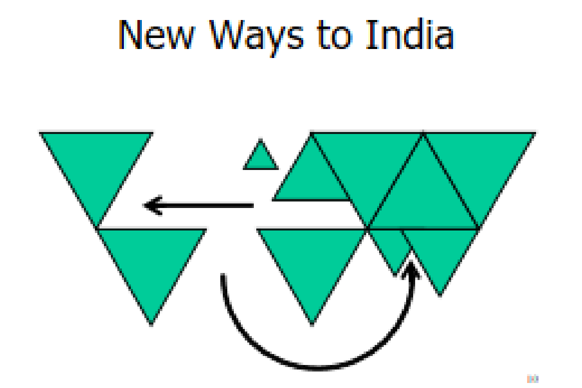
4 QUALITY EDUCATION

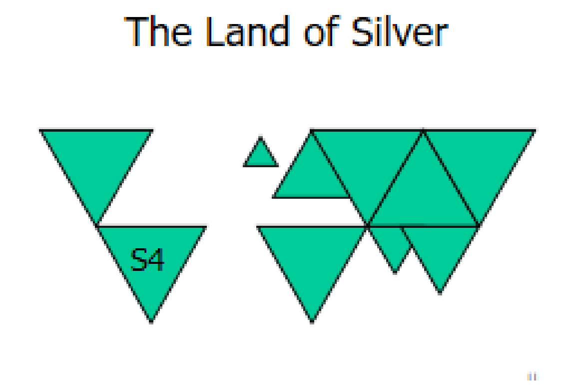
Integrating History in STEM or STeN

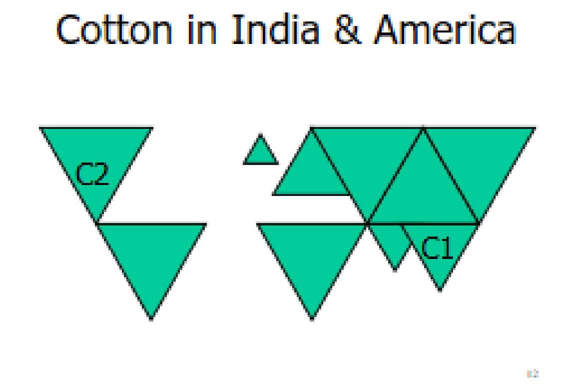
including economy & Numeracy as Math with Units

- to make all Youth Numerate by 2030

Allan Tarp
MATHeCADEMY • net
Denmark • 2025







Economy gives a core Understanding and Use of Numbers and Calculations in the World

Their basic meanings show geometry and algebra as rooted in economics. So, **STEM** should change to **STeN** including economics and Numeracy.

In Greek, geometry means to measure earth. And in Arabic, algebra means to reunite numbers. So, they have a common root in the basic economic question "How to divide the earth, and what it produces?"

A hunter-gatherer needs not tell the different degrees of **many** apart.

But a farmer does since here you produce to a market. And there, you need to be **numerate** to answer the question "How **many** in **total**?"

Which at once leads to the answer "That depends on the **unit**."

Units Matter. STeN and children all use units. **Math** does not - and must go.

Units Change, at Workplaces and at Markets

At the workplace we use our hands and muscles to transform input to output placed on a row as single items. For a market, we need the items to be **Bundled** in, e.g., **2s**, **5s**, **tens**, **dozens, scores,** etc.

At the market, a buyer may want to buy **7s**, or to pay 5$ per 4 kg.

So, **Changing Units by ReCounting** is a core task in Numeracy:

- '2 **3s** = ? **5s**', and
- '6 **7s** = ? **tens**', and
- '3 **tens** = ? **6s**'.
- 'With 4kg per 5$, 12$ = ? kg' and ? $ = 10kg'

With units, we can solve a facebook Puzzle

Question | Answer

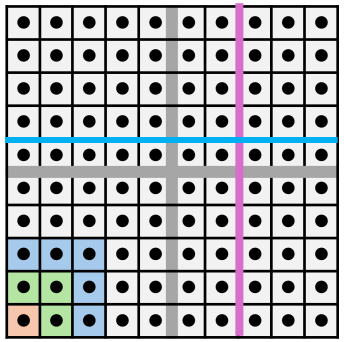
Without units	With units
1 + 4 = 5	1 **1s** + 4 **1s** = 5
2 + 5 = 12	2 **1s** + 5 **2s** = 12
3 + 6 = 21	3 **1s** + 6 **3s** = 21
8 + 11 = ?	8 **1s** + 11 **4s** = **52**

Numeracy? We ask a 3year-old “How many years next time?” The answers is 4, with 4 fingers shown

But, with 2 bundled as **2s**, the child says “No, that is not 4, that is 2 **2s**.”

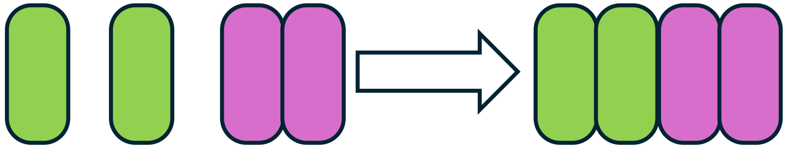
The educated sees the **essence**, 4, the un-educated sees the **existence**, 2 **1s** **bundled** as 1 **2s** in space, and 2 of them when **counted** in time.

Children understand Numbers as 2D on a **BundleBundleBoard** with a **bundle-unit** below, and a **counting-numbers** going up.



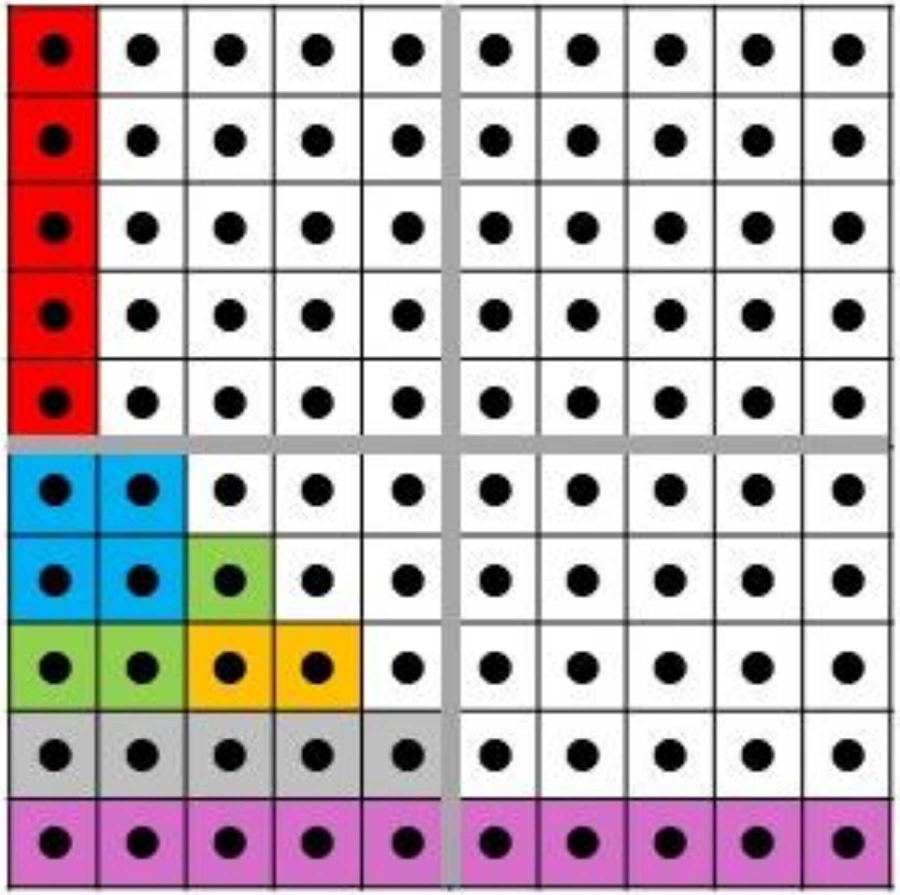
BBM BundleBundleMath, or Existence-math describes **Many** by the child's own **Counting-numbers with Bundle-units.**

