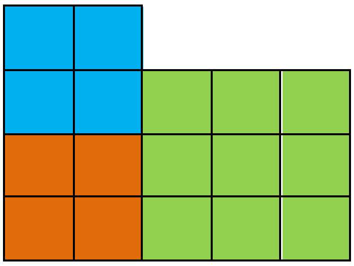
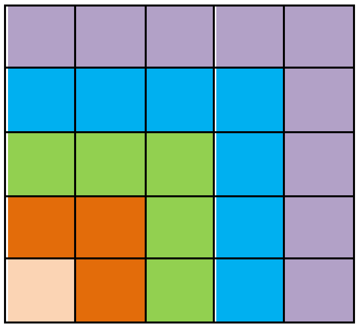
BundleBundle Math
on a BundleBundle Board

T = 5 = ll l l l = 1**B** 3 **2s** **T = 5** = ll ll l = 2**B** 1 **2s** **T = 5** = ll ll ll = 3**B** -1 **2s** **T = 5** = ll ll l = 1**BB** 0**B** 1 **2s**	***Numeracy*** = *Math with units, where* ***Addition folds*** *while* ***Multiplication holds****, using* *Children's own CountingNumbers with* ***BundleU****nits.* ***A paradigm-shift*** *from* ***HalfMath*** *to* ***FullMath***

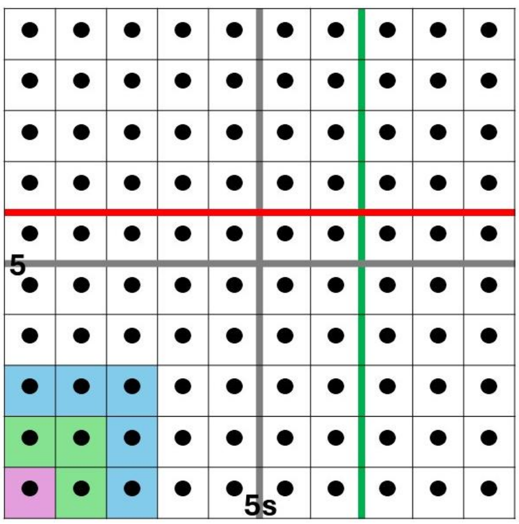
Existence before **Essence** means **Counting** before **Adding**





4 **2s**, 2**BB** **2s** 1**BBB** **2s** 2 **2s**, 1**BB** **2s** 1 **2s**, 1**B** **2s**	3 **3s**, 1**BB** **3s** 1 **3s**, 1**B** **3s**	1**BB** **5s** = 1**BB**2**B**1 **4s** 1**BB** **4s** = 1**BB**2**B**1 **3s** 1**BB** **3s** = 1**BB**2**B**1 **2s**

4 **2s** plus 3 **3s** add next-to as 3**B**2 **5s** or 3 2/5 **5s** or 4**B**-3 **5s**
as an example of Integral Calculus adding areas

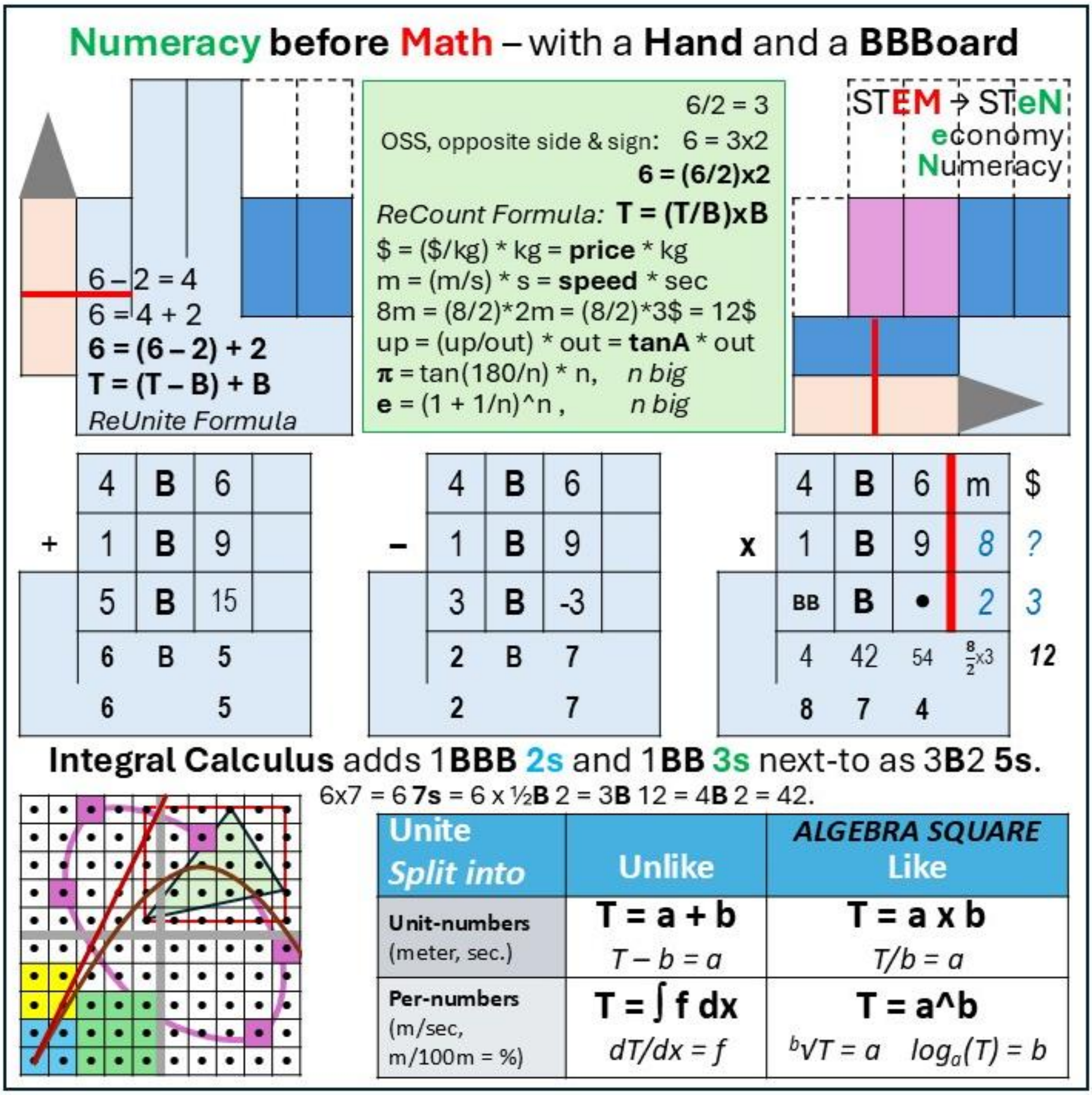


A 10x10 **B**undle-**B**undle **B**oard, a **BB**Board with

- 6 **7s**
- 4 **tens**
- ten **3s**
- 4 **3s**

6*7= (**B**-4)*(**B**-3) (= 6*½**B** 2 = 3**B**12 = 4**B**2)
= 10**B** – top4**B** – side3**B** + 4*3
= 3**B**12 = 4**B**2 = 42

