then model must be trusted, and a fictional if-then model must be supplemented with alternative scenarios, and fake what-then models must be rejected. Instead, it enforces that all

models are simplifying approximations of the world. And that 8 competences are needed to model instead of just 2: count and add in time and space.

Conclusion and Recommendations

The fourth UN Sustainable Development Goals wants to ensure that all youth and most adults achieve literacy and numeracy. However, two different views exist on how to reach the goal numeracy. One recommends using an institutionalized essence, mathematics, as a means, the other recommends developing the children's already existing numeracy created when adapting to the physical fact Many by counting and adding in time and space. The controversies in three grand theories illuminate the choice between the two. Philosophy offers the two core concepts, outside objective existence versus inside subjectively constructed and institutionalized essence. Sociology warns against an institutional goal displacement where a means uses not reaching the goal as a means to become the goal itself. And psychology provides two constructivist learning methods where one sees learning as adapting to inside essence, and the other as adapting to outside existence.

The core question thus is: In mathematics education, is the goal to master the institutionalized essence, mathematics, to later master the outside existence, Many, in time and space, or the other way around? And are there other ways to numeracy than the institutionalized mathematics?

It turns out, that before school children develop a numeracy using two-dimensional bundle-numbers with units as 2 3s that are different from the school's one-dimensional numbers without units built upon the assumption that $1+1 = 2$ despite a collapsing V-sign falsifies this. Also, developing the children's already existing numeracy by using their numbers means also learning core mathematics at the same time. So, letting existence precede essence offers an alternative way to the UN end goal. And, how ethical is it to colonize the children's own numbering with instructions on how to math? Maybe it is finally

time for a Kuhnian (1962) paradigm shift in mathematics education also that will allow a communicative turn in the number-language education as the one that has taken place in the word-language education in the 1960s (Widdowson, 1978).

Therefore, think things. Or, in the Viking version: “Derfor, tænk ting”.

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Table 1

The Algebra Square shows how to unite and split our four number-types, and how to solve equations by moving 'to opposite side with opposite sign'.

Calculations unite/ split Totals in	Unlike	Like
Unit-numbers m, s, kg, \$	$T = a + n$ $T - n = a$	$T = a * n$ $\frac{T}{n} = a$
Per-numbers m/s, \$/100\$ = %	$T = \int f \, dx$ $\frac{dT}{dx} = f$	$T = a^n$ $\sqrt[n]{T} = a \quad \log_a T = n$

Figure 1

Bundle-Bundles are squares with the formulas $T = 1BB2B1$ and $T = 1BB-2B1$ to produce the following and previous square. This makes square numbers easy to find: 1, 4, 9, 16, 25 and 81, 64, 49, 36, 25

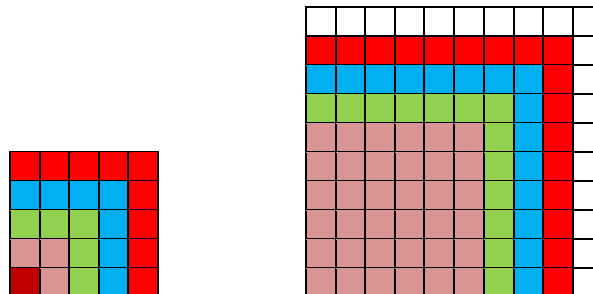


Figure 2

*A rectangular stack is recounted in a square (almost) with the square root as the side by moving half the surplus from the top to the side. Next to a BBBoard showing 6*7 as 6 7s*

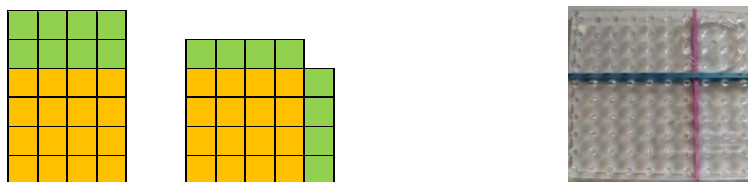


Figure 3

Students reacting against adding without units by demonstrating the importance of including the units

	
$\begin{array}{r} 1 \quad 1s \\ + 1 \quad 1s \\ \hline 2 \quad 2s \\ 1 \quad 2s \end{array}$	$\begin{array}{r} 2 \quad 1s \\ + 1 \quad 2s \\ \hline 3 \quad 3s \\ 1 \quad 4s \end{array}$
$2 \quad 2s$	$3 \quad 3s \quad \text{calculus}$
$1 \quad 2s$	$1 \quad 4s \quad 2*1 + 1*2 = 4$

Figure 4

The digits shown as icons with as many strokes as they represent

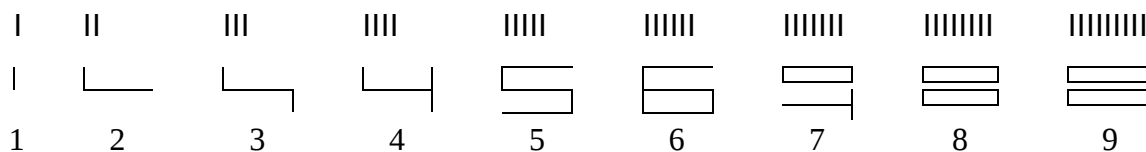


Figure 5

Including the unit, B, allows using overloads and underloads in standard calculations thus making the place value system and carrying unneeded

Overload	Underload	Overload	Overload
65 + 27	65 - 27	7 x 48	336 /7
6 B 5 + 2 B 7	6 B 5 - 2 B 7	7 x 4 B 8	33 B 6 /7
8 B12 9 B 2	4 B-2 3 B 8	28 B 56 33 B 6	28 B 56 /7 4 B 8
92	38	336	48

Figure 6

*A combined equation as '3*u + 2 = 14' be solved by a song*

$3*u + 2 = 14$ $(3*u) + 2 = 14$ $3*u = 14 - 2$ $u = (14 - 2)/3$ $u = 4$	Equations are the best we know; they're solved by isolation. But first the bracket must be placed, around multiplication. We change the sign and take away, so only u itself will stay. We just keep on moving, we never give up. So feed us equations, we don't want to stop.
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Figure 7

*A ten-by-ten Bundle-Bundle Board shows 6 7s as 6*7, and as 6*½B2 = 3B12 = 4B2 = 42, and as (B-4)*(B-3) = BB - 3B - 4B + 4*3 = 3B12 = 4B2 = 42, and why minus*minus must be plus*