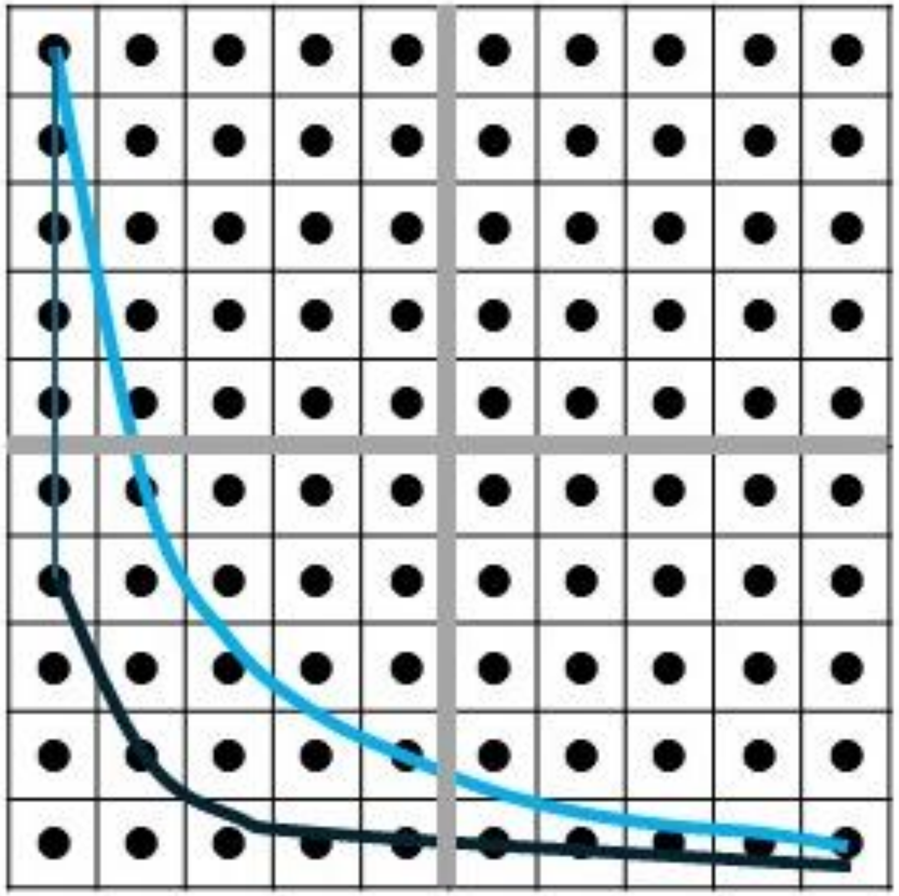
Essence-math adds without units. And **colonizes** **Many** with **Half-Matics** using counting-numbers only. And becoming ‘**Mathema-tism**’ by claiming that 2+1 = 3 despite here, 2 **1s** +1 **2s** = 1 **4s** , not 3 **3s**.



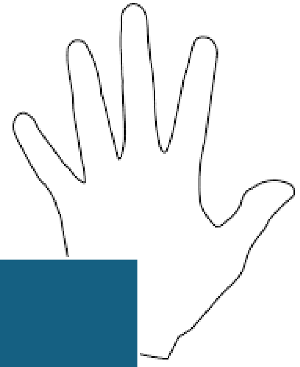
So, **Units Matter!**

ReCounting 10 & 4 1s in 2s & 3s & 4s & 5s & 10s
A Carnot Cycle with the Energy in a Heat Engine

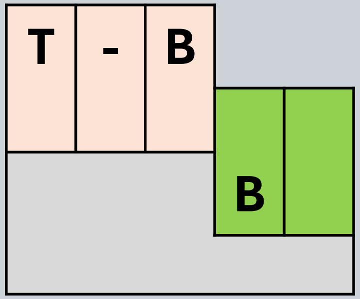


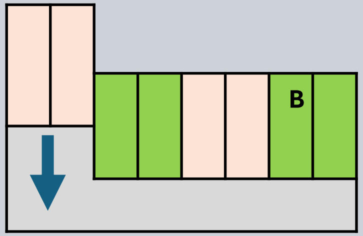


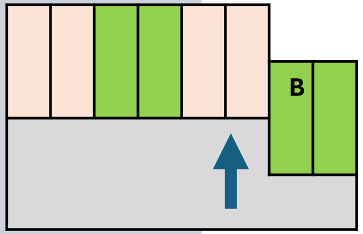
ReUniting and **ReCounting** Totals



ReUnite Totals	**ReCount** Totals
Iconize: **–** pull-away 'rope' **+** pull-back 'double-rope'	"How many **Bs** in T?" "From T, push-away **Bs**" **Iconize:** **/** push-away 'broom' **x** push-back 'lift'
T = (T-B)+B	**T = (T/B)xB**
The ReUnite Formula Solves uniting equations:	**The ReCount Formula** Solves counting equations:
u + 2 = 7 *But, 7 = (7 – 2) + 2* ***u*** = 7 – 2	***u*** x 2 = 8 *But, 8 = (8/2) x 2* ***u*** = 8/2
u – 2 = 7 *But, u = (u – 2) + 2* ***u*** = (***u*** – 2) + 2 = 7 + 2 = 9	***u*** / 2 = 4 *But, u = (u/2) x 2* ***u*** = (***u*** /2) x 2 = 4 x 2 = 8



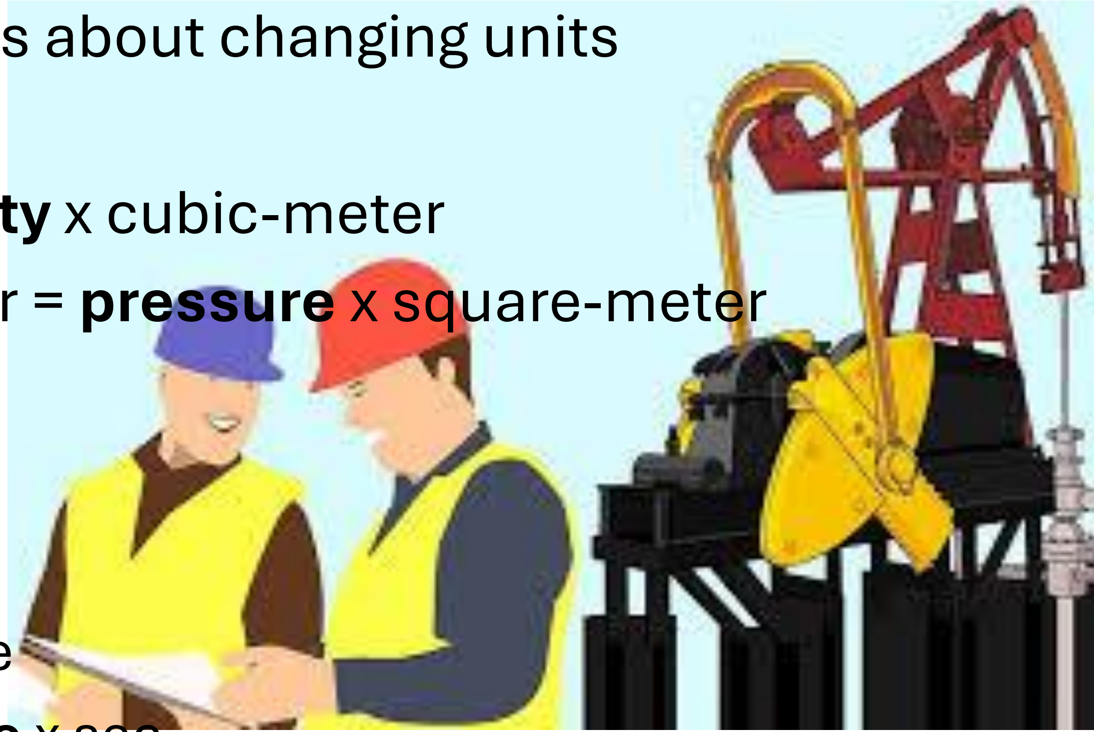




The **ReCount Formula** and **per-numbers** are the core of **STeN** (economy & Numeracy included)