Allan.Tarp@MATHeCADEMY.net, Denmark, January 2026, beta 07

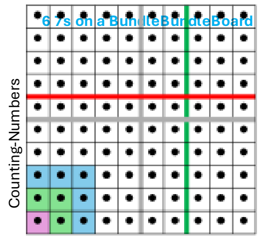


Backside of a BundleBundleBoard

STeN solves Problems on a BundleBundleBoard to make all Youth Numerate by 2030

NUMERACY

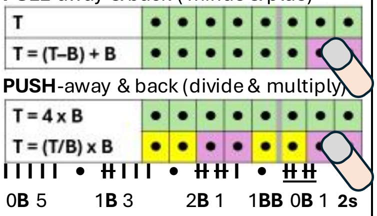
BBM • BundleBundleMath with units *(Kids: II II "Not 4, but 2 2s")*



Bundle-Numbers

PULL-away & back (minus & plus)

PUSH-away & back (divide & multiply)



Ten = 2BB 0B 2 = 1BBB 0BB 1B 0 **2s**

• **ReUnite formula:** **T = (T–B) + B**

***u* + 3 = 5.** But, 5 = (5-3) + 3, so *u* = 5-3 = 2

***u* - 3 = 5.** But, *u* = (*u*-3) + 3 = 5 + 3 = 8

5 - *u* = 3. But, 5 = (5-*u*) + *u* = 3 + *u*, so, *u* + 3 = 5

• **ReCount formula:** (changes units in **STEM**) **T = (T/B) x B**

***u* * 3 = 12.** But 12 = (12/3) * 3, so *u* = 12/3 = 4

***u*/3 = 5.** But *u* = (*u*/3) * 3 = 5 * 3 = 15

12/*u* = 3. But 12 = (12/*u*) * *u* = 3 * *u*, so *u**3 = 12

Next **B**undle**B**undle: 1**BB** 2**B** 1

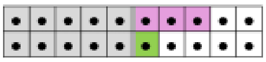
Before **BB-square:** 1**BB** -2**B** 1

7 = 0**B** 7 = 1**B** -3 = ½**B** 2 **tens**

Squares:	01	04	09	16	25
	81	64	49	36	

ΔB^2 = 2**B**+1 Later, (*x*^2)' = 2**x*

Add & Subtract etc.



8 + 6 = (½**B** 3) + (½**B** 1) = 1**B** 4 = 14

28+36 = 2**B** 8+3**B** 6 = 5**B** 14 = 6**B** 4 = 7**B** -6

8 x 46 = 8 x 4**B** 6 = 32**B** 48 = 36**B** 8 = 368

368 / 8 = 36**B** 8 / 8 = 32**B** 48 / 8 = 4**B** 6 = 46

T = 8 – 6 = (½**B** 3) – (½**B** 1) = 0**B** (3 – 1) = 0**B** 2 = 2

T = (1**B** -2) – (1**B** -4) = 0**B** (-2 + 4) = 0**B** 2 = 2

(notice - - is +)

Place value & carry & borrow unneeded

Video: Many before Math

T = 6*7 = 6*(½**B** 2) = 3**B** 12 = 42 (Overload)

T = 6 * 7 = (**B** - 4) *(**B** - 3) — **BB**M FOIL Down & Cross

= **BB** - 3**B** - 4**B** - - 12 (notice - - is +)

= (10 - 3 - 4)**B** + 12 = 3**B** 12 = 4**B** 2 = 42

6 * 7 = (**B**-4) * (**B**-3) = (½**B** 1) * (½**B** 2)

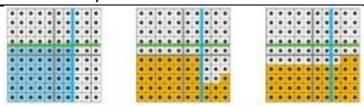
With fingers :

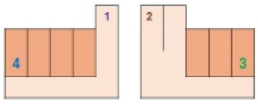
T = (1+2+5) * ½**B** + (1*2) = 8 * ½**B** 2 = 4**B** 2

T = (1+2)**B** (4*3) = 3**B** 12 = 4**B** 2 = 42

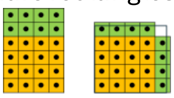
T = 6 * 7 = 6 * (6+1) = 6^2+ 6 = 36+6 = 42

ReCount 6 **7s** into 5**B** -8 **tens** and 4**B** 2 **tens**





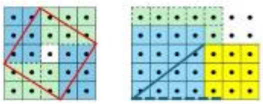
Square rectangles



To square the total T = 6 **4s**, half of the (6-4) **4s** move from the top to the side to get a 5 x 5 square, and an unfilled square in the upper corner.

This we fill with a 4**u*** slice of the top and the side. Here ***u*** is found by the equation 2**u**4 = 1, or 8**u* = 1, or ***u*** = 1/8 = 0.125, So, 5-0.125 = 4.88. Calculator: √24=4.90

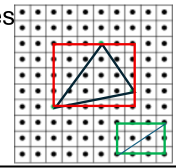
Add Squares



On a **BBB**oard four 2 **3s** arranged as a 5-by-5 square contains 2 squares (3 **3s** and 2 **2s**) as well as 2 stacks. But it also contains one square formed by the stacks' diagonals as well as four half stacks.

So, a 4- and a 3-square add as the square formed by the mutual **B**ottom-**T**op **BT** line thus having the length as the square-root of the sum, i.e., √(4^2 + 3^2) = 5. Pythagoras or Gougu rule.

Triangles



A triangle has the points **A**(3,4) and **B**(6,8) and **C**(8,5) packed inside at 4 **5s** stack. We find its angles and sides. **Ar** is **A**'s right angle.

• π = *n**tan(180/*n*) for *n* large

In a stack with a diagonal, up = (up/out)*out = TanAngle*out.

In a 2 **3s** stack, Tan**A** = 2/3. A calculator shows **A** = 33.7

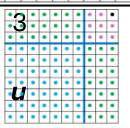
Tan**Ar** = 1/5 gives **Ar**= 11.3 and **AC** = √(1+25) = 5.1

Tan**Br** = 4/3 gives **Br**= 53.1 and **AB** = √(16+9) = 5.0

Tan**Cr** = 2/3 gives **Cr** = 33.7 and **BC** = √(4+9) = 3.6

A = 90 - 11.3 - (90 - 53.1) = 41.8 & **B** = 70.6 & **C** = 67.6

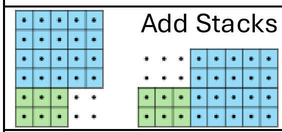
Solve quadratic equations



On a **BBB**oard, (***u***+3)*(***u***+3) is a square with four parts, two squares (***u***^2 and 3^2), and two stacks, 2*3****u***, so that T = ***u***^2 + 6****u*** + 9.

The quadratic equation ***u***^2 + 6****u*** + 8 = 0 then makes the whole square go away except for 9-8 = 1. So (***u***+3)^2 = 1. This gives two solutions, ***u*** = -2 and ***u*** = -4.

Add Stacks



OnTop: **Proportionality** makes units like

2 **3s** + 4 **5s** = 1**B** 1 **5s** + 4 **5s** = 5**B** 1 **5s**

2 **3s** + 4 **5s** = 2 **3s** + 6**B** 2 **3s** = 8**B** 2 **3s**

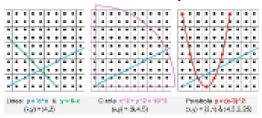
NexTo: **Calculus** adds or splits areas

2 **3s** + 4 **5s** = 4**B** -6 **8s** = 3**B** 2 **8s**

2 3**s** + ? **5s** = 4 **8s**

? = (4 **8s**–2 **3s**)/5 = 5**B** 1 **5s** = (T2-T1)/5 = ΔT/5

A BBBoard as a trip-board



On a 2/4 trip from the (0,0)-dot to the (*x*,*y*)-dot we have that *y*/*x* = 2/4, or *y* = 2/4**x* = ½**x*. Another line has *y* = 6-*x*. Where the two lines meet, we have *y* = 1/2**x* = 6-*x*. This gives *x* = 12-2**x*, or 3*x* = 12, or *x* = 4. Here, *y* = 6 – 4 = 2. So, they meet in point (4,2).

