4 QUALITY EDUCATION

From STEM to STeN with economy & Numeracy

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Denmark

The ancient Viking Empire

called Istanbul ‘Miklagaard’

Economy gives a core Understanding and Use of Numbers and Calculations in the World

Their basic meanings show geometry and algebra as rooted in economics. So, **STEM** should change to **STeN** including **e**conomy and **N**umeracy.

In Greek, geometry means to measure earth. And in Arabic, algebra means to reunite numbers. So, they have a common root in the basic economic question "How to divide the earth, and what it produces?"

A hunter-gatherer needs not tell the different degrees of **many** apart.

But a farmer does since here you produce to a market. And there, you need to be **numerate** to answer the question "How **many** in **total**?"

Which at once leads to the answer "That depends on the **unit**."

Units Matter. STeN and children all use units. **Math** does not - and must go.

Units Change, at Workplaces and at Markets

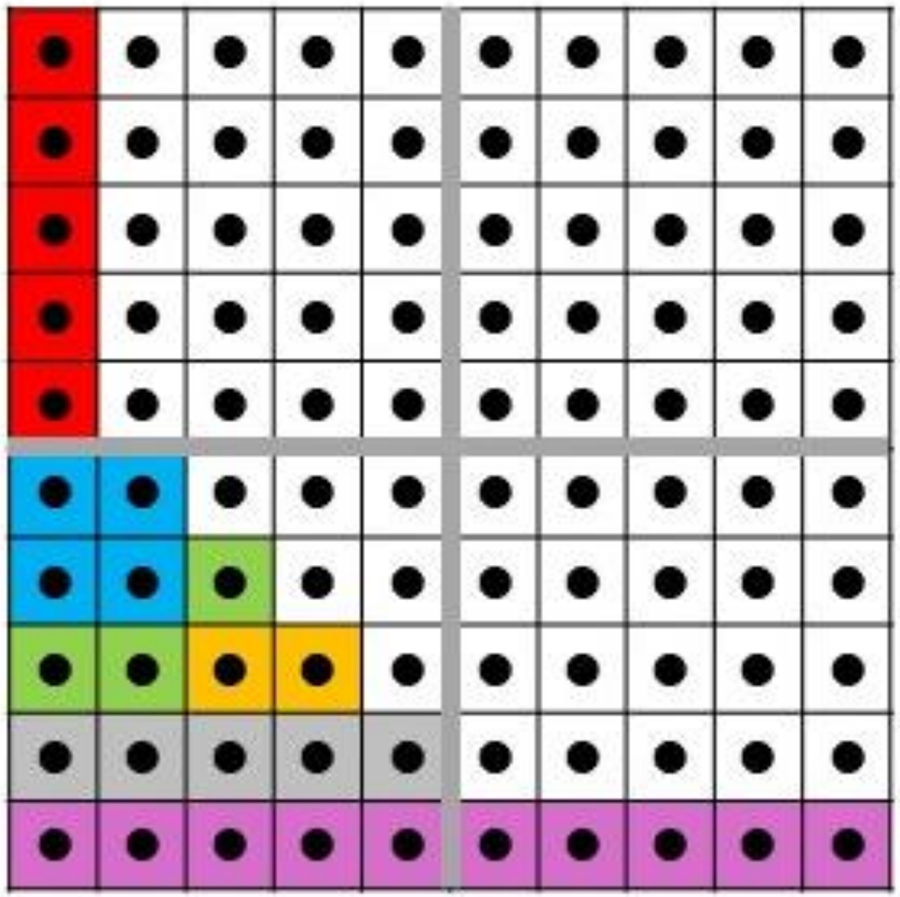
At the workplace we use our hands and muscles to transform input to output placed on a row as single items. For a market, we need the items to be **Bundled** in, e.g., **2s**, **5s**, **tens**, **dozens, scores,** etc.

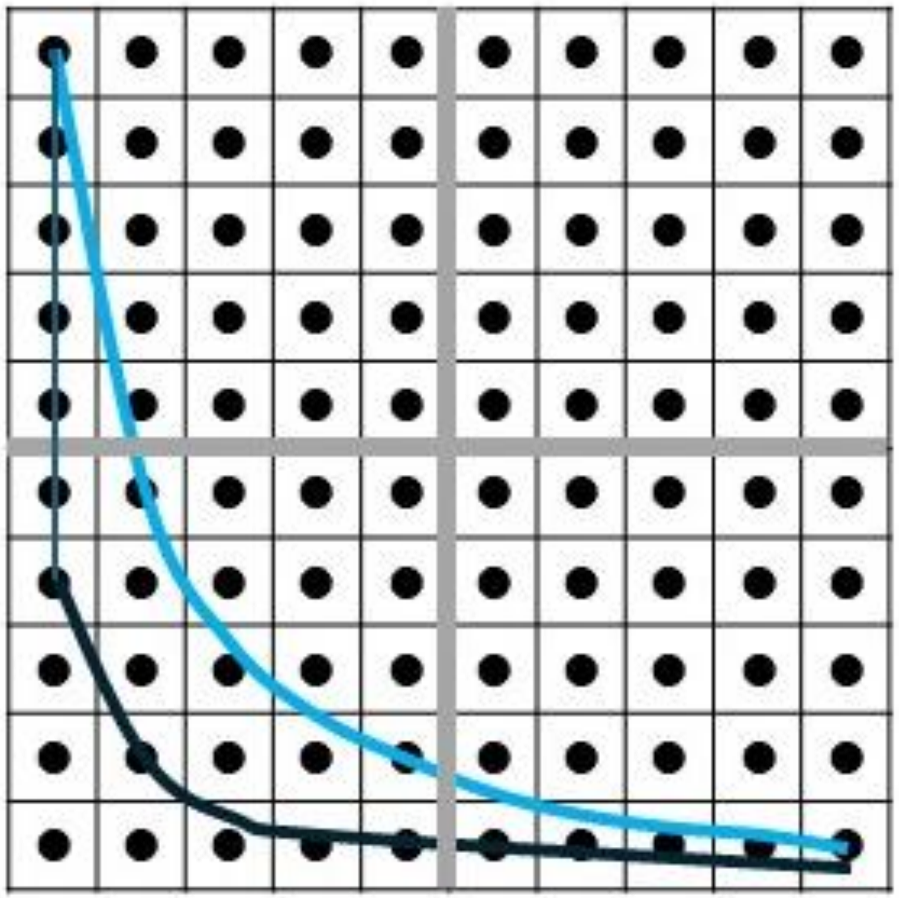
At the market, a buyer may want to buy **7s**, or to pay 5$ per 4 kg.

So, **Changing Units by ReCounting** is a core task in Numeracy:

- '2 **3s** = ? **5s**', and
- '6 **7s** = ? **tens**', and
- '3 **tens** = ? **6s**'.
- 'With 4kg per 5$, 12$ = ? kg' and ? $ = 10kg'

ReCounting 10 & 4 1s in 2s & 3s & 4s & 5s & 10s
A Carnot Cycle with the Energy in a Heat Engine





With units, we can solve a facebook Puzzle

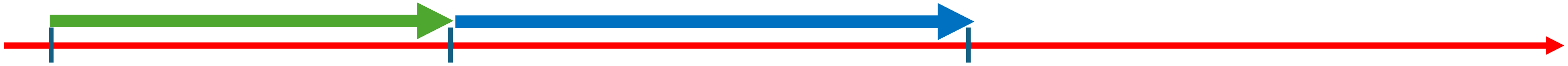
Question	Answer
Without units	**With units**
1 + 4 = 5	1 **1s** + 4 **1s** = 5
2 + 5 = 12	2 **1s** + 5 **2s** = 12
3 + 6 = 21	3 **1s** + 6 **3s** = 21
8 + 11 = ?	8 **1s** + 11 **4s** = **52**

No more **PlaceValues** or **Carrying** or **Borrowing** when **Bundling** deModels **essence** into **existence**

Overload	Underload	Overload	Overload
65 + 27	65 – 27	7 x 48	336 /7
6**B** 5 + 2**B** 7	6**B** 5 – 2**B** 7	7 x 4**B** 8	33**B** 6 /7
8**B** 12 9**B** 2	4**B** **-2** 3**B** 8	28**B** 56 33**B** 6	28**B** 56 /7 4**B** 8
92	**38**	**336**	**48**

Adding Numbers with like Units, 7 + 9 = ?

Inside the 'essence paradigm', numbers add **serial** next-to on the number line. We find the result by counting on 9 times from 7.

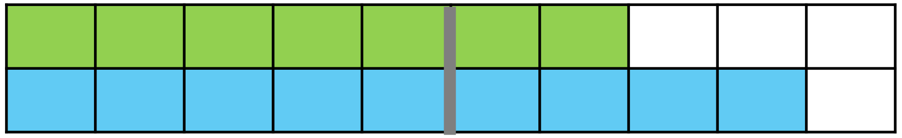


Outside, in the 'existence-paradigm', numbers add **parallel** on-top.

We see that T= 7 + 9 = 2**B** 2 **7s** = 2**B** -2 **9s** = 2**B** -4 **tens** = 1**B** 6 **tens** = 16

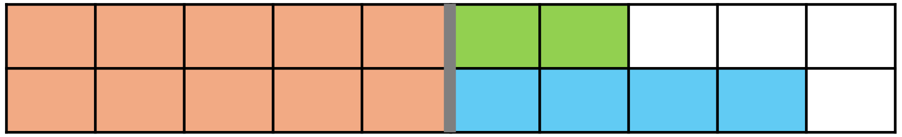
- Added directly as less-numbers:

T = 7 + 9 = **B** -3 + **B** -1 = 2**B** -4 = 16



- Added directly as half-bundles

T = 7 + 9 = ½**B** 2 + ½**B** 4 = 1**B** 6 = 16



BundleCount in Bundle-Units **2s**

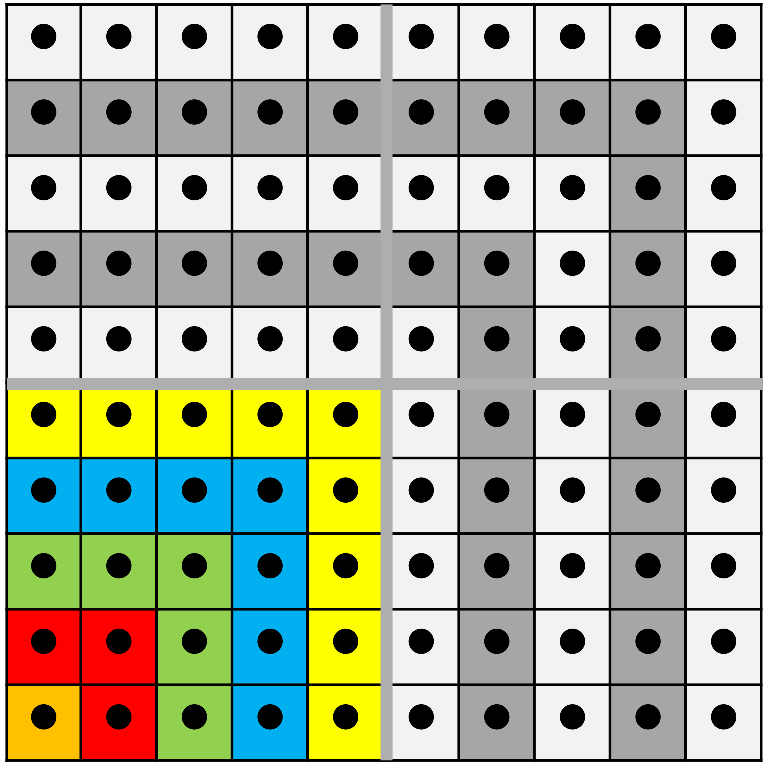
IIIII ● IIII ● IIII ● IIII ● IIII I

0**B** 5 1**B** 3 2**B** 1 3**B** -1 1**BB** 0**B** 1

Ten fingers = 2**BB** 0**B** 2 = 1**BBB** 0**BB** 1**B** 0

We see, that a **BB** is a square, where 2 **2s**

- with 2**B**+1 is 3 **3s** that
- with 2**B**+1 is 4 **4s** that
- with -2**B** +1 is 3 **3s**



1**BB** 2**B** 1 is the **next-BB formula**

1**BB** -2**B** 1 is the **before-BB formula**

Later, (x^2)' = 2*x

UN: "By 2030, ensure that all youth achieve Numeracy" Will replacing STEM with STeN make a difference?

- Oops, two different definitions of '**numerate**' exist:

SUSTAINABLE DEVELOPMENT GOALS

The UK Oxford Dictionary: Competent in the basic principles of **mathematics**, esp. arithmetic.

The US Merriam-Webster Dictionary: Having the ability to understand and work with **numbers**.

- Why this difference? We ask the three **Grand Theories**

Philosophy: To prevent inside **essence** from colonizing it, outside **existence** must come first.

Sociology: Institutionalizing **essence** may lead to a **goal displacement** where a means becomes the **goal** by working, not for, but **against** reaching the original goal, **existence**.

Psychology: **Self-referring** systems construct **essence** as **inside reductions of outside complexity**. So, as a **social system,** math cannot transfer **its essence** to **psychic systems** that construct their **own essence** when **disturbed** by outside **existence** of **Many** while **Counting & Adding in Time & Space**.

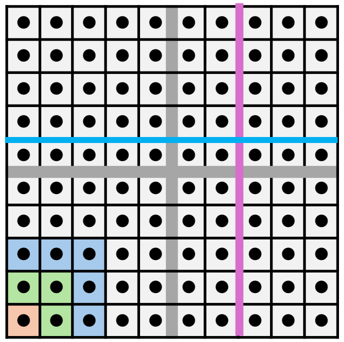
Math first? Or **existence** first? Well, let us ask: **How Numerate are Children before School**?

We ask a 3year-old "How many years next time?"
The answers is 4, with 4 fingers shown

But, with 2 bundled as **2s**, the child says "No, that is not 4, that is 2 **2s**."

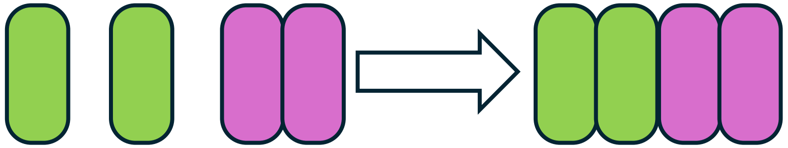
The educated sees the **essence**, 4, the un-educated sees the **existence**, 2 **1s** **bundled** as 1 **2s** in space, and 2 of them when **counted** in time.

Children understand Numbers as 2D on a **BundleBundleBoard** with a **bundle-unit** below, and a **counting-numbers** going up.



BBM BundleBundleMath, or Existence-math describes **Many** by the child's own **Counting-numbers with Bundle-units.**

Essence-math adds without units. And **colonizes** **Many** with **Half-Matics** using counting-numbers only. And becoming '**Mathema-tism**' by claiming that 2+1 = 3 despite here, 2 **1s** +1 **2s** = 1 **4s** , not 3 **3s.**



So, **Units Matter!**

Bundling in Space, Children create Units to Count with in Time. Adults do the same

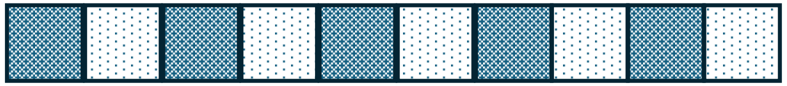
47: *1 number?*

No, 3 numberings:

Numbering the **B**undles & the **B**undle-size & the un**B**undled.

4 ty 7 = 4 **tens** 7

4 **B**undles 7 = 4 **B** 7

4  7

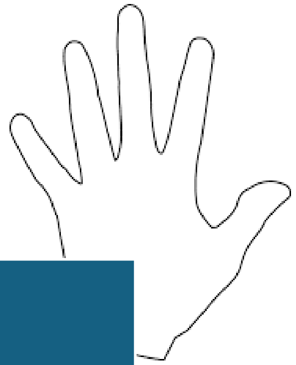
4 **B**undles, at ten-per-**B**undle, and 7 un**B**undled

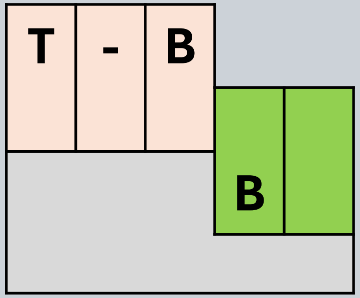
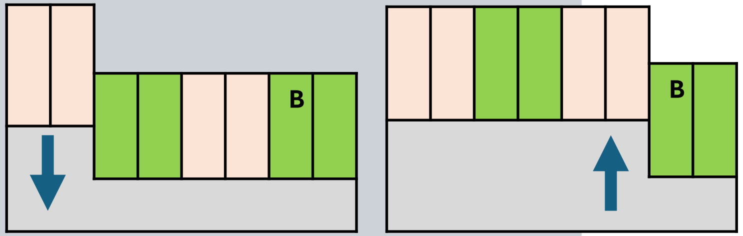
47 = 4**B** 7 = 3**B** 17 = 5**B** -3 (overload & underload)

407 = 4**BB** 0**B** 7

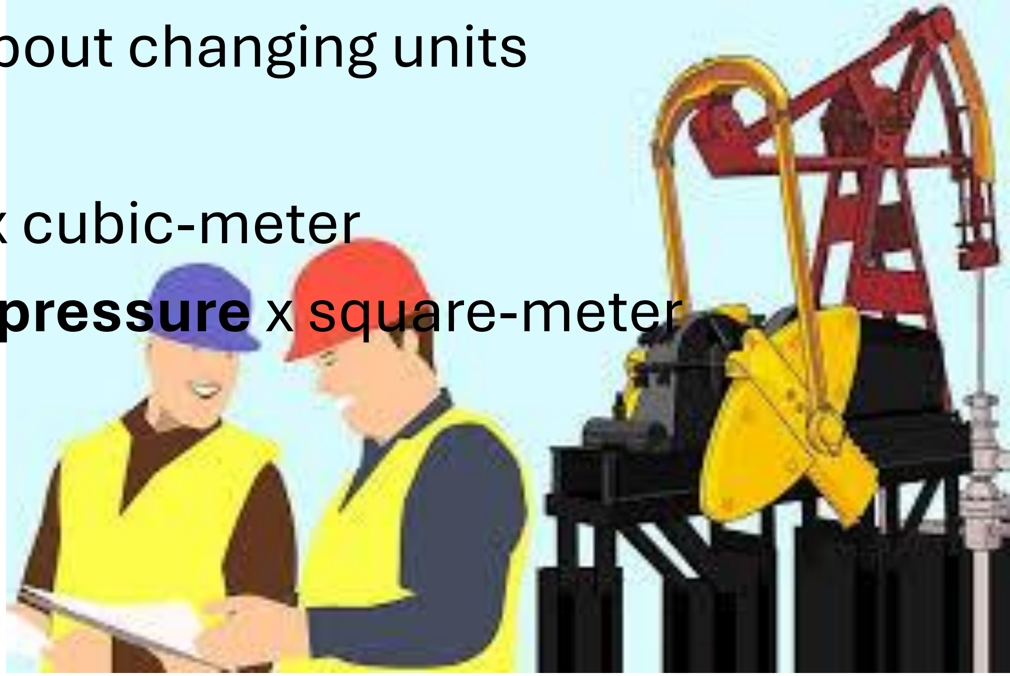
*With units: No more **PlaceValues, Carrying, Borrowing***

ReUniting and **ReCounting** Totals



ReUnite Totals	**ReCount** Totals
Iconize: **–** pull-away 'rope' **+** pull-back 'double-rope'	"How many **Bs** in T?" "From T, push-away **Bs**" **Iconize:** **/** push-away 'broom' **x** push-back 'lift'
 	
T = (T-B)+B	**T = (T/B)xB**
The ReUnite Formula Solves uniting equations:	**The ReCount Formula** Solves counting equations:
u + 2 = 7 *But, 7 = (7 – 2) + 2* ***u*** = 7 – 2	***u*** x 2 = 8 *But, 8 = (8/2) x 2* ***u*** = 8/2
u – 2 = 7 *But, u = (u – 2) + 2* ***u*** = (***u*** – 2) + 2 = 7 + 2 = 9	***u*** / 2 = 4 *But, u = (u/2) x 2* ***u*** = (***u*** /2) x 2 = 4 x 2 = 8