STeN typically contains multiplication formulas about changing units

- $ = ($/hour) * hour = salary * hour
- kg = (**kg/cubic-meter**) x cubic-meter = **density** x cubic-meter
- force = (**force/square-meter**) x square-meter = **pressure** x square-meter
- meter = (**meter/sec**) x sec = **speed** x sec
- energy = (**energy/sec**) x sec = **Watt** x sec
- energy = (**energy/kg**) x kg = **heat** x kg
- gram = (**gram/mole**) x mole = **molar mass** x mole
- Δ momentum = (**Δ momentum/sec**) x sec = **force** x sec
- Δ energy = (**Δ energy/ meter**) x meter = **force** x meter = work
- **energy/sec** = (**energy/charge**) x (**charge/sec**) or **Watt** = **Volt** x **Amp**



ReCounting Sides in a Stack halved by its Diagonal gives **Trigonometry** before Geometry, and π

In Greek, geo-metry means to earth-measure. The earth may be divided in triangles; that may be divided in right triangles; that may be seen as a stack halved by its diagonal. This 'half-stack' has three sides: the base *b*, the height *h*, & the diagonal *d*, connected with the angle ***A*** by per-number formulas recounting the sides pairwise.

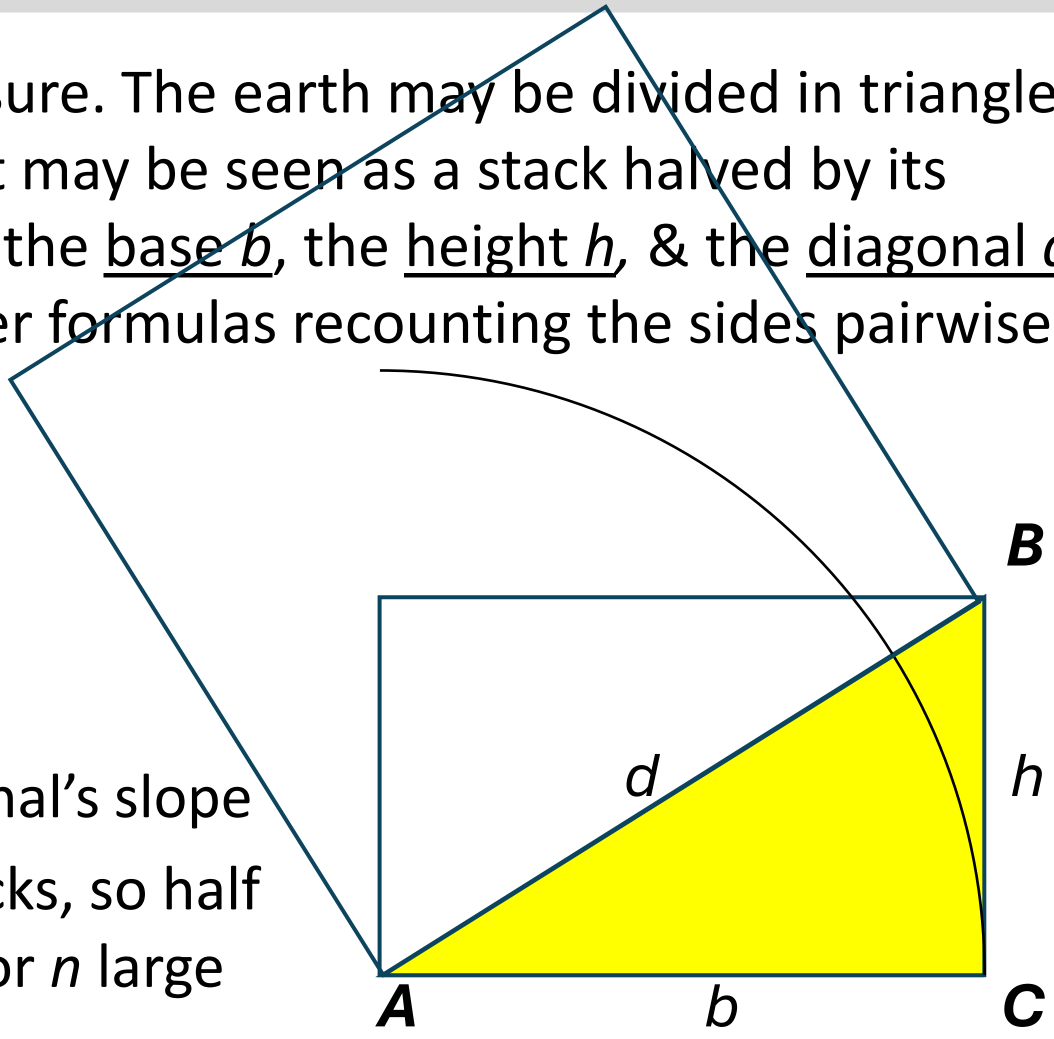
$h = (h/b) \times b = \tan A \times b$

$h = (h/d) \times d = \sin A \times d$

$b = (b/d) \times d = \cos A \times d$

tan***A*** = *h*/*b* = Δ*y*/Δ*x* = rise/run = the diagonal's slope

A circle contains very many small half-stacks, so half the circumference is: $\pi = n \times \tan(180/n)$ for *n* large



Triangles on a BundleBundleBoard

Point A is at the (2,3) dot. Point B is at the (3,8) dot. And point C is at the (7,5)dot.

To find length, angles and area of the triangle ABC we enclose it in a 5 **5s** stack.

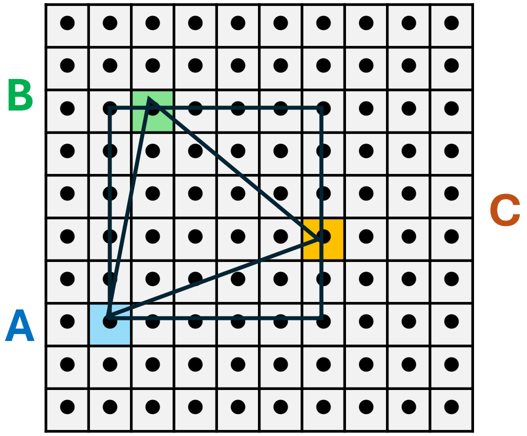
All three angles are split in two outer, and one inner angle.

We find the left angles using tangent, and the sides by using sine

Tan A = 1/5, so A = 11 degrees.

And, sin A = 1/c. But, 1 = (1/c)*c, so, 1 = sin 11.3 * c, c = 1/sin11.3 = 5.1

Likewise, tan B = 3/4, so B = 37 degrees. And, tan C = 5/2, so C = 68 degrees



The side a = 5.7, and the side b = 5.4. The area of the three outer half-stacks are ½*(1*5 + 3*4 + 5*2) = 13.5.

So, the area of the inner triangle ABC is 5*5 – 13.5 = 11.5

To find the angles in the triangle ABC we begin with 90 degrees for A and 180 degrees for B and C.