On a circle with radius 10 and center in the (0,0)-dot, *x*^2 + *y*^2 = 10^2. On its way the *y* = 1/2**x* line meets the circle. Here *y* = ½*x* makes *x*^2 + *y*^2 = *x*^2 + (½*x*)^2 = 100, or *x*^2 + 1/4*x*^2 = 100. This gives *x* = 2*4.5 = 9. Here *y* = 2/4*9 = 4.5. So, they meet in point (9,4.5).

A trip where *y* = (*x*-3)^2 is a bent line called a parabola. It meets the *y* = 1/2**x* line in point (*x*,*y*). Here *y* = ½*x* makes ½**x* = *x*^2 – 6**x* + 9, or *x*^2 – 6.5**x* + 9 = 0. There are two solutions, *x* = 2 and *x* = 4.5. This gives *y* = 1 and *y* = 5.25. So, they meet in points (2,1) and (4.5, 5.25).

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Poster at the STEMPD 2025 Conf. in Istanbul: Numeracy as BBM, BundleBundle-Math with units.

CHILDREN'S OWN BUNDLE-NUMBERS WITH UNITS
may Reach the UN Development Goal: All are Numerate by 2030

Allan Tarp, MATHeCADEMY.net, Allan.Tarp@gmail

A01. Understanding & using NUMbers, Children are NUMERATE before school using BBM, BundleBundleMath

II II "No, that is not 4, that is 2 twos". Said a 3year old child when asked "How many years next time?" As educated, we only see the **essence**. As uneducated, the child sees the **existence**, bundles of **twos** in **space**, and 2 of them when counted in **time**.
The number 'two' exists both in **space** and in **time**. In **space**, 2 exists as **twos**, a **space-number**, a **B**undle-number of **twos**, a two-**B**undle.
In **time**, 2 exists together with the unit that was counted, as 2 units, **a time-number**, or a counting-number.
V II: Bundle-numbers falsify '1+1 = 2' with 2 V-signs showing that 1 **1s** + 1 **1s** = 1 **2s** and 2 **1s** + 1 **2s** = 1 **4s**, and not 3 **3s** as expected if **1+1 = 2**.

A02. Counting fingers in space

Space-count five fingers in **2s**, **3s** and **4s** using 'flexible **B**undle-numbers'. 5 = 1**B**3 = 2**B**1 = 3**B**-1 = 1**BB** 0**B** 1 **2s**.
Ten = 2**BB** 0**B** 2 = 1**BBB** 0**BB** 1**B** 0 **2s**. T = 38 = 3**B** 8 = 2**B** 18 = 4**B** -2. **Demodel**: 35+46 = 3**B** 5 +4**B** 6 = 7**B** 11 = 8**B** 1 = 81
6*28 = 6*2**B** 8 = 12**B** 48 = 16**B** 8 = 168. And T = 4567 = 4**BBB** 5**BB** 6**B** 7, T = 4***B**^3 + 5***B**^2 + 6***B** + 7*1. **No carry or place values**.

A03. Add and subtract 1digit numbers counted in half-bundles

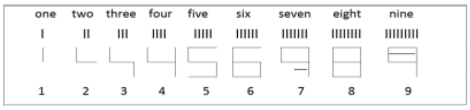
T = 6 + 7 = ½**B** 1 + ½**B** 2 = 1**B** 3 = 13. T = 4 + 7 = ½**B** -1 + ½**B** 2 = 1**B** 1 = 11. T = 3 + 4 = ½**B** -2 + ½**B** -1 = 1**B** -3 = 7
T = 8 - 6 = ½**B** 3 - ½**B** 1 = 3 - 1 = 2. T = 6 - 4 = ½**B**1 - ½**B** -1 = 1 - - 1 = 2 (- - = +). T = 6 - 8 = ½**B** 1 - ½**B** 3 = 1 - 3 = - 2

A04. Time-counting fingers

Time-count fingers in ½**B**, "1,2,3,4,5,6" no, "0**B** 1, 0**B** 2, 0**B** 3, 0**B** 4, 0**B** 5, or ½**B** 0, ½**B** 1". Time-count from 88 to 102: "8**B** 8, 8**B** 9, 8**B** ten or 9**B** 0, ..., 9**B** 9, 9**B** ten or ten**B** 0 or ten-ten 0ten 0 or 1**BB** 0**B** 0, 1**BB** 0**B** 1, 1**BB** 0**B** 2".

A05. Digits are icons

Four sticks in the 4-icon: I I I I -> IIII -> 4 Etc.



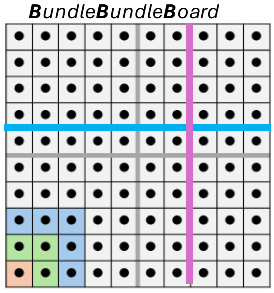
A06. Operations are icons, ReCount & ReUnite formulas

Push-away & -back to stack, (division-broom & multiplication-lift), 6 = 3x2 = (6/2) x 2, **T = (T/B) x B** (**ReCount formula**)
Pull-away & -back (minus-rope and plus up-or-on cross), 5 = (5-2) + 2, **T = (T-B) + B** (**ReUnite formula**)
On-top of a stack, the unbundled become decimals, fractions or negatives. 7 = 3**B** 1 = 3½**B** = 4**B** -1 **2s**.

A07. Recounting between from icon and tens

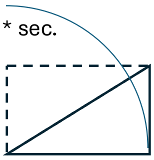
How many **5s** in 40. ReCount 40 in **5s**: *u**5 = 40 = (40/5)*5, so *u* = 40/5, "To Opposite Side with Opposite Sign".
6 **7s** = ?tens. 6*7 = 6*½**B** 2 = 3**B** 12 = 4**B** 2 = 42. Or, 6*7 = (**B**-4)*(**B**-3) = From **BB**, pull-away 4**B** & 3**B** & pull-back the 4*3 pulled-away twice = 3**B** 12 = 4**B** 2 = 42. So (**B**-4)*(**B**-3) = **BB** - 4**B** - 3**B** + 4*3, and - * - is +.

***B**undle**B**undle**B**oard*



A08. The recount-formula gives per-numbers as 2$/5kg

20kg = (20/5) * 5kg = (20/5) * 2$ = 8$. Meter = (meter/sec) * sec = speed * sec.
Fractions with like units: 2$/5$ = 2/5. Trigonometry in a stack:
up = (up/out) * out = tan(Angle) * out. π = *n* * tan(180/*n*) for *n* large.

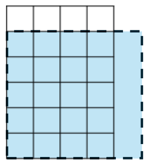


A09. Bundle-bundles are squares

2 **2s** = 1**BB** 0**B** 0 **2s**. 3 **3s** = 1**BB** 0**B** 0 **3s**, etc.
T = 1**BB** 2**B** 1 = next **BB**, T = 1**BB** -2**B** 1 = before **BB**.

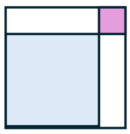
A10. Squaring stacks

T = 6 **4s** = 1**BB** what? (where **B**=√(6*4)). Guess 1: '(6-1) **(4+1)s**' or '5 **5s**', since ½(6-4) = 1. The empty 1-corner needs two 't **4s**' stacks, and t*4 = ½ gives t = 1/8. Guess 2: '4.9 **4.9s**'. **B**ingo, so √(6*4) = √24 = 4.9



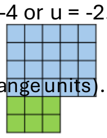
A11. Solving quadratics

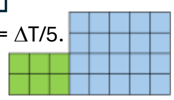
A (*u*+3) square has two squares and two stacks: (*u*+3)^2 = *u*^2 + 3^2 + 2*3**u* = *u*^2 + 6**u* + 9.
If *u*^2 + 6**u* + 8 = 0, all disappears but 1, so, (*u*+3)^2 = 1, so *u* = -4 or u = -2.