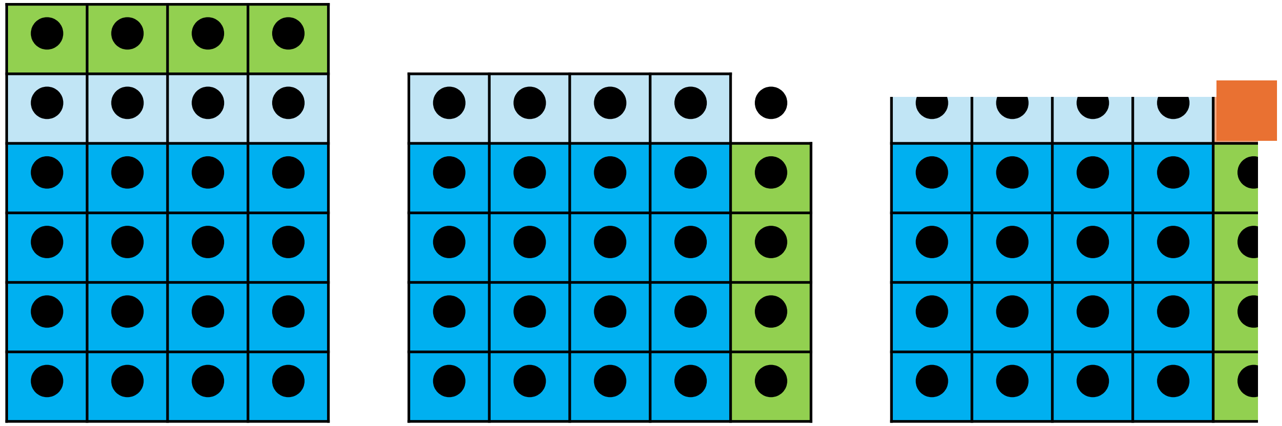
The **ReCount Formula** and **per-numbers** are the core of STeN (economy & Numeracy included)



STeN typically contains multiplication formulas about changing units

- $ = ($/hour) * hour = salary * hour
- kg = (**kg/cubic-meter**) x cubic-meter = **density** x cubic-meter
- force = (**force/square-meter**) x square-meter = **pressure** x square-meter
- meter = (**meter/sec**) x sec = **speed** x sec
- energy = (**energy/sec**) x sec = **Watt** x sec
- energy = (**energy/kg**) x kg = **heat** x kg
- gram = (**gram/mole**) x mole = **molar mass** x mole
- Δ momentum = (**Δ momentum/sec**) x sec = **force** x sec
- Δ energy = (**Δ energy/ meter**) x meter = **force** x meter = work
- **energy/sec** = (**energy/charge**) x (**charge/sec**) or **Watt** = **Volt** x **Amp**

Squaring Stacks with a Square root



A **BBB**oard has a stack of 6 **4s** that we can square to a **BB**-square with the square root as the side.

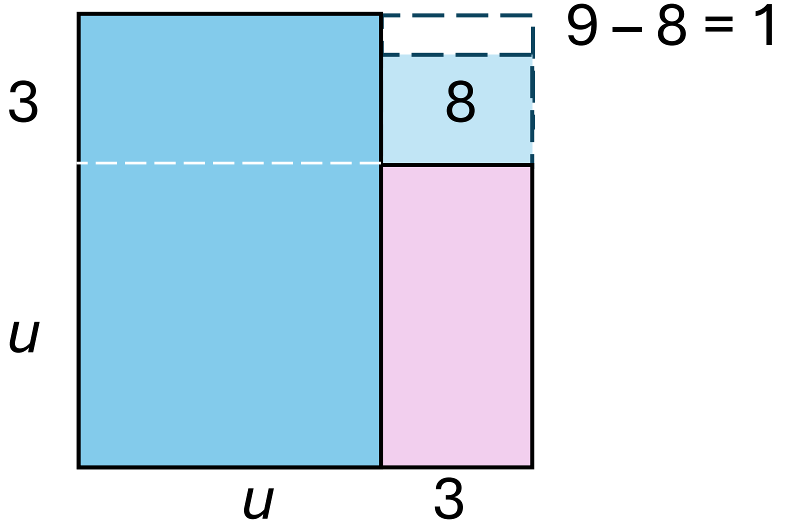
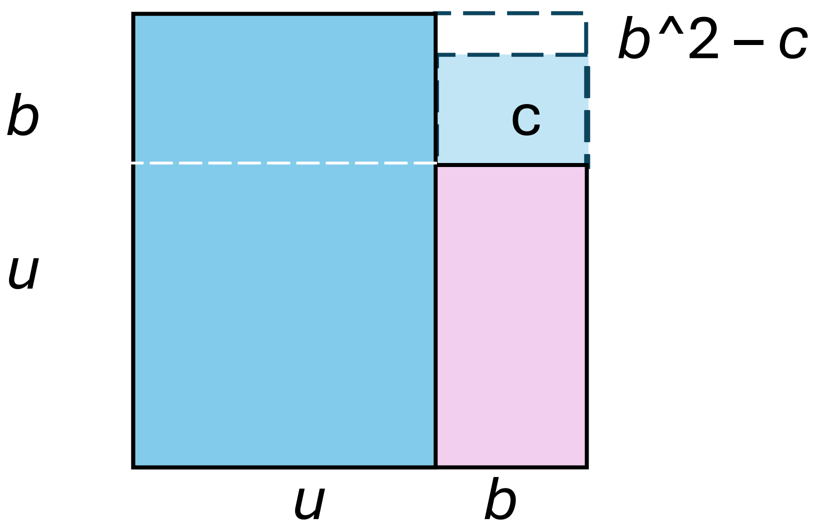
- First, we move half the top-surplus to the side.
- Next, we take *t*-slices from the top and the side to fill the empty corner.
- 2*4*t = 1, or 8*t = 1 = (1/8)*8, so t = 1/8 giving, 5-1/8 = 4.875 ≈ **4.9**

Our guess then is that 6 **4s** can be squared as a 4.9 square.

A calculator gives the answer: the square-root of 6*4 is √(6*4) = 4.90.

Math through NUMERACY showing how to UNDERSTAND and USE children's innate BundleNumbers with Units to Count & Add in Time & Space, CATS

Bundle-Bundles as Squares Ease Algebra

Quadratic Equations with Numbers	Quadratic Equations with Letters
$u^2 + 6*u + 8 = 0$	**$u^2 + 2*b*u + c = 0$**
	
$(u+3)^2 = u^2 + 6*u + 8 + 1$ $(u+3)^2 = 0 + 1$ $u = -3 \pm 1$ **$u = -4$ & $u = -2$**	$(u+b)^2 = u^2 + 2*b*u + c - c + b^2$ $(u+b)^2 = 0 + b^2 - c$ **$u = -b \pm \sqrt{(b^2 - c)}$**

ReCounting from **tens** to **digits:** 30 = ?**4s**

A **Division Table** changes Units & solves Equations

Predicted on a Calculator

30/4
7.more

30 – 7x4
2

Two bands after **4s** and over 3 allows changing units from 3 **tens** to some **4s**, thus solving the equation '*u* * 4 = 3 **tens**'.

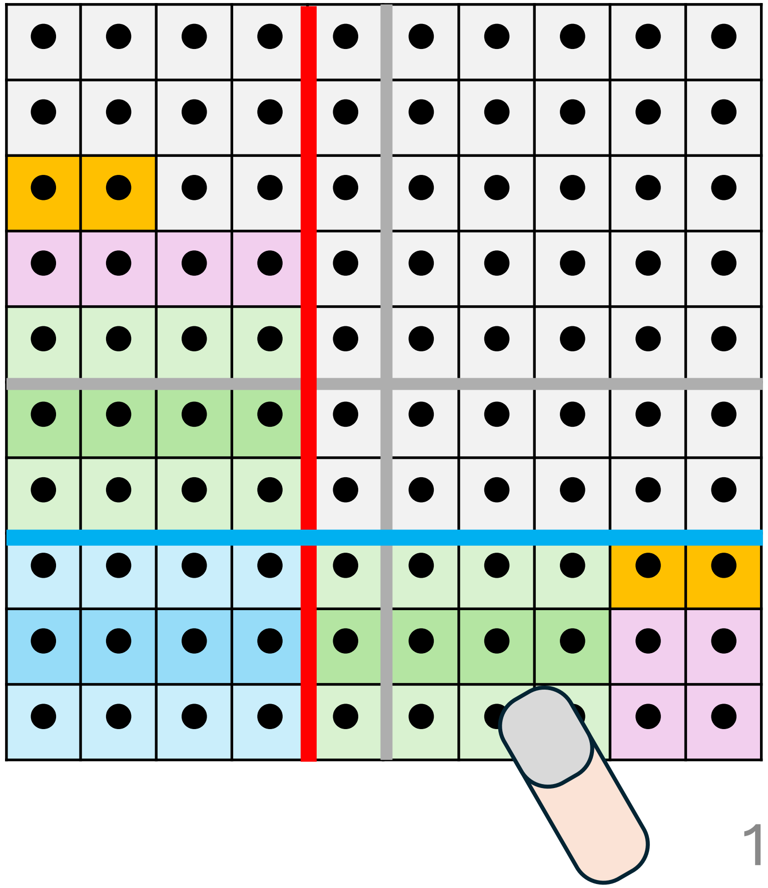
With a finger I find the **4s** to move. First 3, then 1, giving 7.

The unbundled **2s** become a decimal, so 3 **tens** = 7**B** 2 **4s**.

I now predict the result on a calculator, recounting 30 in **4s**:

30/4 = 7.more
30 – 7*4 = 2

30 = 7**B** 2 **4s**



The equation is solved the "**OPPOSITE side & sign**" way.

u * 4 = 30
u = 30/4

but 30 = (30/4) * 4

ReCounting from **digits** to **tens:** 6 * 7 = ? **tens**
A **Multiplication Table** with **Algebra** on a **BB**Board

Two bands after 6 & **7s** allow feeling & seeing & recounting 6 **7s** in **tens**.