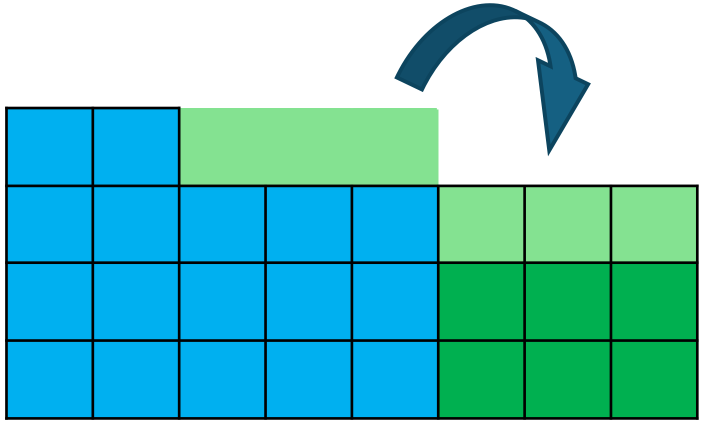
Then we pull away the two neighbor angles in the outer right triangles and get:

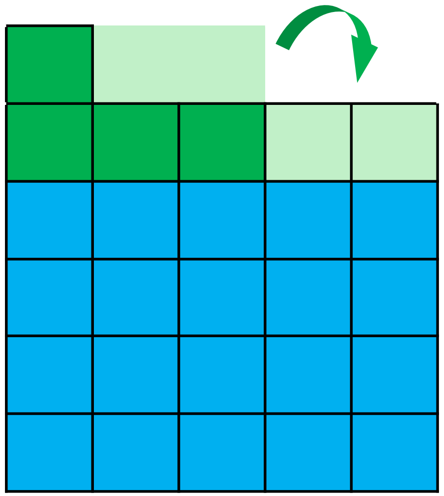
A = 90 – 22 – 11 = 57 degrees, and B = 180 – 79 – 37 = 64 degrees, and C = 180 – 68 – 53 = 59 degrees.

Finally, we test the results by adding the three angles: 57 + 64 + 59 = 180.

Once Counted & ReCounted, Totals may Add

BUT: NextTo →	**or OnTop** ↑
4 **5s** + 2 **3s** = 3**B**2 **8s**	4 **5s** + 2 **3s** = 5**B**1 **5s**
The areas are integrated *Adding areas = Integration*	The units are changed to be the same *Change unit = ReCounting = Proportionality*



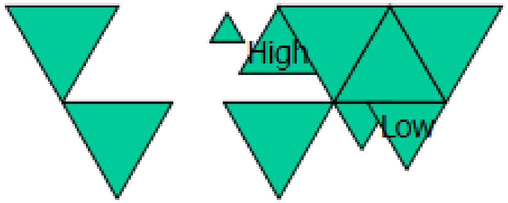


World Trade History I
Europe **SILVER** for Eastern Silk & Pepper

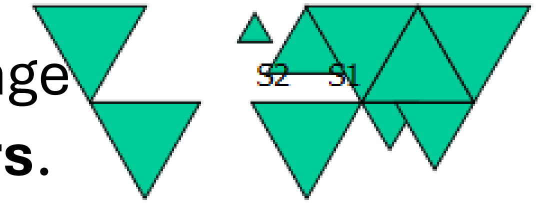
The West Highland wanted Eastern Lowland's silk & pepper.
The East only wanted Western **SILVER**.
S1.SILVER in Greece created Mathematics & letter numbers.
S2. SILVER in Spain created **Roman numbers** for administration.
When Vandals took the Roman **SILVER**-mines came Dark Middelage
S3.German Harz **SILVER** via Italy imported **Hindu-Arabic numbers**.
Renaisance Art and German Music was financed by Italy creating
Banks making money from interest on loans:

- 10$ + 10$ = 20$, but
- 10% + 10% = 20% + 1% compound **interest**
- 10% 10times = 100% + 159% comp. interest
- **(1+r)^n = 1+R** (110%^10 = 259%)

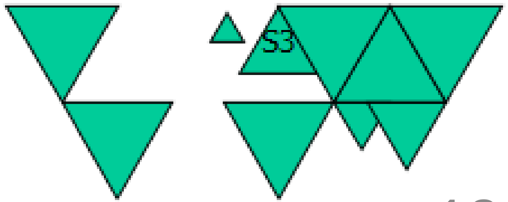
Highland & Lowland



Silver in Greece and Spain



German Silver



Illustrations from the MrAlTarp video" A Postmodern Deconstruction of World History, https://youtu.be/xQAdrI_CvyY

UnSystematic & Systematic **B**undling

Roman Numbers use un-systematic **B**undling

IIIII -> V • VV -> X • XXXXX -> L • LL -> C • CCCCC -> D • DD -> M

Addition is easy, multiplication is not

XXX V III + XX V III = XXXXX VV IIIIII = LX X V I = LXXVI

XXX V III * XX V III = ? Oops, difficult, **STOP**

HinduArabic Numbers use systematic **B**undling

Ten **1s** -> 1 **tens** • ten **tens** -> 1 **Hundreds** • ten **Hundreds** -> 1 **Thousands**

a **B**undle of 1s -> 1 **B** • a **B**undle of **B**s -> 1 **BB** • a **B** of **BB**s -> 1 **BBB**

Systematic Bundling creates **Power** and **Logarithm**: 8 = 1**BBB 2s** = 2^3, so, log2(8) = 3

Addition is easy, multiplication is easy

38 + 28 = 3**B** 8 + 2**B** 8 = 5**B** 16 = 6**B** 6 = 66

38 * 28 = 3 **B** 8 (**BB**-FOIL: Down & Cross)

* 2 **B** 8 = 6**BB** (24+16)**B** 64 = 6**BB** 40**B** 64 = 6**BB** 46**B** 4 = 10**BB** 6**B** 4 = 1064

Before: the Renaissance Miracle, **Regula de Tri**
Now: Change Units by **ReCounting**, T = (T/B)*B

Renaissance Italy used 'Regula de Tri' (the rule of three) to change units:

Question:

"With the per-number, 2$ per 3kg, what is the price for 9kg?"

First, they arranged the three numbers with alternating units:

'9kg, 2$, 3kg'.

Then they found the answer by multiplying and dividing:

9*2/3 = 6$.

Today we may use proportional thinking - or simply recount 9 kg in **3s**:

9kg = (9/3)*3kg = (9/3)*2$ = 6$

World Trade History II: From **SILVER** to **COTTON** in East & West

Portugal's searoute around Africa to India avoided Arabic middlemen and made Italy go bankrupt.

S4. Spain found a westly searoute to the **SILVER**-land, Argentine.

England robbed Spanish **SILVER**, and sailed on open sea to India by the moon "That obeys the Lord's **unpredictable** force". Pray, said the Pope.

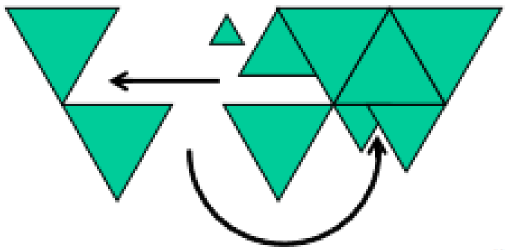
No, said Newton, calculate. "As the apple, also the moon falls to the earth, obeying nature's **predictable** force, gravitational attraction.

"A force **upholds order** said Aristotle, and **Arabic Algebra** gives predicting calculations" said the Pope.

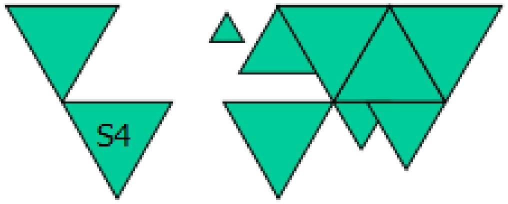
No, said Newton. "A force **changes order**, so I have to invent change calculation, **calculus**.

In India, England robbed **COTTON** to be planted in North America.