A12. Adding next-to and on-top, and reversed

2 **3s** + 4 **5s** = ? **8s or 5s**. (Integral calculus add areas, recounting change units). 2 **3s** + ? **5s** = 3 **8s**, ? =(T2-T1)/5 = ΔT/5.





A13. Adding per-numbers and fractions, and reversed

2kg at 3$/kg + 4kg at 5$/kg = (2+4)kg at (3*2 + 5*4)/(2+4) $/kg. Integral calculus adds (locally constant) per-numbers.

A14. The Algebra Square

ReUnites Unlike and Like Unit- & Per-numbers

Unite/ *Split Totals in*	**Unlike**	**Like**
Unit-numbers m, s, kg, $	**T = a + n** *T – n = a*	**T = a * n** *T/n = a*
Per-numbers m/s, $/100$ = %	**T = ∫ f dx** *dT/dx = f*	**T = a^n** *ⁿ√T = a* *log_a(T) = n*

A15. Fact, Fiction & Fake Models

MrAlTarp Videos:

Fact models are 'since-then' stories that quantify & predict predictable quantities by using factual numbers and formulas. Typically, they model the past and the present. They need to be checked for correctness and units.

Fiction models are 'if-then' stories that quantify and predict unpredictable quantities by using assumed numbers and formulas. Typically, they model the future. They need to be supplied with scenarios, built on alternative assumptions.

Fake models are 'what-then' stories that quantify and predict unpredictable qualities, using fake numbers & formulas. Typically, they add without units, or they hide alternatives. Here, number-stories need to be replaced by word-stories.

Poster at the EARCOME 9 2025 conf in Korea: Children's Own Bundle-Numbers with Units may Reach the UN Sustainable Development Goal: All Youth are Numerate by 2030.

A Reaction to a BBM BundleBundle Math Curriculum

At the 9th ICMI-East Asia Regional Conference on Mathematics Education (EARCOME 9) in 2025 in Korea, the Special Sharing Groups accepted my paper "Can a Decolonized Mathematics Secure Numeracy for All?" with the following announcement:

> This proposal tackles an urgently needed conversation in mathematics education by challenging deeply ingrained assumptions about number systems and arithmetic instruction and proposing a truly decolonized approach that foregrounds learners' intuitive "bundle-number" language.
>
> Its strength lies in weaving together a compelling theoretical critique - drawing on Habermas's colonization concept and rich philosophical underpinnings - with concrete instructional innovations like the Algebra Square that reframe operations as intuitive spatial and bundling processes.
>
> By aligning this reconceptualization directly with SDG 4's numeracy targets and illustrating how multiplication-centered reasoning better reflects real-world number use, the paper promises to make a bold and impactful contribution to both research and practice.
>
> We look forward to seeing how this work can reshape numeracy instruction and foster truly inclusive mathematical literacy for all.

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Introduction

Looking at four fingers bundled in twos, an educated person only sees four fingers, the ESSENCE. But, as uneducated before school, children see the EXISTENCE, bundles of twos in space, and two of them when counted in time. This textbook in Numeracy uses the child's own counting-numbers with bundle-units as 2 3s and 4B1 5s visible and tangible on a ten-by-ten BundleBundleBoard. We may now ask how mathematics may be taught to children if using their own two-dimensional numbers with bundle-units instead of the school's one-dimensional line-numbers without units. In other words, we may ask how children may learn mathematics by meeting existence before essence to use the two core concepts of Existentialism holding that existence must precede essence.

Counting outside Totals in Bundles then must precede adding them inside. In this 'Bundle-math' approach, mathematical concepts are re-rooted in outside existing examples instead of being defined as examples itself inside. Now tens, hundreds and thousands become bundles, bundle-bundles, and bundle-bundle-bundles, as does 2, 4 and 8 when counting in twos instead.

Bundle-counting, one-dimensional lines on a ruler are replaced by two-dimensional rectangles on a BBBoard, containing the outside existing subjects that is linked to inside essence predicates in a number-language sentence as in a word-language sentence. Here units are always included in counting sequences as 0Bundle1, 0B2, …, 1B0. Here digits become icons with as many sticks as they represent. Here also operations become icons created in the counting process. An outside Total may be split by pulling away a Bundle, using a rope as an icon for subtraction. Pulled back, the total is restored in a Reunite-formula, T = (T – B) + B, using a double-rope as an icon for pulling back. Or, the outside Total may be counted in Bundles by pushing away Bundles, using a broom as an icon for division. Pushed back, the total is restored in a Recount-formula, T = (T/B) x B, using a lift as an icon for pushing back. The Recount formula is used all over STEM to change units.

Recounting from tens to icons, the equation, u*2 = 8, together with another basic equation, u+2 = 8, are solved by recounting and reuniting, both moving numbers to 'opposite side with opposite sign':

u*2 = 8 = (8/2)x2, so u = 8/2, and, u + 2 = 8 = (8 – 2) + 2, so u = 8 – 2.

Recounting from icons to tens leads to early algebra when including the Bundle-unit. Then 6*7 becomes (B-4) * (B-3) placed on a BBBoard and found by pulling-away the top 4B and side 3B, and adding the 4*3 pulled away twice. Or by writing 6*7 = 6* ½B2 = 3B12 = 4B2. And, bundle-bundles squares allow rectangular stacks to be recounted in squares with the square root as the side.

Recounting in physical units creates per-numbers as 4$/5kg bridging the units by recounting: 20kg = (20/5)*5kg = (20/5)*4$ = 16$. With like units, per-numbers become fractions: 4$/5$ = 4/5.

Recounting the sides in a stack halved by its diagonal leads to trigonometry before geometry.

Recounting now is followed by reuniting (called 'Algebra' in Arabic). Stacks may add on-top after recounting has made the units like, or next-to as areas, i.e., as integral calculus (or differential calculus if reversed), also used to add per-numbers and fractions that must be multiplied to unit-numbers to add. Squares add as the square formed by their mutual Bottom-Top line.

Units create an 'Algebra Square' to reunite our four number-types. Add and multiply unite unlike and like unit-numbers, where integrate and power unite unlike and like per-numbers. That are split by subtraction, division, differentiation, and the factor-finding root or the factor-counting logarithm.

Counting before adding thus gave us a number-language to tell inside number-tales about outside totals using the same three genres, fact and fiction and fake, as the word-language uses.

Can mathematics be decolonized? Well of course, since mathematics is a socially constructed essence that will always be a colonization of the natural existence it came from and reduces.

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A Future? From STEM to STeN to make all Youth Numerate by 2030

From STEM to STeN, why?

The fourth of the 17 UN Sustainable Development Goals, Quality Education, has as a goal target to “By 2030, ensure that all youth achieve literacy and numeracy”. Replacing STEM with STeN also including economics and Numeracy may reach the UN goal. STEM integrates mathematics with its roots in science, technology and engineering, all using formulas from algebra and trigonometry to predict the behavior of predictable physical quantities, and to model unpredictable quantities by scenarios. But economics is missing despite mathematics’ historical root is the marketplace with production, trade and consumption where quantities always carry units. Without units, mathematics must go and be replaced by numeracy that always uses units.