- Count the half-**B**undles, ½**B**s

6 * 7 = (5 + 1 + 2) * ½**B** + 2 = 8 * ½**B** + 2 = 4**B** 2 = 42

- Recount 7 as ½**B** 2

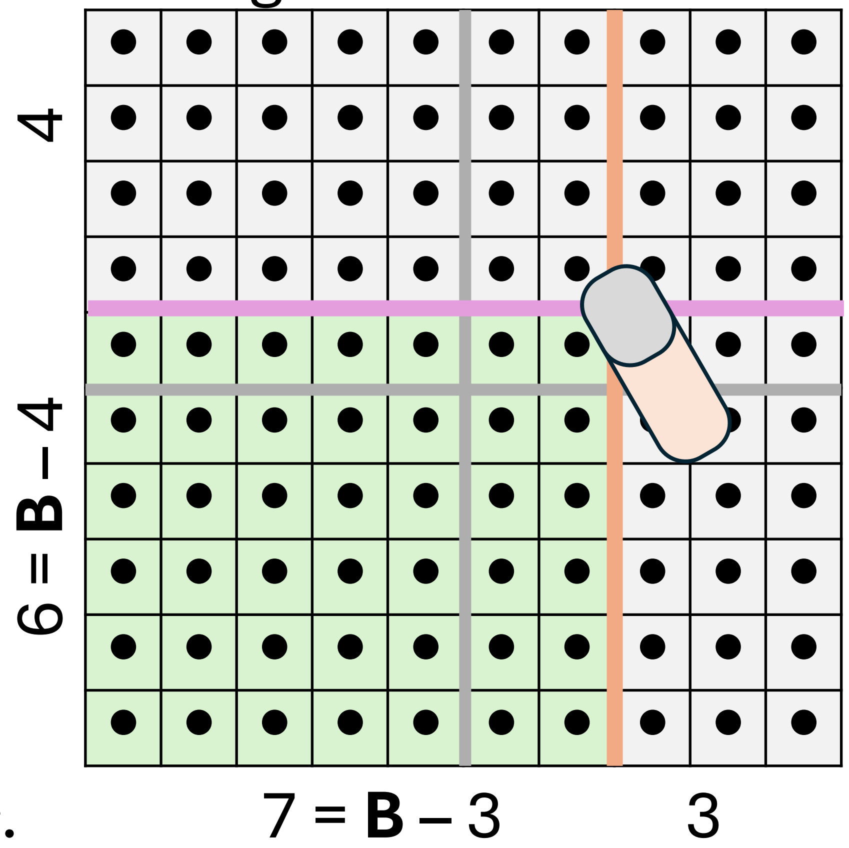
6 * 7 = 6 * ½**B** 2 = 3**B** 12 = 4**B** 2 = 42 *(an overload)*

- Pull-away the outside **B**undles

6 * 7= (**B** – 4)*(**B** – 3) = **BB** – 3**B** – 4**B** - - 3 * 4

= (10 – 3 – 4)**B** + 12 = 3**B** 1**B** 2 = 4**B** 2 = 42.

*12 is pulled-away twice, so minus * minus is plus.*



ReCounting Goods gives **PerNumbers** and **Fractions**

A **per-number** 4kg/5$ recounts goods in kg's and dollar's.

ReCounting in the per-number changes units (proportionality)

- Question: 20kg = ? $.
- Answer: 20kg = (20/4) * 4kg = (20/4) * 5$ = 25$.
- Question: 20$ = ? kg.
- Answer: 20$ = (20/5) * 5$ = (20/5) * 4kg = 16kg.

Footnote.
With like units, **per-numbers become fractions**: 4$/5$ = 4/5, and 40$/100$ = 40%

Question: 8$ = ?% ***with 40$ = 100%,***	**Question: 80% = ?$** ***with 40$ = 100%***
Answer: 8$ = (8/40)*40$ = (8/40)*100% = 20%	**Answer: 80% = (80/100)*100% = (80/100)*40$ = 32$**

With PerNumbers, No More **Proportional Reasoning** & **Multiplicative Thinking**

ReCounting Sides in a Stack halved by its Diagonal gives **Trigonometry** before Geometry, and π

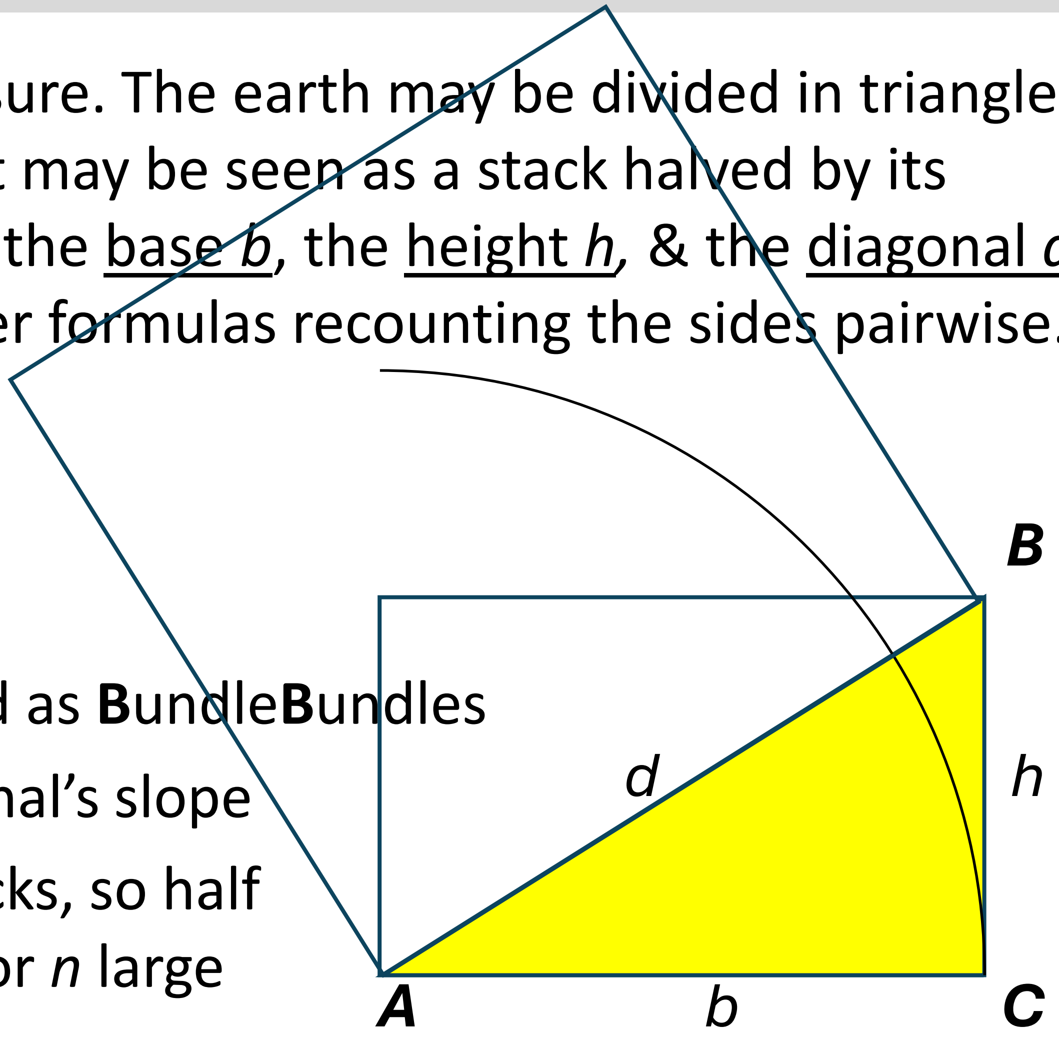
In Greek, geo-metry means to earth-measure. The earth may be divided in triangles; that may be divided in right triangles; that may be seen as a stack halved by its diagonal. This 'half-stack' has three sides: the <u>base *b*</u>, the <u>height *h*</u>, & the <u>diagonal *d*</u>, connected with the <u>angle ***A***</u> by per-number formulas recounting the sides pairwise.

$h = (h/b) \times b = \tan \mathbf{A} \times b$
$h = (h/d) \times d = \sin \mathbf{A} \times d$
$b = (b/d) \times d = \cos \mathbf{A} \times d$

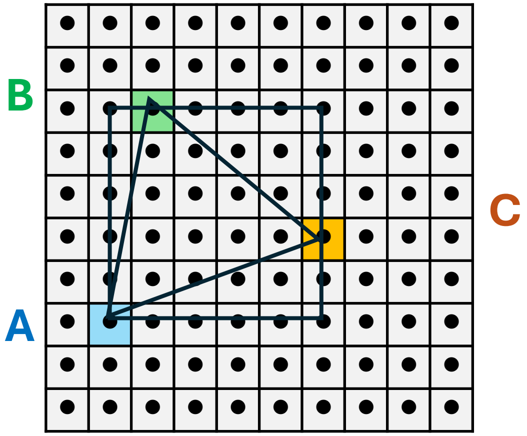
$\underline{h \times h + b \times b = d \times d}$, so the sides add as **B**undle**B**undles

tan***A*** = $h/b = \Delta y/\Delta x$ = rise/run = the diagonal's slope

A circle contains very many small half-stacks, so half the circumference is: $\boldsymbol{\pi = n \times \tan(180/n)}$ for n large



Triangles on a BundleBundleBoard



Point **A** is at the (2,3) dot. Point **B** is at the (3,8) dot. And point **C** is at the (7,5)dot.

To find length, angles and area of the triangle ABC we enclose it in a 5 **5s** stack.

All three angles are split in two outer, and one inner angle.

We find the left angles using tangent, and the sides by using sine

Tan A = 1/5, so A = 11 degrees.

And, sin A = 1/c. But, 1 = (1/c)*c, so, 1 = sin 11.3 * c, c = 1/sin11.3 = 5.1

Likewise, tan B = 3/4, so B = 37 degrees. And, tan C = 5/2, so C = 68 degrees

The side a = 5.7, and the side b = 5.4. The area of the three outer half-stacks are ½*(1*5 + 3*4 + 5*2) = 13.5.

So, the area of the inner triangle ABC is 5*5 – 13.5 = 11.5

To find the angles in the triangle ABC we begin with 90 degrees for A and 180 degrees for B and C.

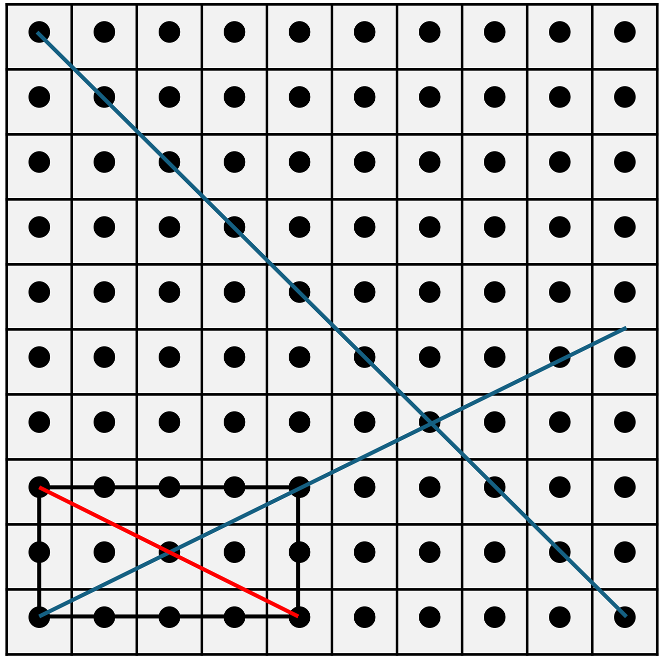
Then we pull away the two neighbor angles in the outer right triangles and get:

A = 90 – 22 – 11 = 57 degrees, and B = 180 – 79 – 37 = 64 degrees, and C = 180 – 68 – 53 = 59 degrees.