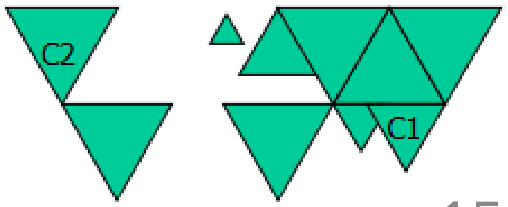
New Ways to India



The Land of Silver

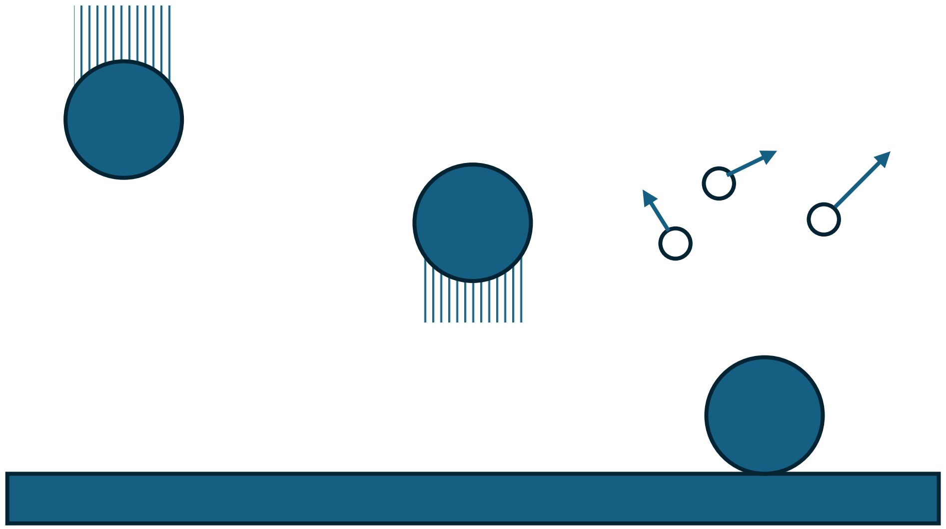


Cotton in India & America

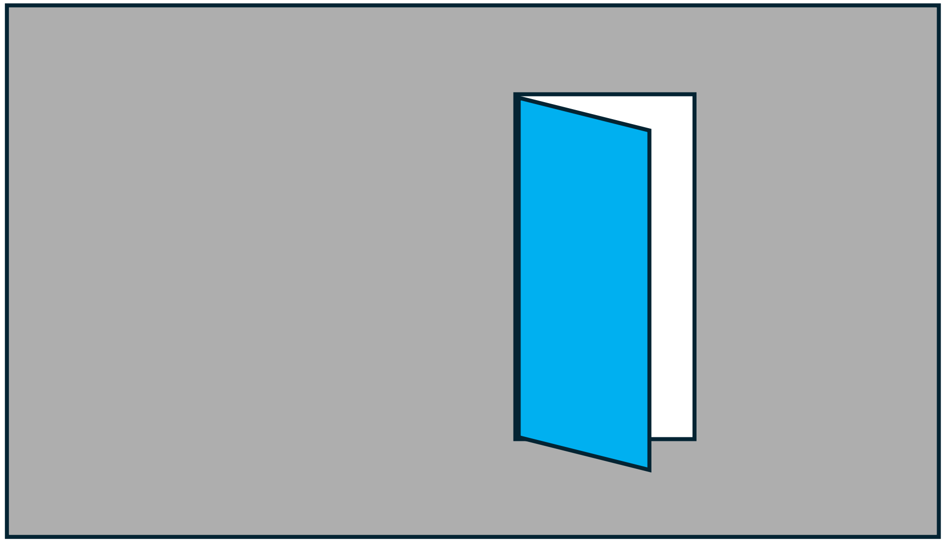


Calculus: **Change** Calculations
Forces Change Motion: $F*\Delta t = \Delta(m*v) = m*\Delta V$

Moving down and up, gravity pumps motion in and out of the ball, loosing motion gradually when colliding with small balls, molecules.



A force's Impulse gives a **change** in the Momentum, and the acceleration is inverse proportional to the mass: Push to an open and a closed door.



World Trade History III:
Engines, Closed Markets & Free Trade

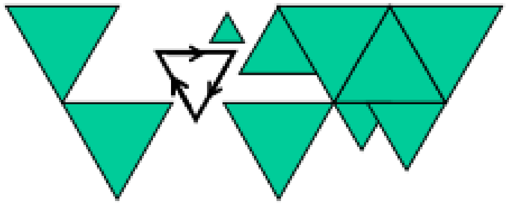
US **COTTON**, African workers & English maschines created **Triangular Trade**.

Engines in the North created closed economies with the South as colonies supplying raw materials, and becoming a market for the **surplus production**.

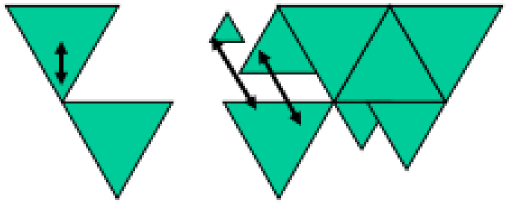
It took **two world wars** to create an open global economy with free trade.

Supplying multiple armies in WW2 let to the invention of an artrificial brain, a **COMPUTER**, programmed by the binary numbers children learn in grade one when bundle-counting fingers in **2s**: 5 = II II I = 1**BB** 0**B** 1 = 101 **2s**

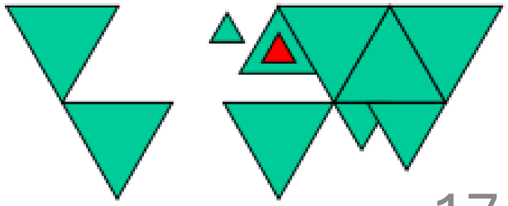
Triangular Trade



Closed Economies



WW1, WW2 & Free Trade



World Trade History IV: **POWER** via Steam & Electrons

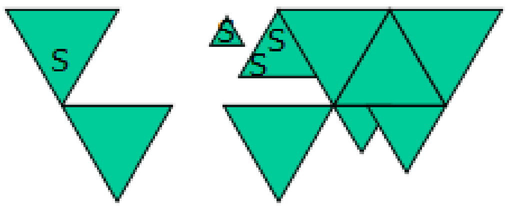
POWER creating motion in maschines may come from:

- The natural gravitation creates water & wind mills
- Heating and cooling a cylinder creates high and low pressure moving a piston up and down in steam engines (**centralized power**).
- Heating and cooling a left and right cylinder creates a wind of steam rotating a turbine rotating a wire in a magnetic field to create a current of **electricity** carrying motion from a central power plant to local consumers (**decentralized power**).

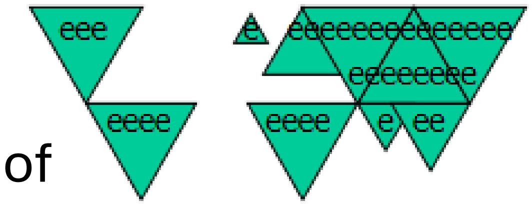
Turning **electricity** on and of creates binary signals, 101 = on-off-on, so computers can store & process information.

ELECTRONS carry **ENERGY** and **INFORMATION**.