Two different Definitions of ‘Numerate’ exist

The English Oxford Dictionary defines it as being “competent in the basic principles of mathematics, esp. arithmetic”. In contrast, the American Merriam-Webster dictionary defines it as “having the ability to understand and work with numbers.” The word ‘competent’ is a predicate, a non-action word, I cannot ‘competent’ something, I can only be judged as competent by someone who is competent. In contrast, ‘work’ is an action word, a verb, since with my body and mind I can work on something and test the result to see if it works. Also, there is a difference between the words ‘mathematics’ and ‘numbers.’ Again, mathematics is a non-action word, I cannot ‘mathematics’ a thing. In contrast, ‘number’ is both a verb and a noun since I can number different degrees of Many to produce a number for later calculations.

Existentialist philosophy holds that existence must precede essence to prevent colonization. With Numbers as existence and math as essence, numeracy thus should precede math. In fact, numeracy may be defined as ‘math with units where addition folds and multiplication holds’. Which is observed when the foundation of mathematics, ‘2+1 = 3’, is falsified by an open and closed V-sign showing that 2 1s plus 1 2s unite as 1 4s and not as 3 3s as mathematics says without units.

Economics gives a Fundamental Understanding of Numbers and Calculations

The basic meanings of geometry and algebra show that they are both rooted in economics. In Greek, geometry means to measure earth, and in Arabic, algebra means to reunite numbers, so they have a common root in the basic economic question “How to divide the earth and what it produces?” Hunter-gatherers need not tell the different degrees of many apart, but farmers do since farmers produce to a market to survive, and here they need to be numerate to answer the question “How many here?”. This immediately leads to the answer “That depends on the unit.” Economics thus begin at once by reusing the number-names when using bundling to count. The Romans unsystematically gave names to the bundles 5s, 10s, 50s, 100s, 500s and 1000s. This worked well for administrative addition and subtraction jobs but not for multiplication. So, when German silver reopened the trade between India and Renaissance Italy, Hindu-Arabic numbers named only the unbundled, the bundles, the bundle of bundles (BB or B^2), the bundle-bundle-bundles (BBB or B^3), etc. With tens, 234 is not 1 but 3 numbers, 2BB 3B 4.

At a market, goods are sold in bundles with units as 2 3s, the same numbers that a 3year old child use when seeing four fingers bundled in twos: “No, that is not four, it is two twos”. But a buyer may want to have 5s, or to trade 4 per 5, or to pay 4$ per 7. So, changing units is a core job using the proportional recount-formula, T = (T/B)*B, saying “From T, push-away B’s to count them”. Thus, recounting from tens gives equations solved by moving to opposite side with opposite sign:

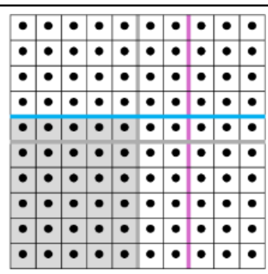
Find u, so u*2 = 8. Here, 8 recounts in 2s as 8 = (8/2)*2, so u = 8/2.

Recounting into tens leads to early algebra:

6 7s = 6*7 = 6*(½B2) = 3B 12 = 4B 2 = 42. Or,

6*7 = (B-4)*(B-3) = BB-3B-4B+4*3 = (10-3-4)B 12 = 4B 2 = 42

This is seen on a ten-by-ten BundleBundleBoard when pulling away the top and the side and adding what was pulled twice.



Likewise, when changing the units for length, weight, and currency. And, when changing from the quantity to the price. Here, Renaissance Italy used ‘regula detri’, the rule of three.

Asking “With the per-number 2$ per 3kg, what is the price for 9kg?”, first they arranged the three numbers with alternating units: ‘9kg, 2$, 3kg’. Then the answer comes from multiplying and dividing: 9*2/3 = 6$. Now we recount in the per-number: 9kg = (9/3)*3kg = (9/3)*2$ = 6$.

Trigonometry precedes geometry when recounting the height in the base in a stack as 4 5s:

height = (height/base) * base = tan(A) * base, giving π = n * tan(180/n) for n high enough.

Once counted and recounted, 2 3s & 4 5s may add next-to as 8s using calculus to add areas. Or add on-top as 5s using recounting for like units.

In a bill as ‘2kg at 3$/kg + 4kg at 5$/kg’, kg’s add directly but per-numbers add by their areas using calculus after being multiplied to $.

With economics we learn core mathematics by counting in time and space before adding.

Numeracy as Math Counting Totals in Units before Adding them with Units

In a numeracy education using the children’s bundle-numbers with units, Tarp (2018) shows that ‘counting before adding’ give the same concepts but with different identities and order.

Bundle-counting in 3s leads to 9 as a bundle-bundle, a B^2, which leads on to squares, square roots, and quadratics.

Counting transforms the operations into icons where division and multiplication become a broom and a lift that pushes-away bundles to be stacked as shown when recounting 8 in 2s as 8 = (8/2)x2, which creates per-numbers when recounting $ = ($/kg) * kg.

Subtraction is a rope that pulls-away the stack to find the unbundled that, placed on-top of the stack as part of an extra bundle, become decimals, fractions, or negatives, e.g., 9 = 4B1 = 4½ = 5B-1 2s.

Finally, addition is a cross, showing the two ways to add stacks, on-top using the linearity of recounting to make the units like, or next-to creating integral calculus by adding areas, which is also used when adding per-numbers needed to be multiplied to areas before adding.

Including units provides an ‘Algebra Square’ showing how to unite the four types of numbers:

Multiplication and addition unite like and unlike unit-numbers, and power and integration unite like and unlike per-numbers.

And how to split totals with the opposite operations: division and subtraction, together with the factor-finding root or the factor-counting logarithm and differentiation.

Table 01. The Algebra Square shows the ways to reunite unlike and like unit- and per-numbers

Uniting *splitting Totals in*	**Unlike**	**Like**
Unit-numbers m, s, kg, $	**T = a + n** *T – n = a*	**T = a * n** *T/n = a*
Per-numbers m/s, $/100$ = %	**T = ∫ f dx** *dT/dx = f*	**T = a^n** $\sqrt[n]{T} = a$ $\log_a(T) = n$

References

Tarp, A. (2018). Mastering Many by counting, re-counting and double-counting before adding on-top and next-to. *Journal of Mathematics Education 11*(1), 103–117.

Tarp, A. (2025). Math is fun with bundle-numbers on a bundle-bundle-board. In Kwon, O., Kaur, B., Pang, J., Noh, J., Lee, S., Han, S., Yeo, S., & Lim, M. (Eds.). (2025). *Proceedings of the 9th ICMI-East Asia Regional Conference on Mathematics Education (Vol. 1)*. Seoul National University, Siheung Campus, Korea: EARCOME9, 363-392.

Micro Curricula in BBM BundleBundleMath, Counting before Adding

Children use numbers with Bundle-units. So, we now design micro-curricula, MC, to develop a number-language by working with totals existing as outside things and actions on a BBBoard that may be supplied with centimeter cubes placed on-top of the BBBoard to meet the UN Sustainable Development Goal 4.6 that by 2030, all youth must be Numerate.

MC01. Digits as icons in space, IIIII = 5

The total here exists as sticks to be rearranged at a table and reported by a drawing on paper. The 'T=?' question is answered in two ways, as a collection of single ones, I I I I I, and as one bundle of ones, ~~IIII~~, that may be rearranged into an icon, 5, called a digit containing the number of sticks that it represents if written in a less sloppy way, and thus similar to the digits on a calculator. Zero is iconized by a looking glass finding nothing. Each time a folding ruler is folded to look like the icon.