Finally, we test the results by adding the three angles: 57 + 64 + 59 = 180.

A Trip in Time changes the place on a BBBoard

A Line meets a Line



On a space-board, the first dot is number 1.
But on a time-board it is number 0 since we have not yet changed place.
In a 2 **4s** stack, the diagonals slope up 2 per 4, 2/4, or down -2 per 4, -2/4.
We take a 2 per 4, 2/4, trip from the (0,0) dot to an unknown (x,y) dot.
If the angle hasn't changed, we will have that
$Tan\ A = y/x = 2/4$, or $y = 2/4 * x$, or $y = ½ * x$, called the line's equation.
This formula can find one number if the other number is known.
Another line goes from the (0,9)-dot to the (9,0)-dot.
Inside its 9 **9s** stack the diagonal slopes -9/9 or -1/1.
So, after x steps y have decreased to $y = 9 – x$.
The lines then meet where $y = ½ * x = 9 – x$.
Here, we change the unit by recounting x in 2s as $x = (x/2) * 2 = 2 * u$ with $u = x/2$ and $x = 2*u$
Now, $½ * x = 9 – x$ *gives,* $½ * 2 * u = 9 – 2 * u$, or $u = 9 – 2 * u$, or $u + 2 * u = 9$, or $3 * u = 9$, or $u = 9/3 = 3$.
This gives $x = 2 * u = 2 * 3 = 6$.
Here, $y = 9 – 6 = 3$, so the two lines meet at the (6,3)-dot.

BBBoard as a TimeBoard for Trips with Meeting Points

A Line meets a Circle

The circle with radius 10 and center in the (0,0)-dot contains the (x,y)-points, where $x^2 + y^2 = 10^2$.

On its way the $y = 1/2*x$ line meets the circle in the (x,y)-dot that is placed both on the line and on the circle.

So, $y = ½*x$ makes $x^2 + y^2 = x^2 + (½*x)^2 = 100$,

Or, $x^2 + ¼*x^2 = 100$.

We change the unit by recounting x in **2s** as

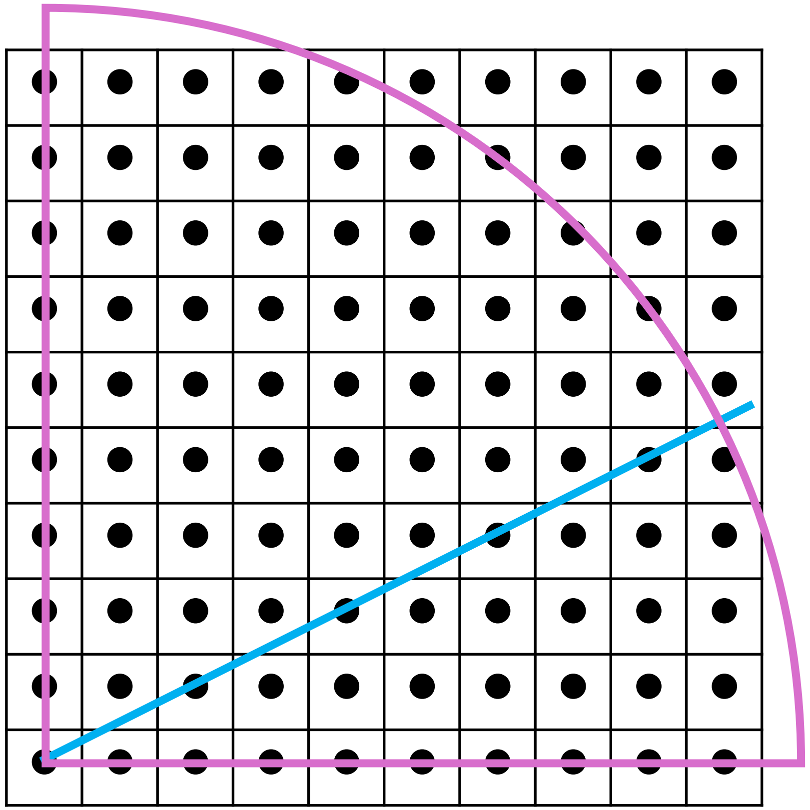
$x = (x/2)*2 = 2*u$ with $u = x/2$.

Then $(u*2)^2 + ¼*(u*2)^2 = 100$, or $4*u^2 + u^2 = 100$,

or $5*u^2 = 100$, or $u^2 = 100/5 = 20$, or $u = \sqrt{20}$, or $u = 4.5$.

Which gives $x/2 = 4.5$, or $x = 2*4.5 = 9$. Here $y = 2/4*9 = 4.5$.

So, they meet in point (9, 4.5).



Circle $x^2 + y^2 = 10^2$

Meeting point $(x,y) = (9, 4.5)$

BBBoard as a TimeBoard for Trips with Meeting Points

A Line meets a Parabola

A trip where $y = (x\text{-}3)^\wedge 2 = x^\wedge 2 - 6*x + 9$ is a bent line called a parabola. It meets the $y = 1/2*x$ line in point (x,y) that is placed both on the line and on the parabola.

So, $y = ½*x$ makes $½*x = x^\wedge 2 - 6*x + 9$, or $x^\wedge 2 - 6.5*x + 9 = 0$.

On the board we see they meet in point (2,1).

To find the other meeting point we write

$x^\wedge 2 - 6.5*x + 9 = 0$ as $(x - 2) * (x - u) = 0$. And,

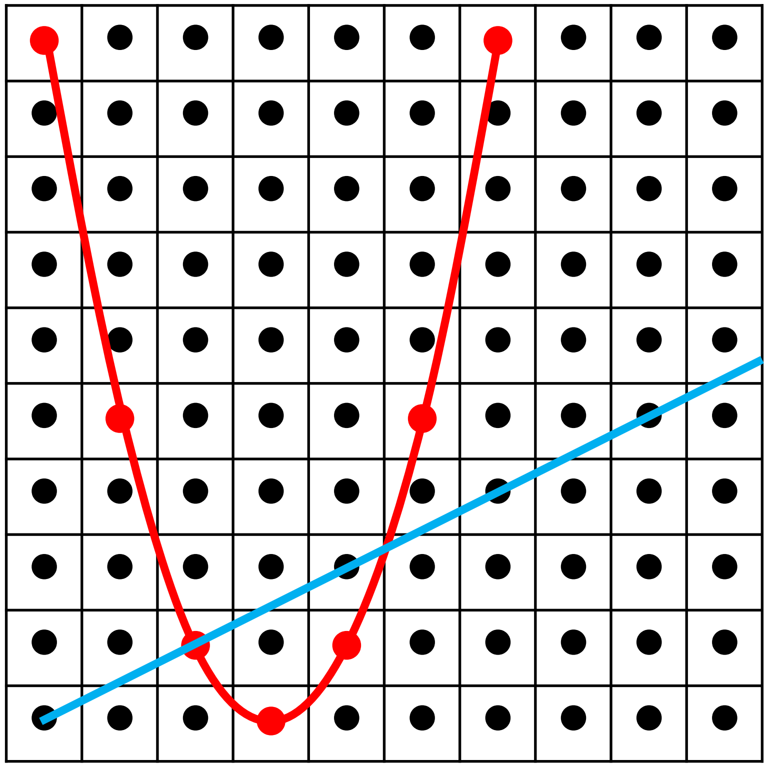
$(x - 2) * (x - u) = x^\wedge 2 - u*x - 2*x + 2*u = x^\wedge 2 - (u+2)*x + 2*u$.

We see that, $u+2 = 6.5$, and $2*u = 9$, both give $u = 4.5$

And that, $(x - 2)*(x - 4.5) = 0$ gives $x = 2$ and $x = 4.5$.

Here $y = ½*2 = 1$ and $y = ½*4.5 = 2.25$.

So, the line meets the parabola in (2, 1) and (4.5, 2.25).



Parabola **y = (x-3)^2**
Meeting points (x,y) = (2, 1) & (4.5, 2.25)

STeN: Technology on a BundleBundleBoard

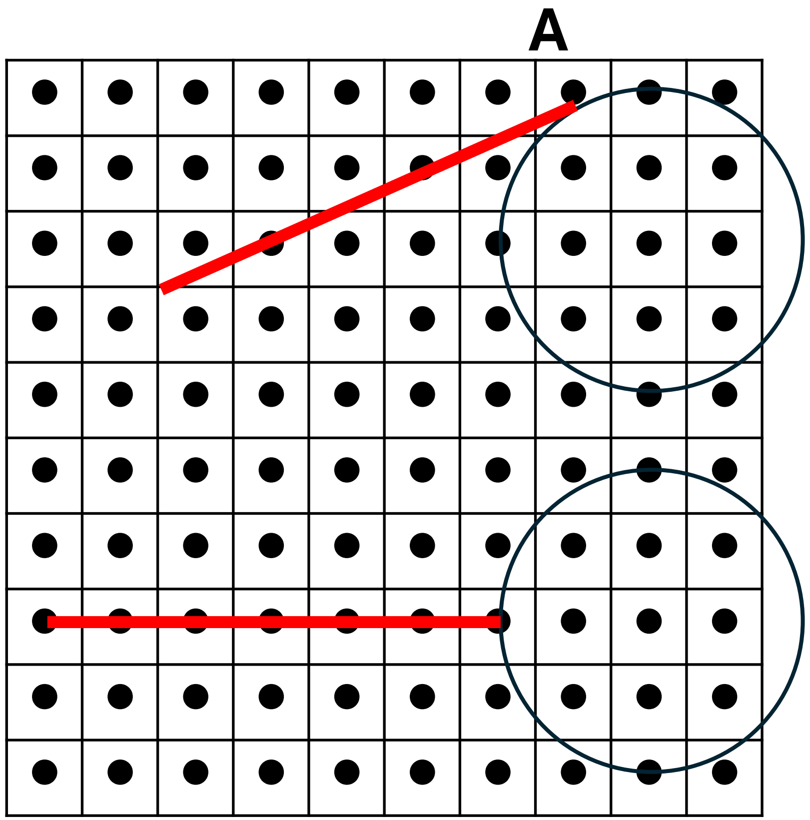
Motion is transferred from a circle to a line by a piston with the length 3.

On the BBBoard with 2 as the unit we like to find the orbit of its endpoint when the angle with its contact point is A.

We soon find the formula for the distance between the endpoint and the circle to be

√ [9 – (sin(A)^2)] – 1 – cos(A).

So, with A as 0, 90 and 180 degrees, the distances are 1, 1.83, and 3.