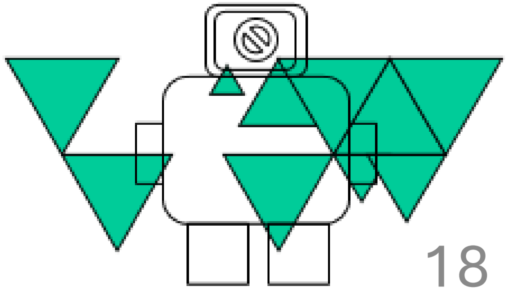
Steam makes Motion



Electrons carry Motion



Robot in Action

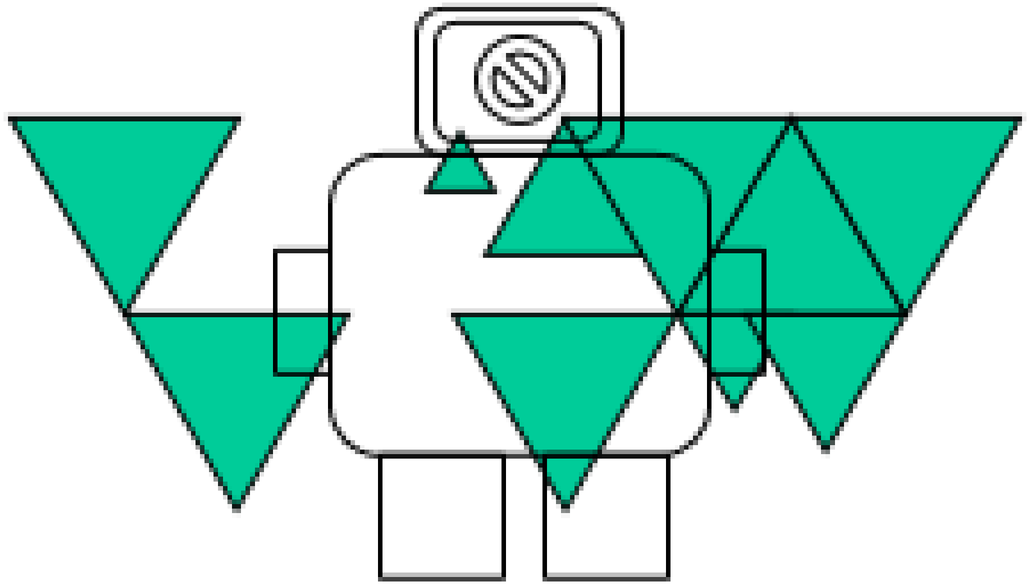


The **COMPUTER** also let to more examples of new applied math called **OPERATIONAL RESEARCH**

Examples of OR, **OPERATIONAL RESEARCH**

- Queueing Theory
- Linear Programming
- Decision Theory
- Information Theory
- Graph Theory
- Game Theory
- Artificial Intelligence

Robot in Action



From Math to Numeracy so Science, Technology, Engineering & Economics can increase Productivity

Discovery/Inventions	Increase Productivity (Production/man/hour)
Iron	Robust and effective tools
Wheel	No need to use energy to lift and lower the center of gravity
Steam engine	Heating and cooling creates alternating pressure diffrence to push & pull
Explosion motor	From solid to gas multiplies the pressure
Electro-magneticism	Rotating a wire in a magnetiv fiels creates electricity
Nuclear power	Matter+ anti-matter <-> enegy (Big Bang formula)
Trancistors and chips	Fixed If-then deciscieons may be put on a printplate
Artificial intelligence	Flexible If-then deciscieons may be programmed into a computer
Solar cells, wind mills	Hravesting nature's enegy
Molecule engeniring	Medcine industry

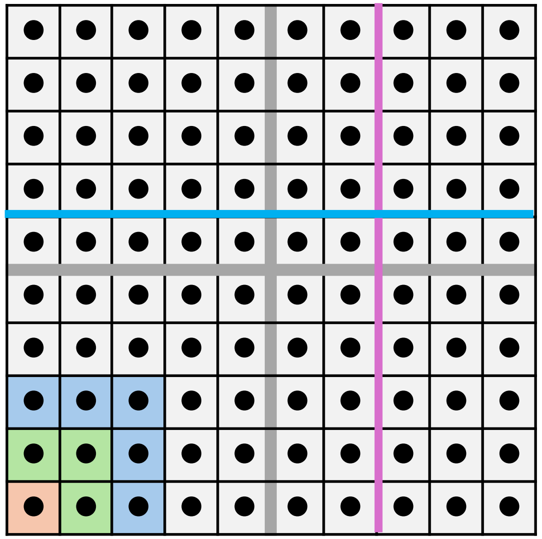
UN: "By 2030, ensure that all youth achieve Numeracy"
Will replacing STEM with STeN make a difference?

By 2030 all Youth "understand and work with numbers" **if we change**:

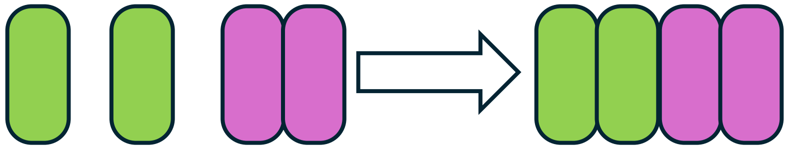
- From **HalfMath** using **CountingNumbers** in time only to **FullMath** using **BundleNumbers** in space also.

So, No Math in K-3 (we can't math, it is not an action word) only **CATS Numeracy** where Children develop their own **Numbers with Bundle-units** to **C**ount and Re**C**ount before **A**dding next-to or on-top using a **B**undle**B**undle**B**oard.

So, Math through Numeracy, not before!

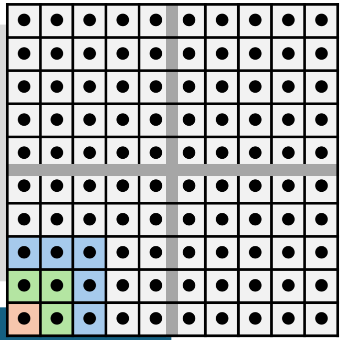


Numeracy = **Math with units**
where addition folds,
but multiplication holds.

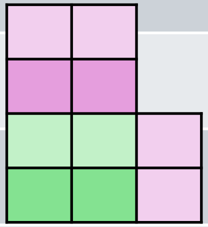


Existence before Essence makes Children

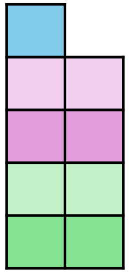
BundleCount in BundleNumbers with Units on a BundleBundleBoard



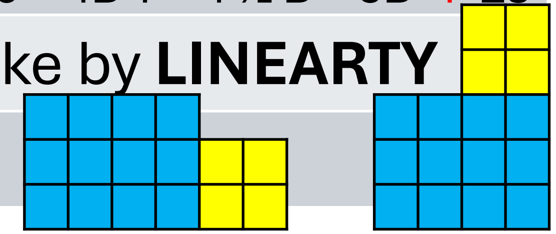
This Reverses	the Operation order
POWER	**Bundles Bundles**
LOG	Counts the number of **Bundlings**
ROOT	Finds the side in a **BundleBundle**
DIVISION	PUSH-away **Bundles**
MULTIPLICATION	PUSH-back **Bundles** to stack
SUBTRACTION	PULL-away **Bundles** to find the **unBundled** to place on-top
ADDITION	PULL-back **Bundles** to unite
ON-TOP	T = (T/**B**)*$\mathbf{B}$ reCounting makes the units like by **LINEARTY**
NEXT-TO	as areas rooting **CALCULUS**



2 **2s** = 2^2 = 1**BB = 1B^2**
4 **2s** = 1**BBB = 1B^3**
log2(8) = 3, log3(9) = 2
8 = 1**BB**?, √8 < 3 (=2.8)