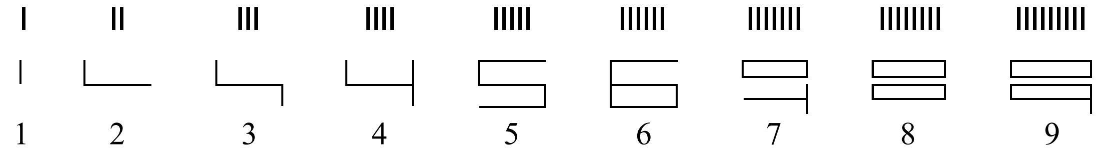


Figure 01. A digit as an icon with the number of sticks as it represents, four in the 4-icon, etc.

If we count in tens, ten sticks are replaced by one stick in a different color or material to allow more bundles to be rearranged as icons also, so that 67 means 6-Bundle-7, 6B7, called 6-ten-7, or more precisely, 6-tens-7, or 6 Bundles with tens per Bundle and 7 unbundled singles. So, 67 is not one number, but three numberings: of the number of Bundles, 6, of the Bundle-size, tens, and of the number of unbundled singles, 7. Ten thus needs no icon since it becomes 'one bundle and no unbundled', written as T = 1B0 tens, or T = 1.0 tens, or T = 10 if leaving out the unit and the decimal point.

Example. One stick is one single. An extra stick added to 1 stick gives two singles that may unite to one 2-icon. And so on. An extra stick added to 8 sticks gives nine singles that may unite to one 9-icon. And an extra stick added to 9 sticks gives ten singles that instead of uniting to one ten-icon is bundled together as one bundle replaced by one stick of a different color or a different material and written as 1B0 since there are no singles left.

An extra stick added to ten sticks gives eleven singles that may unite to one bundle and 1 single left, which made the Vikings call eleven 'one left' and written as 1B1.

Likewise with twelve that the Vikings called 'two left'. There is no 'three left' because of the ancient counting method "one, two, many". So, from 3 we specify both the bundles and the singles.

The name 'twenty' comes from the Vikings' 'twende ti'.

Skill building. Roll some dice twice (physically or virtually) to get the number of bundles and unbundled singles. Then phrase and report the number.

So, with 3 and 5, say '3 Bundles with tens per Bundle and five unbundled', 'three-bundle-five', three-ten-five, and thirty-five; and finally write T = 3B5 = 35.

End test. Roll some dice twice an extra time.

MC02. Tally-counting in time, **** = ~~IIII~~ I**

The total here exists as sticks to be moved one by one on a table and reported by strokes on paper. The 'T=?' question is answered by tally-counting the total in time reported as some 5-bundles and some unbundles singles, e.g., T = 2B1 5s.

Example. In a sentence, count the e's and the a's.

Skill building. Some dice is rolled a dozen times to show Even (1 2s, 2 2s or 3 2s) or Odd (1, 3, 5). The tally counting in 5s is reported with two totals e.g., W = 1B4, and L = 0B3, giving a total T = 2B2, or 1B7, or 3B-3. And giving the difference D = 1B1.

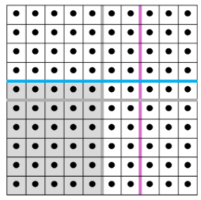
End test. Roll a dice some extra times.

MC03. Bundle-counting in time with units: 0B1, …, 0B5 or 1B0, 3 3s = 1BB

The total here exists as lines or stacks on a BBBoard.

The 'T=?' question is answered in time by moving the finger along the pegs with a counting sequence that by including the bundle as a unit makes the place value system unneeded.

First, we count in lines, then in stacks marked by a vertical rubber band.



Counting 5 fingers in 3s we ask, "What do we have here?" to emphasize that we focus on existence instead of essence. We cannot count one finger as 'one' since 1 3s is 3 1s and we only have one. Instead, we count '0 bundle 1, 0B2, 0B3 or 1B0' since 3 1s is 1 bundle with no unbundled left.

Counting 5 fingers in 2s we notice that four fingers are 1B2, but also 2B0, and 1BB0B0 since 2 2s is a bundle of bundles, a bundle-bundle, a BB, that is a square, as is 3 3s, 4 4s etc.

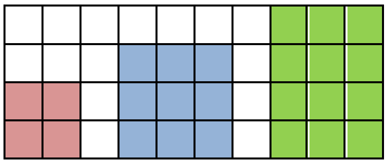


Figure 02. 2 2s, and 3 3s, and 4 4s as bundle-bundle squares

Counting the five fingers on a hand, their essence '5' may exist in various ways:
T = 1 5s = 1B1 4s = 1B2 3s = 1B3 2s = 2B1 2s = 1BB 0B 1 2s = 5 1s

Counting ten fingers in 3s, we get 0B1, 0B2, 0B3 or 1B0, 1B1, … , 2B3 or 3B0, 3B1.
T = 3B1 = 1BB0B1 3s, since 3 3s are a bundle of bundles, a bundle-bundle, a BB, a square.

Counting ten fingers in 2s we notice that 8 as 2BB 0B 0 is 1BBB 0BB 0B 0. So, we may also write ten as 1BBB 0BB 1B 0 2s, or as 1010 if leaving out the units. Finally, when counting hundred on the BBBoard we finish with 1BB 0B 0:
0B1, 0B2, … ,0B9, 0Bten or 1B0, 1B1, … , 9B8, 9B9, 9Bten or tenB0 or 1BB0B0.

1BB0B0	1BB0B1	1BB0B2	1BB0B3	1BB0B4	1BB0B5	1BB0B6	1BB0B7	1BB0B8	1BB0B9	~~1BB0B10~~
~~10B0~~	~~10B1~~	~~10B2~~	~~10B3~~	~~10B4~~	~~10B5~~	~~10B6~~	~~10B7~~	~~10B8~~	~~10B9~~	~~10B10~~
9B0	9B1	9B2	9B3	9B4	9B5	9B6	9B7	9B8	9B9	~~9B10~~
8B0	8B1	8B2	8B3	8B4	8B5	8B6	8B7	8B8	8B9	~~8B10~~
7B0	7B1	7B2	7B3	7B4	7B5	7B6	7B7	7B8	7B9	~~7B10~~
6B0	6B1	6B2	6B3	6B4	6B5	6B6	6B7	6B8	6B9	~~6B10~~
5B0	5B1	5B2	5B3	5B4	5B5	5B6	5B7	5B8	5B9	~~5B10~~
4B0	4B1	4B2	4B3	4B4	4B5	4B6	4B7	4B8	4B9	~~4B10~~
3B0	3B1	3B2	3B3	3B4	3B5	3B6	3B7	3B8	3B9	~~3B10~~
2B0	2B1	2B2	2B3	2B4	2B5	2B6	2B7	2B8	2B9	~~2B10~~
1B0	1B1	1B2	1B3	1B4	1B5	1B6	1B7	1B8	1B9	~~1B10~~
0B0	0B1	0B2	0B3	0B4	0B5	0B6	0B7	0B8	0B9	~~0B10~~

Figure 03. Counting from zero to 109 with units.

Skill building. Count a dozen and a score in 5s, 4s, 3s, and 2s.

End test. Count 30 in 3s.

MC04. Bundles counted in space with over- and underloads, 5 = 1B3 = 2B1 = 3B-1 2s

The total here exists as fingers and sticks. The ‘T=?’ question is answered in space by ‘flexible bundle counting’ that allows unbundled to stay unbundled as an overload, and that allows borrowing extra sticks to fill up an extra bundle with an underload. Using flexible bundle-numbers with units makes carrying and borrowing unneeded.