STeN: Engineering on BundleBundleBoard

On a sloping hill, roads will be more or less steep. On my bike I can make 20 degrees. So, a BBBoard shows that I can make a 30% slope, but not a 40% slope since here the steepness is 16.7 and 21.8 degrees.

My company is asked to plan a road with hairpin turns and a 5 degrees steepness up a hill with a 20 degrees slope.

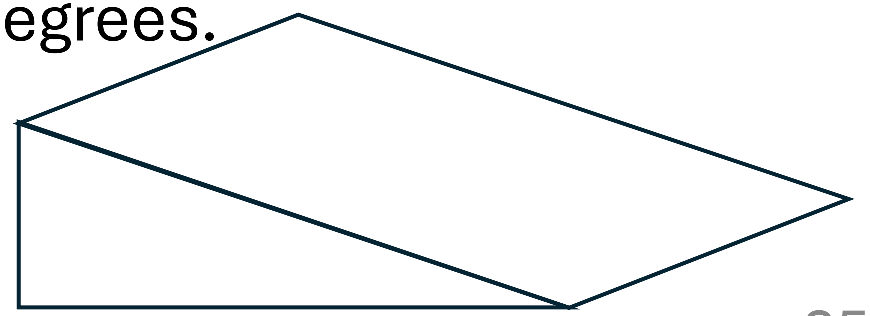
The first guess is a road with √(10^2 + 2^2) = √104 as its length going from (0,0) to the point (10,2) with the height, 2*sin(20).

Here the steepness angle A is found by the equation

sin(A) = 2*sin(20)/√104, which gives A = 3.84 degrees.

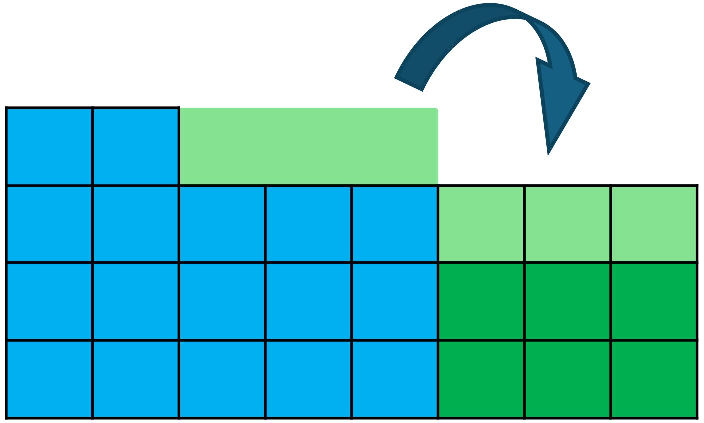
Likewise, a road to (10,3) has the angle A = 5.62 degrees.

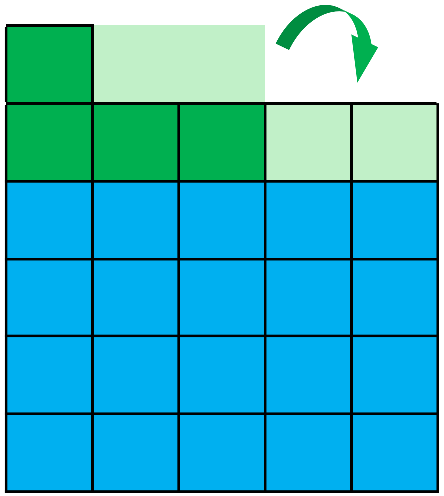
To which point should the road go?



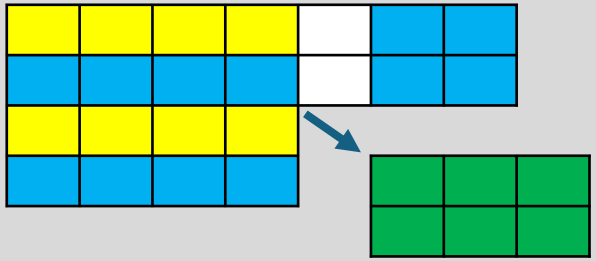
Once Counted & ReCounted, Totals may Add

BUT: NextTo →	**or OnTop ↑**
4 **5s** + 2 **3s** = 3**B**2 **8s**	4 **5s** + 2 **3s** = 5**B**1 **5s**
The areas are integrated *Adding areas = Integration*	The units are changed to be the same *Change unit = ReCounting = Proportionality*





Reversing next-to addition



"If T1 = 2 **3s** and T2 add next-to as 4 **7s**, what is T2?"

We pull away the initial block T1 before recounting the rest in **4s**.

The recount formula predicts the result:

T2 = (T2/**B**) x **B**

= ((4x7 – 2x3)/4) x 4 = 5.2 **4s**

(4x7 – 2x3)/4	**5.some**
(4x7 – 2x3) – 5x4	**2**

Since reversed next-to addition finds area-differences, it is called differential calculus. Here subtraction precedes division; which is natural as reversed integration.

Here, (4x7 – 2x3)/4 = (4x7 – 2x3)/(7 – 3) = ΔT / ΔB is a change per-number

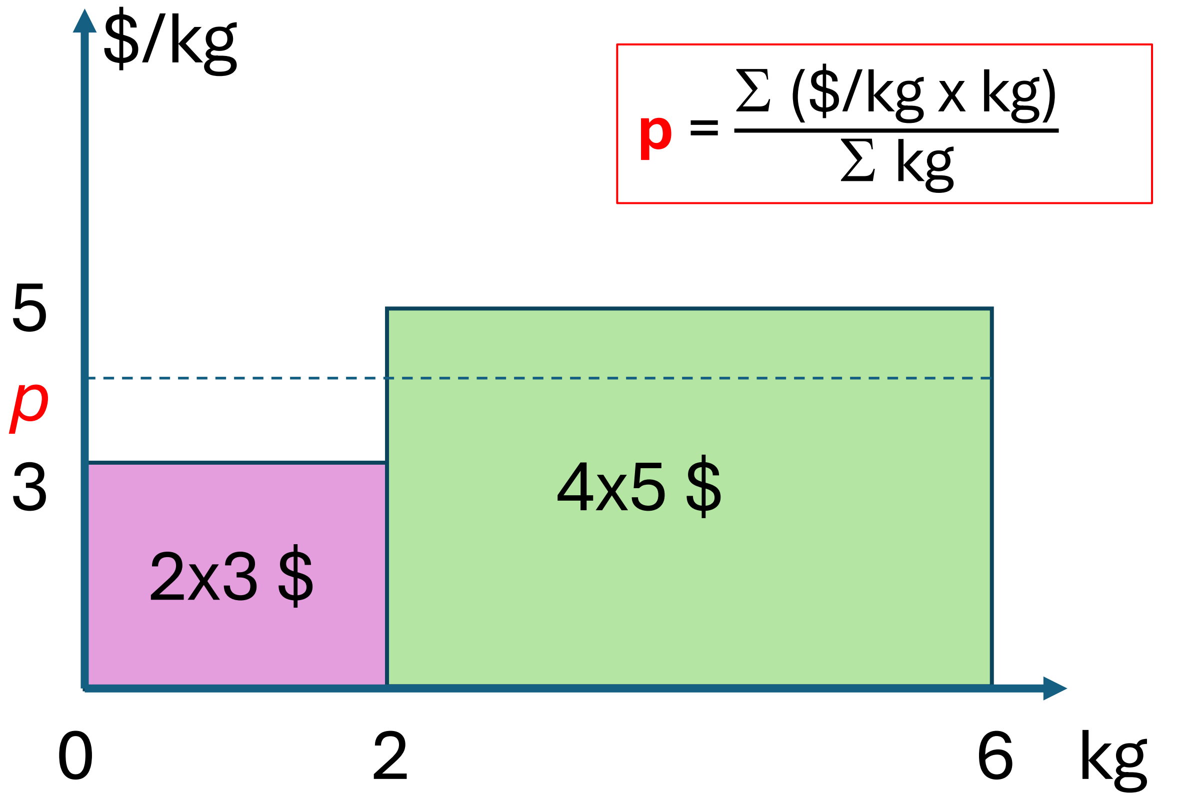
Per-numbers add as Areas (Integral Calculus)

Here, the per-number *p* is **piecewise constant**, which gives the sum Σ (*p**Δ*x*) that becomes ∫*p**dx, if *p* is **locally constant** (by interchanging epsilon & delta)

Question: "2kg at **3$/kg** + 4kg at **5$/kg** = 6kg at **?** **$/kg**?"

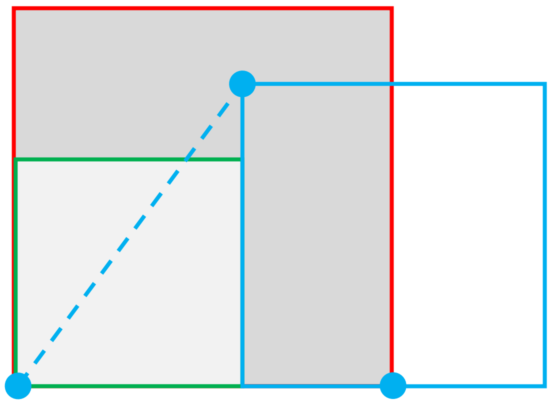
2 kg	at	**3 $/kg**
+ 4 kg	at	**5 $/kg**
(2+4) kg	at	***p*** **$/kg**

- Unit-numbers add directly.
- Per-numbers must be multiplied to unit-numbers, thus adding as **areas** under the per-number curve.
- Here, multiplication before addition
- So, per-numbers and fractions are not numbers, but operators needing numbers to be numbers.