9 = 4**B** 1 = 4 ½ **B** = 5**B** -1 **2s**



STeN: Technology on a BundleBundleBoard

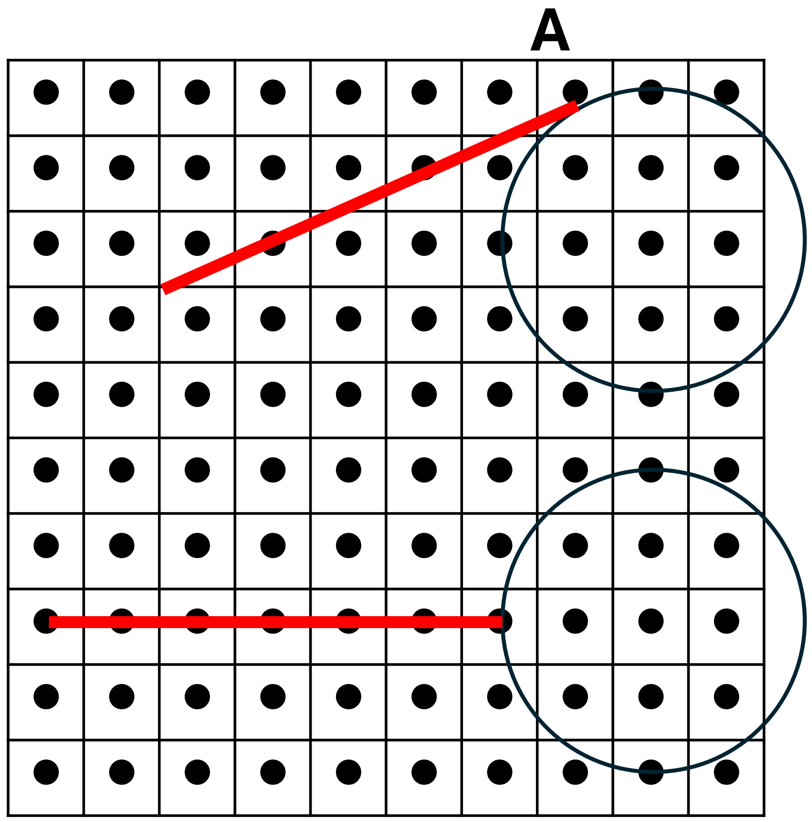
Motion is transferred from a circle to a line by a piston with the length 3.

On the BBBoard with 2 as the unit we like to find the orbit of its endpoint when the angle with its contact point is A.

We soon find the formula for the distance between the endpoint and the circle to be

$\sqrt{[9-(\sin(A)^2)]}-1-\cos(A)$.

So, with A as 0, 90 and 180 degrees, the distances are 1, 1.83, and 3.



STeN: Engineering on BundleBundleBoard

On a sloping hill, roads will be more or less steep. On my bike I can make 20 degrees. So, a BBBoard shows that I can make a 30% slope, but not a 40% slope since here the steepness is 16.7 and 21.8 degrees.

My company is asked to plan a road with hairpin turns and a 5 degrees steepness up a hill with a 20 degrees slope.

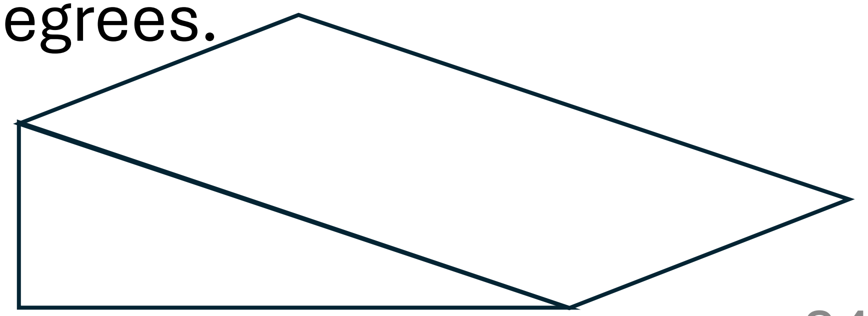
The first guess is a road with √(10^2 + 2^2) = √104 as its length going from (0,0) to the point (10,2) with the height, 2*sin(20).

Here the steepness angle A is found by the equation

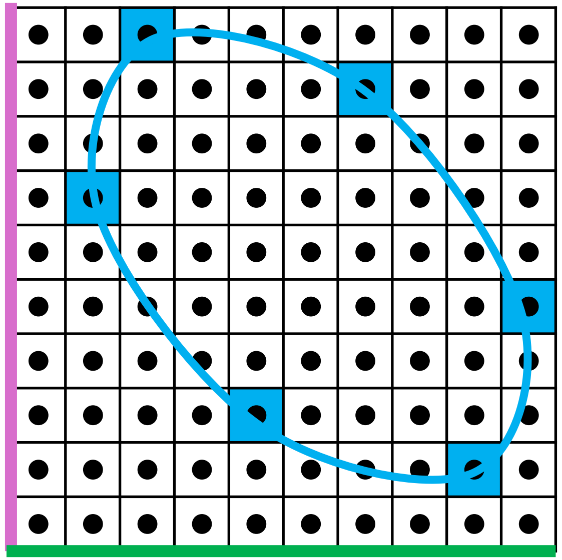
sin(A) = 2*sin(20)/√104, which gives A = 3.84 degrees.

Likewise, a road to (10,3) has the angle A = 5.62 degrees.

To which point should the road go?



STeN: Cats eat Mice, if any

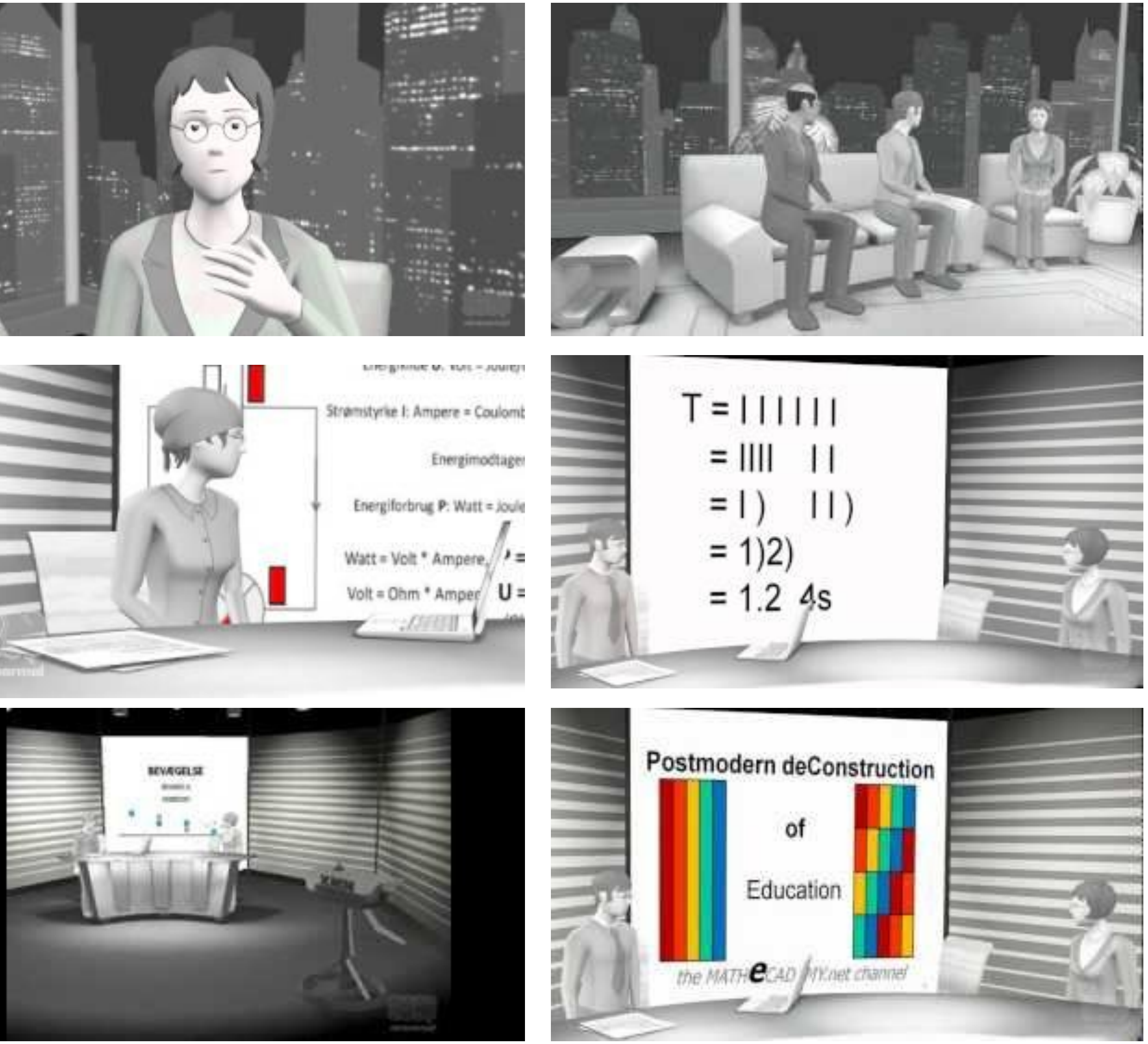


A cats and mice cohabitation on an island is an example of a predator-prey model where cats eat mice. We expect a cycle in time since many cats and many mice leads to many cats and few mice, which leads to few cats and few mice, which leads to few cats and many mice, which leads to many cats and many mice once again.