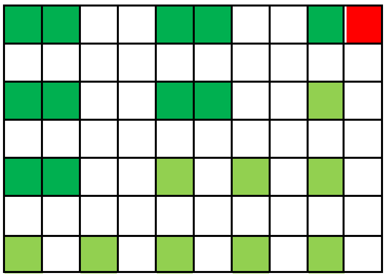


Figure 04. Five fingers may be recounted in 2s as 0B5, or 1B3, or 2B1, or 3B-1

Five fingers may be recounted in 5s as 0B5 (an overload) or 1B0 or 2B-5 (an underload).

Five fingers may be recounted in 4s as 0B5 or 1B1 or 2B-3.

Five fingers may be recounted in 3s as 0B5 or 1B2 or 2B-1.

Ten fingers may be recounted in tens as ½B from 1 to ten:

½B-4, ½B-3, ½B-2, ½B-1, ½B0, ½B1, ½B2, ½B3, ½B4, ½B5 or 1B0 as ten.

This may ease standard calculations.

T = 6 + 3 = ½B1 + ½B-2 = 1B-1 = 0B9 = 9

T = 6 + 7 = ½B1 + ½B2 = 1B3 =13

T = 8 – 3 = ½B3 – ½B-2 = 0B5 = 5, thus showing that -(-2) = +2

T = 4 * 7 = 4 * ½B2 = 2B8 = 28

Overload	Underload	Overload	Overload
65 + 27	65 – 27	7 x 48	336 /7
6 B 5 + 2 B 7	6 B 5 – 2 B 7	7 x 4 B 8	33 B 6 /7
8 B12 9 B 2	4 B-2 3 B 8	28 B 56 33 B 6	28 B 56 /7 4 B 8
92	38	336	48

Figure 05. Bundles and over- and underload make place values, carrying and borrowing unneeded

The number 47 is not one number; there are three numberings:

47 = 4 ty 7 = 4 tens 7 = 4 Bundles, with tens per Bundle, and 7 unbundled ones.

Skill building. The action is repeated with nine fingers arranged counted in 5s, 4s, 3s, and 2s. And with 2-digit numbers, e.g., 67 = 6B7 = 7B-3 = 5B17. Then with cubes, and with sticks.

End test. The action is repeated on a BBBoard, or on an abacus.

MC05. Splitting, 8 = (8-2)+2

The total here exists as a line of pegs on a BBBoard. The ‘T=?’ question is answered by ‘pulling-away’ a bundle hidden under cubes.

A total of 1 8s is split by pulling-away 2. To pull-away just once may be iconized by a rope, -, so that ‘8-2’ means ‘from 8 pull-away 2’ in time, or ‘from 8 pulled-away 2’ in space.

The original 8 now is split in 8-2 and 2 so that $8 = (8-2) + 2$. Here addition is iconized by a cross showing the two directions we can add, next-to or on-top so that '4+2' means '4 with 2 added'. With T for the total and B for the bundle this 'split-formula' may be written as $T = (T–B)+B$.

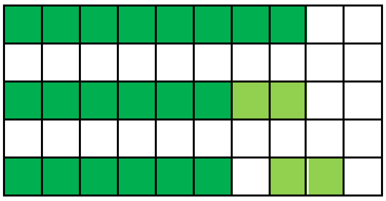


Figure 06. A total of 8 split in two parts by pulling-away 2, so 8 = (8 – 2) + 2

Splitting may be used to solve equations coming from reversed actions. The question "What is the number that with 2 added gives 8" may be shortened to an equation with a letter for the unknown number, '$u+2 = 8$'. Of course, the number is found by reversing the action and pull-away the number that was originally added, so $u = 8-2$, which also comes from splitting 8, '$u+2 = 8 = (8-2) + 2$'. So, we see that the solution is found by moving "to the opposite side with the opposite sign". Also, it follows the formal definition of subtraction: 8-2 is the number u that added to 2 gives 8, or if $u+2 = 8$ then $u = 8-2$.

Skill building. The action is repeated with fingers, sticks, cubes, and an abacus. The action is repeated with other numbers, e.g., $9 = (9 – 3) + 3$.

End test. Pick two numbers.

MC06. Recounting, 8 = (8/2)x2

The total here exists as lines of pegs on a BBBoard. The 'T=?' question is answered by 'pushing-away' bundles hidden under cubes.

A total of 1 8s is recounted in 2s by 4 times pushing-away 2s. To pull-away more times may be iconized by a broom, /, so that '8/2' means 'from 8 push-away 2s' in time, and 'from 8 pushed-away 2s' in space.

With the pushed-away 2s arranged in a stack, 8 contains 2s 4 times or 8/2 times so that $8 = 4*2$, or $8 = (8/2)x2$. Here multiplication is iconized by a lift so that '4x2' or '4*2' means '4 times stacking 2s'. With T for the total and B for the bundle this 'recount-formula' may be written as $T = (T/B)xB$.

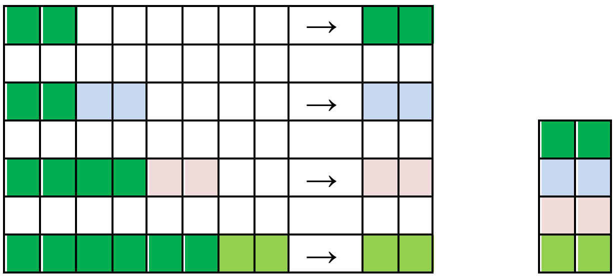


Figure 07. A total of 8 is recounted by pushing-away 2s and lifted into a stack, so 8 = (8/2)x2

Recounting may be used to solve equations coming from reversing actions: The question "What is the number of 2s in 8" may be shortened to an equation with a letter for the unknown number, "$u*2 = 8$". Of course, the number is found be reversing the action and push-away the 2s that was originally united, so $u = 8/2$, which also comes from recounting 8, '$u*2 = 8 = (8/2)*2$'. So, we see that the solution is found by moving "to the opposite side with the opposite sign". Also, it follows the formal definition of division: 8/2 is the number u that multiplied with 2 gives 8, or if $u*2 = 8$ then $u = 8/2$.

Skill building. The action is repeated with 12 counted in 2s and 3s using a finger to hide a bundle.
End test. 18 counted in 2s, and in 3s.

MC07. Including the unbundled, 8 = (8/3)*3 = 2B2 = 2 2/3 = 3B-1 3s

The total here exists as cubes. The 'T=?' question is answered by pushing-away bundles to a stack that is then pulled-away to find the unbundles that then are included on-top of the stack.

Recounting 8 in 3s, first 2 times we push-away 3s, then we pull-away the stack of 2 3s and find 2 unbundled that are placed on-top of the stack. Here they may be seen as singles in a bundle described by a decimal number, 8 = 2B2 3s, or as a fraction part when also recounted in 3s as 2 = (2/3)*3, 8 = 2 2/3 3s. Or we may write 8 = 3B-1 to show that in space 1 is missing in the next bundle, or that in time 1 is pulled-away from it.