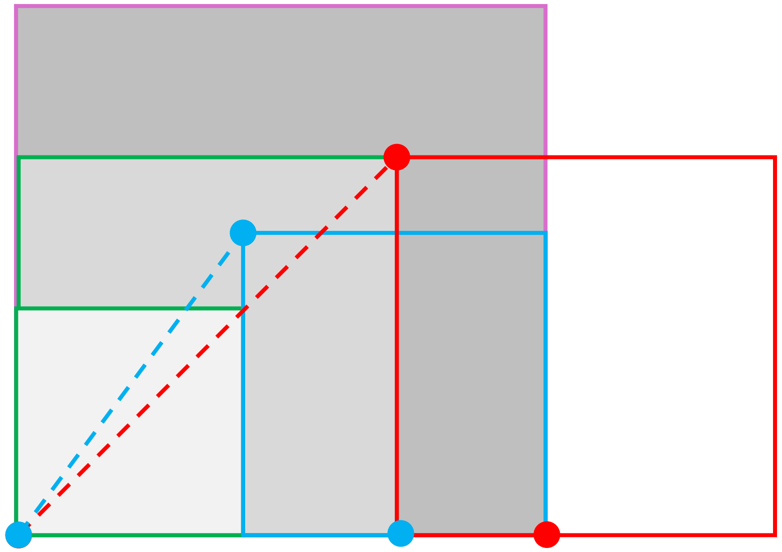


Math through NUMERACY showing how to UNDERSTAND and USE children's innate BundleNumbers with Units to Count & Add in Time & Space, CATS

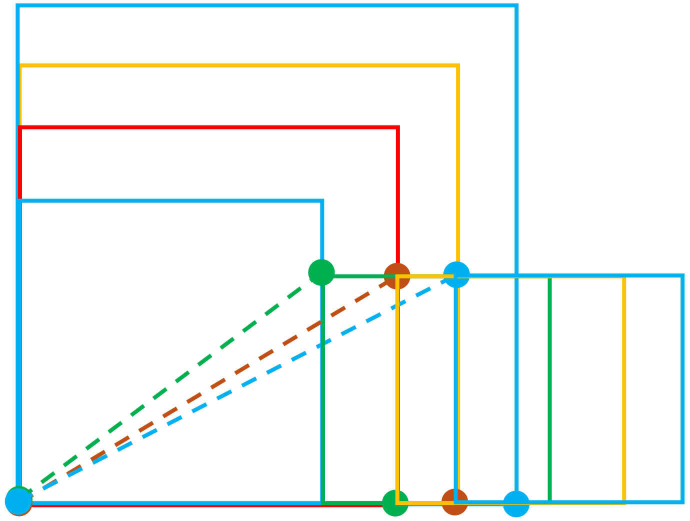
Squares add as Squares via Bottom-Top lines



Here a **3-square** adds a **4-square** and becomes a **5-square**



Here a **3-square** adds a **4-square** and adds a **5-square** and becomes a **7-square**

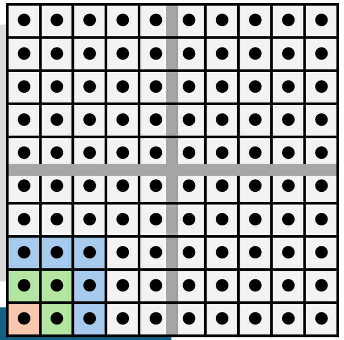


Here a **4-square** adds a **3-square** and adds a **3-square** and adds a **3-square** and becomes a **6.6-square**

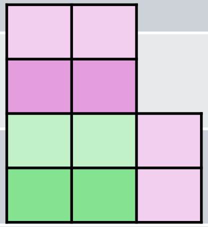
Math through NUMERACY showing how to UNDERSTAND and USE children's innate BundleNumbers with Units to Count & Add in Time & Space, CATS

Existence before Essence makes Children

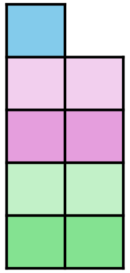
BundleCount in BundleNumbers with Units on a BundleBundleBoard



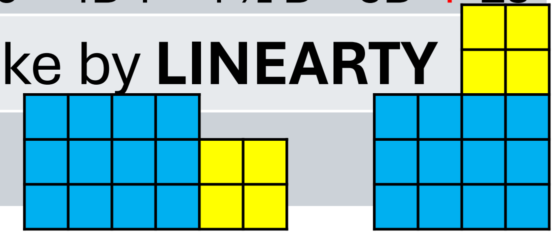
This Reverses	the Operation order
POWER	**Bundles Bundles**
LOG	Counts the number of **Bundlings**
ROOT	Finds the side in a **BundleBundle**
DIVISION	PUSH-away **Bundles**
MULTIPLICATION	PUSH-back **Bundles** to stack
SUBTRACTION	PULL-away **Bundles** to find the **unBundled** to place on-top
ADDITION	PULL-back **Bundles** to unite
ON-TOP	T = (T/**B**)***B** reCounting makes the units like by **LINEARTY**
NEXT-TO	as areas rooting **CALCULUS**



2 **2s** = 2^2 = 1**BB = 1B^2**
4 **2s** = 1**BBB = 1B^3**
log2(8) = 3, log3(9) = 2
8 = 1**BB**?, √8 < 3 (=2.8)



9 = 4**B** 1 = 4 ½ **B** = 5**B** -1 **2s**



The **Algebra Square** ReUnites World's 4 Number-types: Unlike & Like, Unit- & Per-numbers

The Arabic word Algebra means to reunite, to unite and split. Numbers are united in four ways: **Addition** unites unlike unit-numbers. **Multiplication** unites like unit-numbers. **Integration** unites unlike per-numbers. **Power** unites like per-number.

The opposite is to split into. **Subtraction** splits into unlike unit-numbers. **Division** splits into like unit-numbers. **Differentiation** splits into unlike per-numbers. Finally, the **factor-finding root** and **factor-counting logarithm** splits into like per-numbers.

Unite ***Split into***	**Unlike**	**Like**
Unit-numbers (meter, second)	**T = a + b** *T – b = a*	**T = a*b** *T/b = a*
Per-numbers (m/sec, m/100m = %)	**T = ∫ f dx** *dT/dx = f*	**T = a^b** $\sqrt[b]{T} = a$ $\log_a(T) = b$

Modeling has 3 genres, Fact & Fiction & Fake And, not 8, only 2 Competences: Count & Add

- Modeling real world problems is difficult for **essence-math** needing 8 competences; and failing to distinguish between the 3 genres, fact & fiction & fake ('Since-then/If-then/What-then, or 'room/rate/risk' models). All models are said to be approximations.
- By using formulas from the start, **existence-math** avoids modeling problems with its 2 competences, Count & Add, as it sees itself as a number-language parallel to the word-language, both of which have a meta-language (a grammar) and 3 genres.

Fake models are, e.g., mathematism adding numbers without units, as well as averages of numbers that could never be equal.

	WORD language	NUMBER language
Meta-language	*This is a sentence*	*This is a formula*
Inner language	This is a stone	T = 3 4s
Outer world		IIII IIII IIII

Fact & fiction & fake, the 3 genres of both the word-language and the number-language

Once we know how to count and recount totals, and how to unite and split the four number-types, we can now actively use this number-language to produce tales about numbering and numbers; and about totaling and totals in space and time. This is called modeling.

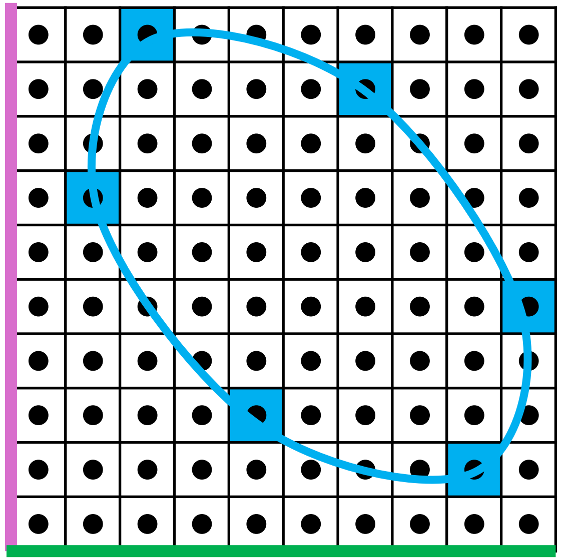
As in the word-language, number-language tales also come in three genres: **fact**, **fiction**, and **fake** models that are also called since-then, if-then, and what-then models, or room, rate, and risk models.

Fact models talk about the past and present, and they only need to have the units checked.

Fiction models talk about the future, and they need to be supplemented with alternative models built upon alternative assumptions.

Fake models typically add without units, e.g., when claiming that '2+3 = 5' always despite 2weeks + 3days = 17days, thus transforming mathematics to 'mathematism'.

STeN: Cats eat Mice, if any

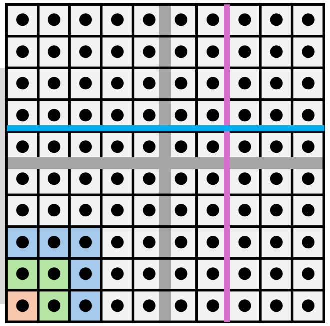


A cats and mice cohabitation on an island is an example of a predator-prey model where cats eat mice. We expect a cycle in time since many cats and many mice leads to many cats and few mice, which leads to few cats and few mice, which leads to few cats and many mice, which leads to many cats and many mice once again.

In a model we assume that a mice-population at 7 and 2 will make the cat-population change with 7-5 and 2-5 respectively. Likewise, a cat-population at 7 and 2 will make the mice-population change with 5-7 and 5-2 respectively. We see that initial populations at the level 5 will give a stable model. Here we assume that the initial populations for the cats and the mice are 8 and 1 respectively. The following period the two populations will then be 8 + (1-5) = 4, and 1 + (5-4) = 2 respectively.

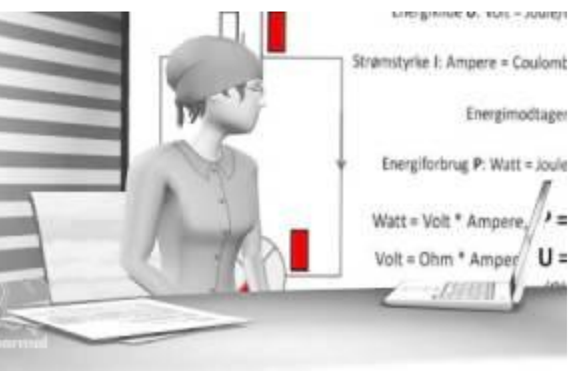
Continuing, we see that the cat population will change as 8, 4, 1, 2, 6, 9, 8; and that the mice population will change as 1, 2, 6, 9, 8, 4, 1. This allows the points (8,1), (4,2), etc., to be marked on a BBBoard, showing a cycle continuing again and again. Different initial numbers will give different cycles.