In a model we assume that a mice-population at 7 and 2 will make the cat-population change with 7-5 and 2-5 respectively. Likewise, a cat-population at 7 and 2 will make the mice-population change with 5-7 and 5-2 respectively. We see that initial populations at the level 5 will give a stable model. Here we assume that the initial populations for the cats and the mice are 8 and 1 respectively. The following period the two populations will then be 8 + (1-5) = 4, and 1 + (5-4) = 2 respectively.

Continuing, we see that the cat population will change as 8, 4, 1, 2, 6, 9, 8; and that the mice population will change as 1, 2, 6, 9, 8, 4, 1. This allows the points (8,1), (4,2), etc., to be marked on a BBBoard, showing a cycle continuing again and again. Different initial numbers will give different cycles.

More MrALTarp YouTube videos



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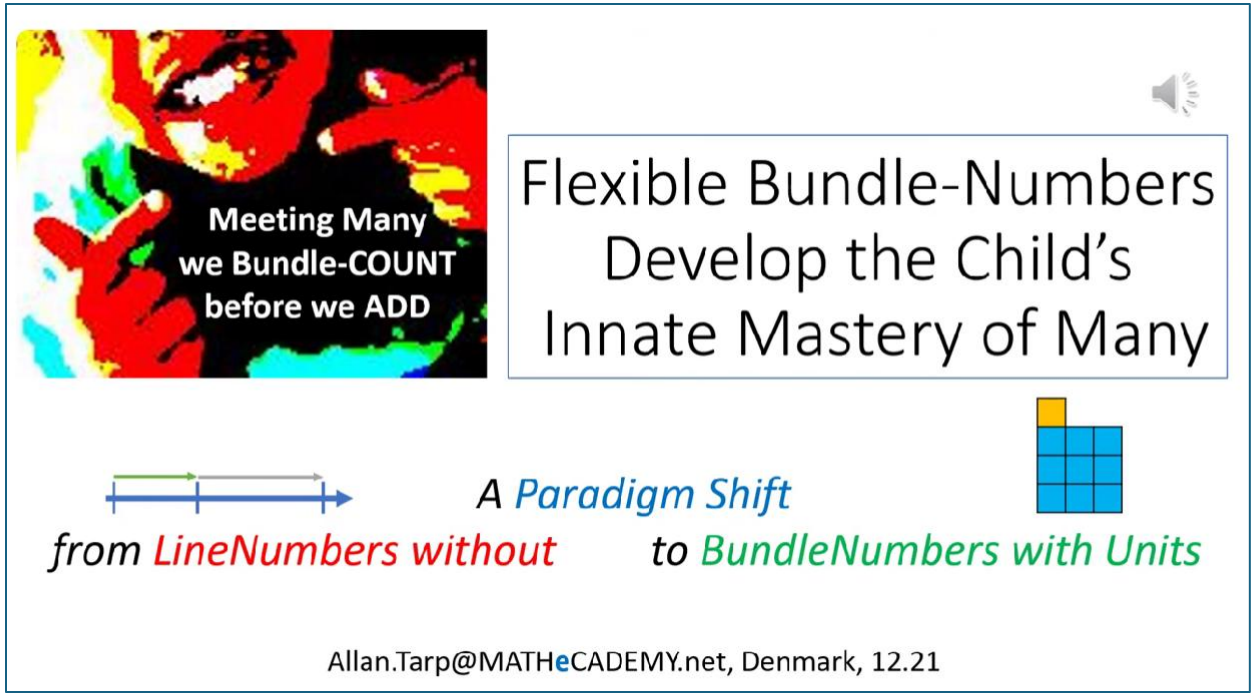
What happened next is seen in this workshop↓ & this textbook→



Workshop: Flexible Bundle Numbers
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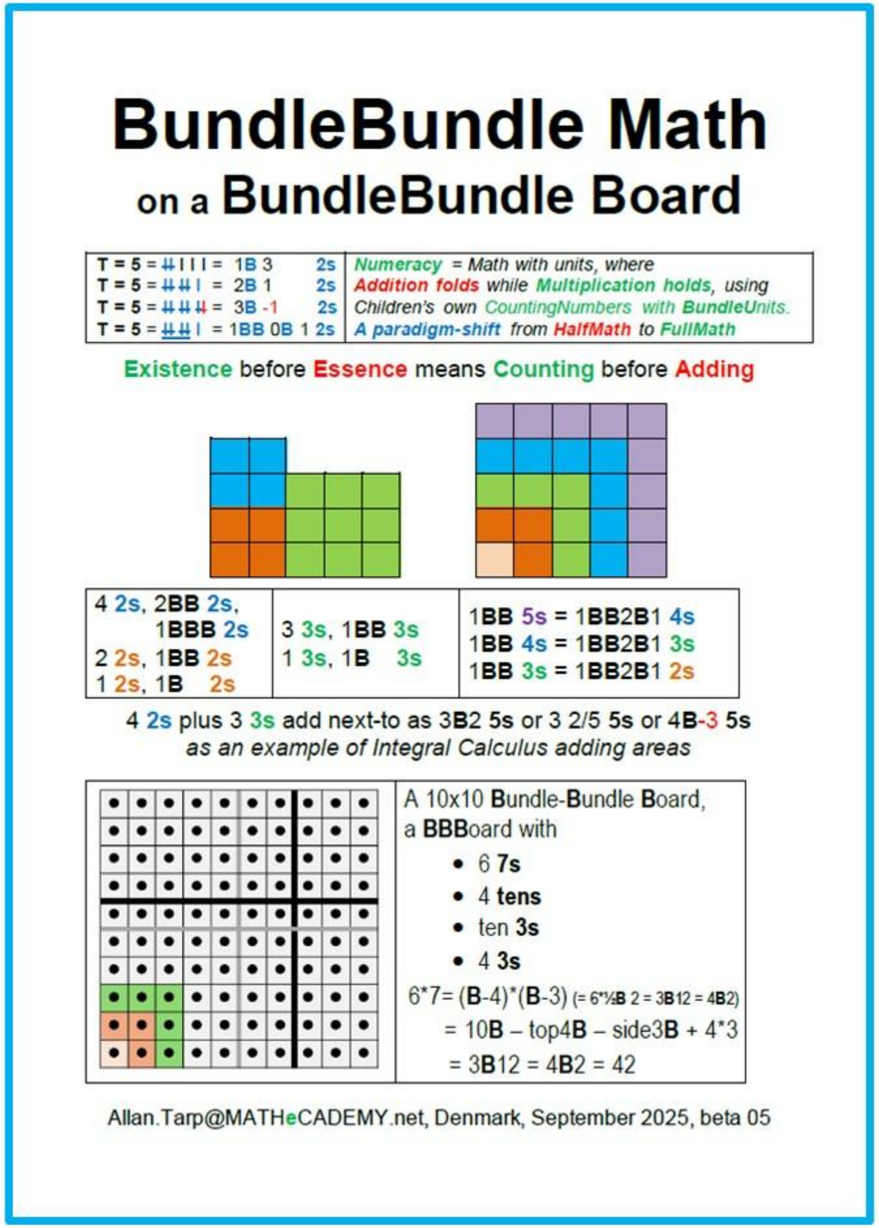

http://mathecademy.net/bundlebundlemath-on-a-bbboard/

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