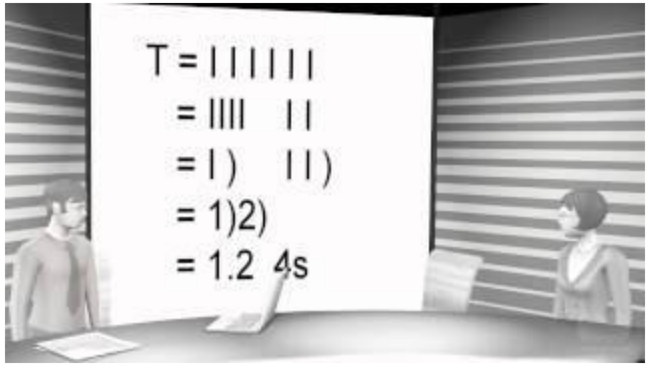
Numeracy as 'Math with Units where Additon Folds but Multiplication holds': Existence replaces Essence

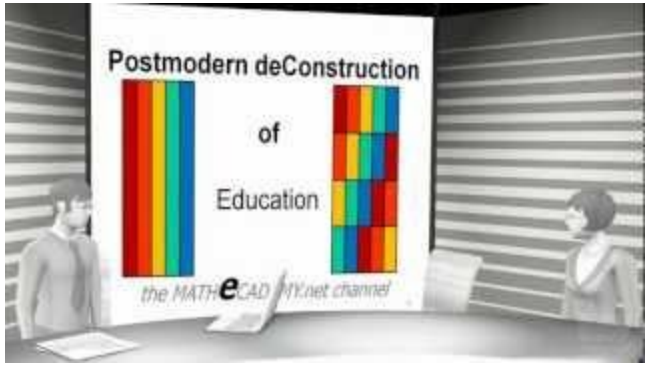


	Numeracy, EXISTENCE	Math, ESSENCE
Digits	Icons	Symbols
345	T = 3**BB** 4**B** 5, **BB** = **B**^2, **BBB** = **B**^3	The place value system tells it
Operations	Icons, order ^ / x - +	Functions, order + - x / ^
3 + 4	Meaningless without units	3 + 4 = 7
3 * 4	3 * 4 = 3 **4s**	3 * 4 = 12
9 = ? **2s**	9 = 3**B** 3 = 5**B** -2 = 4**B** 1 = 4½ **2s**	Meaningless, only ten counting
8 = ? **2s**	8 = (8/2) * 2, T = (T/**B**) * **B**, proportionality	Meaningless, only ten counting
2 * *u* = 8	2**u* = 8 = (8/2) * 2, so *u* = 8/2	(2**u*)*½ = 8*½, so (*u**2)*½ = 4, so *u**(2*½) = 4, so *u**1 = 4, so *u* = 4
6 * 7 = ?	(**B**-4)*(**B**-3) = (10-4-3)***B**+12 = 3**B** 12 = 4**B** 2 =42	eh 44, eh 52, eh 42? OK
4kg = 5$, 6kg=?	6kg = (6/4) * 4kg = (6/4) * 5$	1kg = 5/4$, 6kg = 5/4*6$
1/2 + 2/3 = ?	½ * 2 + 2/3 * 3 = 3/5 * 5	1/2 + 2/3 = 3/6 + 4/6 = 7/6
2*3 + 4*5	2*3 + 4*5 = 2 **3s** + 4 **5s** = 3B2 **8s**, by integration	2*3 + 4*5 = 10*5 ?? sorry, 6+20 =26
7 + 9 = ?	1**B** -3 + 1**B** -1 = 2**B** -4 = 1**B** 6 = ½**B** 2 + ½**B** 4 = 16	7 + 9 = 16
Tangent = ?	rise = (rise/run)*run, tan(Angle) = rise/run	Tan = sine/cosine

More MrALTarp YouTube videos







Many before Math! Math DeColonized by the Child's own 2D BundleBundle Numbers
Online math opens for a communicative turn in number language education.
AI and Difference Research in Math Education
Continuous means locally constant
From STEAM to STEEM part II

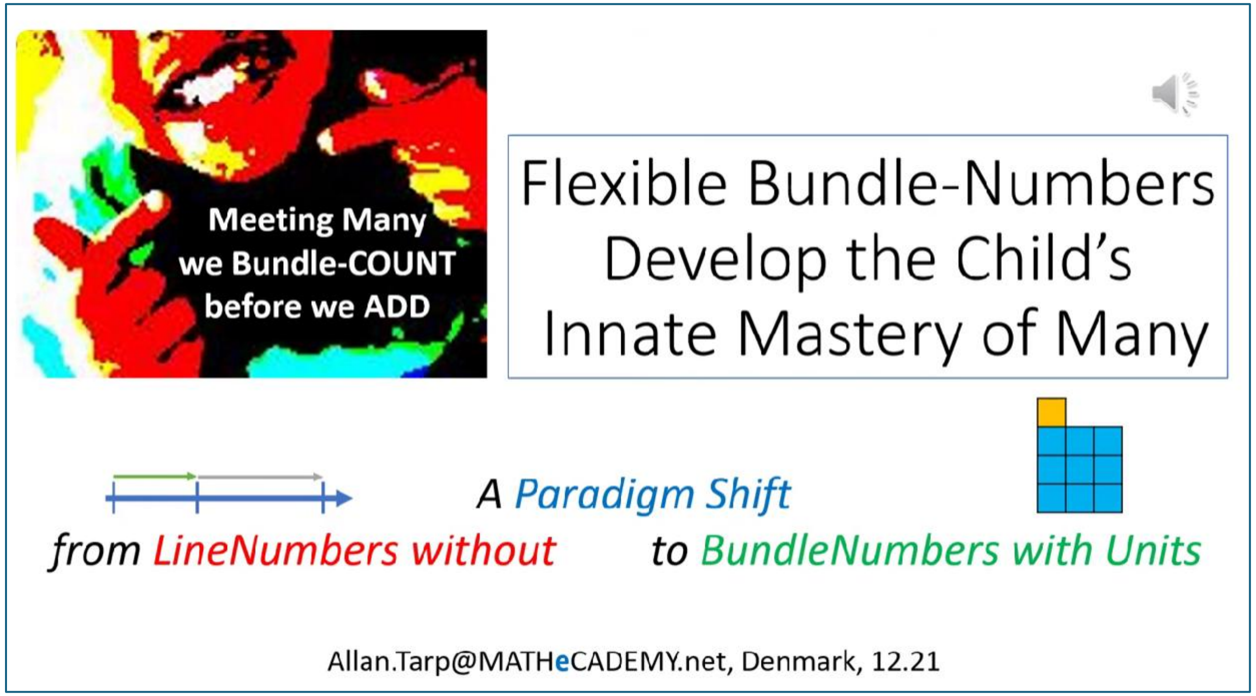
Flexible Bundle Numbers Develop the Childs Innate Mastery of Many
Children's innate Mastery of Many developed by flexible bundle-numbers
To master Many Recount before Adding
Bring Back Brains from Special Education in Mathematics
From STEAM to STEEM

Trigonometry Before Geometry Probably Makes Every Other Boy an Excited Engineer
Introducing the MATHeCADEMY dot net
Mathematics language or grammar
The two infection formulas, part 1
The two infection formulas, part 2

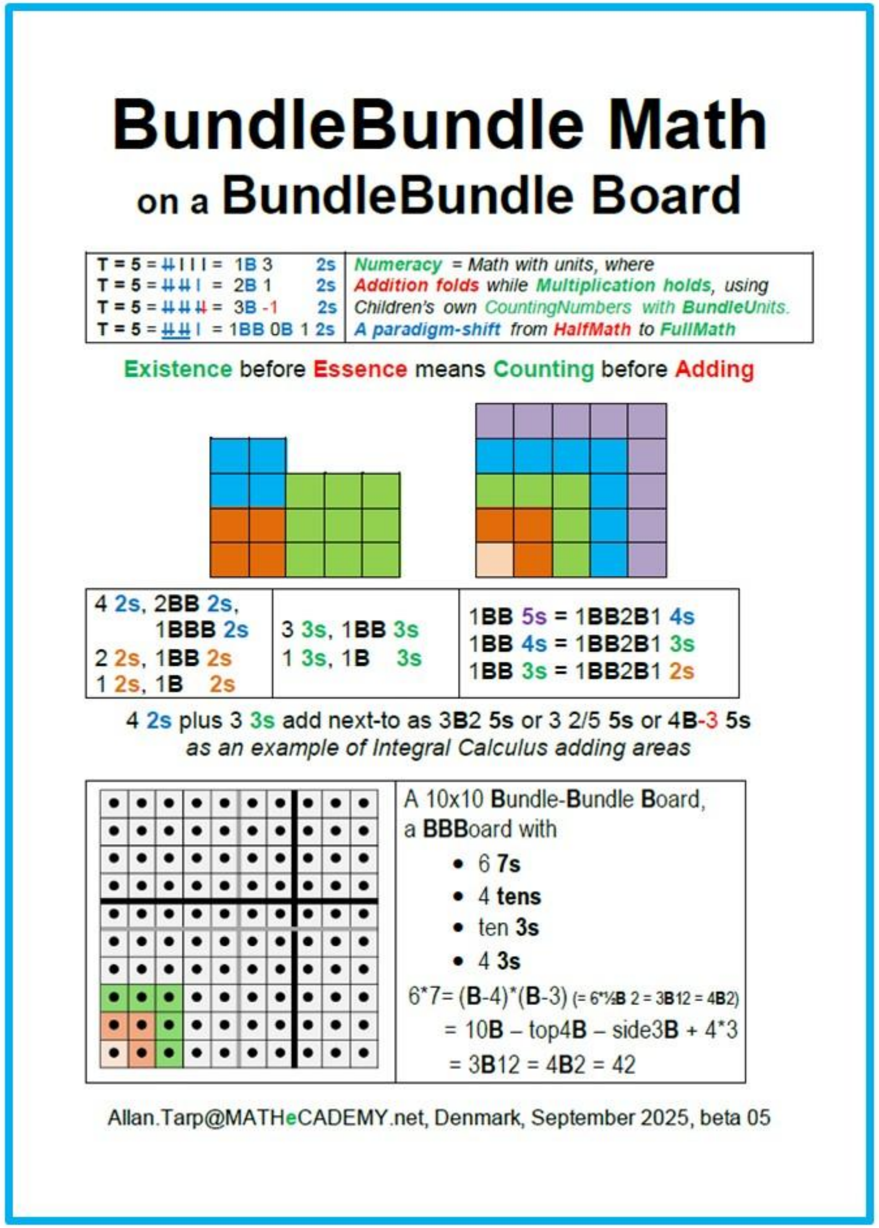
CupCount and ReCount before you Add
Preschoolers learn Linearity & Integration by Icon-Counting & NextTo-Addition
Deconstructing Calculus
Deconstructing PreSchool Mathematics
Deconstructing PreCalculus Mathematics

Deconstructing Fractions
A Postmodern Deconstruction of World History
8 Missing Links of Mandarin Math I
8 Missing Links of Mandarin Math II
A Postmodern Mathematics Education

What happened next is seen in this workshop↓ & this textbook→



Workshop: Flexible Bundle Numbers
Develop the Childs Innate Mastery of Many
https://youtu.be/z_FM3Mm5RmE



http://mathecademy.net/bundlebundlemath-on-a-bbboard/

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