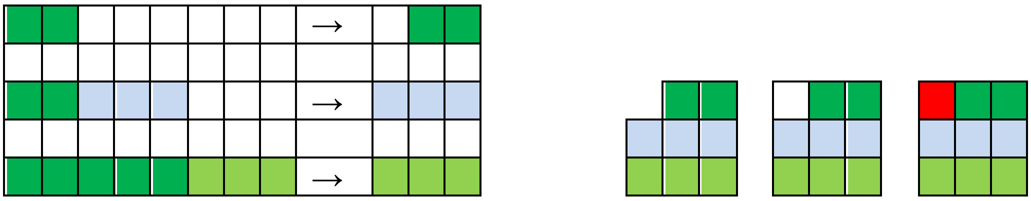


Figure 08. Unbundled become decimals, fractions or less-numbers, 8 = 2B2 = 2 2/3 = 3B-1 3s

With ten as the bundle-number, unbundled occur in the same way: T = 4B3 = 4 3/10 = 5B-7 tens.

Skill building. The action is repeated on a BBBoard with 11 counted in 3s and 4s using cubes or fingers to hide a bundle.

End test. Recount 8 in 5s, and in 3s.

MC08. Recounting in squares, 6 4s = 1 BB ?s

The total here exists as a rectangular bundle-number on a BBBoard. Here the 'T=?' question is answered by working on the upper right corner occurring when half the excess is move from the top to the side to give a first guess about the square root. The rectangle may be shown with rubber bands or with cubes.

If we want to square the total T = 6 4s we move half of the excessing 2 4s from the top to the side to get a 5 x 5 square, and an unfilled square in the upper right corner that we try to fill with a rectangular 4*u slice of the top and the side.

Here u is found by the equation 2*u*4 = 1, or 8*u = 1, giving u = 1/8 = 0.125, and 5-0.125 = 4.875 as our next guess.

However, now there is too much in the corner, so we repeat the process, or consult a calculator showing that the correct answer is √(6*4) = 4.90, which is very close to our third guess.

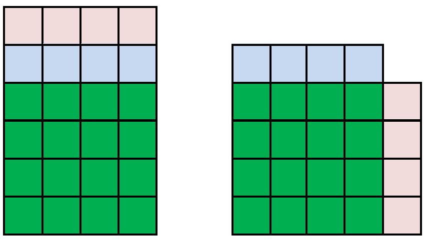


Figure 09. Recounting 6 4s by moving half the excess to the side to try to get a 5 x 5 square

To find the quadratic numbers we see that 5 5s comes from 4 4s by adding 4 twice and 1 for the top right corner. So, with 4 4s as 16, 5 5s is 16 + 4 + 4 + 1= 25.

In this way we may predict the square numbers to be 1, 4, 9, 16, 25, 36, 49, 84, 91 and 100.

And we see that a BB square increases with 2B+1 when B increases with 1.

Skill building. The action is repeated on a BBBoard with other rectangular numbers.

End test. Square 9 5s. Solve the quadratic equation x^2 + 8*x + 12 = 0

MC09. Recounting in another icon, 3 4s = ?5

The total here exists as a rectangular bundle-number on a BBBoard. Here the 'T=?' question is answered on a BBBoard and predicted on a calculator.

With rubber bands on a BBBoard we see that 3 4s may be recounted as 2B2 5s. This may be predicted by a calculator. To find how many 5s there is in 3 4s we enter "3*4/5". The answer is '2.more'.

To find them we pull-away the stack of 2 5s by entering '3*4-2*5' that gives the answer '2'. So, the calculator predicts that 3 4s recount as 2B2 5s, which is validated on the BBBoard.

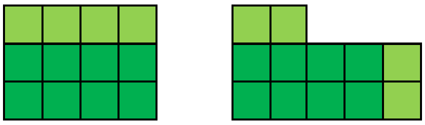


Figure 10. A total of 3 4s recounted in 5s manually. and predicted by a calculator

Skill building. The action is repeated on a BBBoard with other rectangular numbers.

End test. 4 5s = ?6s.

MC10. Recounting from tens to icons, 2 tens = ? 7s

The total here exists as a rectangular bundle-number on a BBBoard. Here the 'T=?' question is answered by recounting.

With rubber bands on a BBBoard we see that 2 tens may be recounted as 2B6 7s. This may be predicted by a calculator. To find how many 7s there is in 2 tens by recounting we enter '20/7'.

The answer is '2.more' found by pulling away the stack predicted by '20-2*7' giving '6'. So, the calculator predicts that 2 tens recount as 2B6 7s, which is validated on the BBBoard.

Alternatively, the question "How many 7s in 20?" leads to the equation u*7 = 20 that is solved moving to opposite side with opposite sign so that again u = 20/7, or u = 2 6/7.

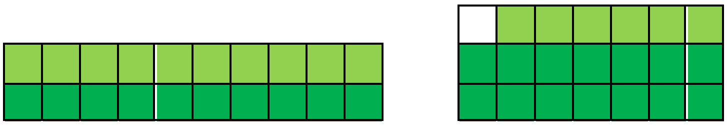


Figure 11. A total of 2 tens recounted in 7s manually, and predicted by a calculator

We notice that decreasing the bundle will increase the height. To study this closer we recount 1 dozen in 6s, 4s, 3s, 2s, and 1s and place a dot in the upper right corner each time.

The points then form a curve called a hyperbola.

Skill building. The action is repeated on a BBBoard with other numbers.

End test. 4 tens = ?8s.

MC11. Recounting from icons to tens, 6 7s = ? tens

The total here exists as a rectangular bundle-number on a BBBoard. Here the 'T=?' question is answered by finding what we must pull-away from the bundle-bundle.

Recounting from icons to tens apparently is another name for the multiplication tables.

With rubber bands on a BBBoard we see that 6 7s is left if from the ten bundles we pull-away 4 top and 3 side bundles, and add the upper right 4 3s that we pull-away twice:

T = 6 7s = 6*7 = (10 – 4 – 3)*B + 4 3s = 3B + 1B2 = 4B2 = 42.

This leads to early algebra if instead we write:

T = 6 7s = 6*7 = (B – 4)*(B – 3) = BB – 4*B – 3*B + 3*4

Here we see that minus times minus must be plus.

So, a quick way to find the answer is to add and multiply the less-numbers and subtract the first and add the latter. With 4 and 3 as the less-numbers here, we quickly learn to say:

“Less (4+3) bundle (4*3)”, or “Less 7 bundle 12”, or “3 bundle 12”, or “4 bundle 2”, or “42”.

We may also write B – 4 and B – 3 on top of each other and then multiply down and across. Or we may use the FOIL method: First, Outside, Inside, Last.

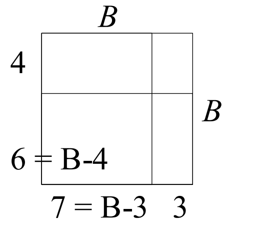
	$T = 6 * 7$ $= (B\text{-}4) * (B\text{-}3)$ $= BB – 4B – 3B + 4*3$ $= 3B12$ = 4B2 = 42	$T = \begin{pmatrix} 1B & -4 \\ 1B & -3 \end{pmatrix}$ $= 1BB – 4B – 3B + 4*3$ $= 10B – 7B +$ 1B2 $= 3B12 = 42$	$T = \begin{pmatrix} 2B & +3 \\ 4B & +6 \end{pmatrix}$ $= 8BB+12B+12B+18$ $= 8BB+24B+18$ $= 10BB\ 5B\ 8$ = 1058

*Figure 12. 6*7 is left when pulling-away 4B and 3B and adding the 4 3s pulled away twice*

Multiplying the two-digit numbers 23*46 as 2B3 4B6s, a vertical and a horizontal rubber band between the bundles and the singles allows a BBBoard to show the four stacks 2B 4Bs, and 2B 6s, below the 3 4Bs, and the 3 6s. With overloads, they add up to 8BB 24 B 18, or to 10BB 5B 8 = 1058 without.

This process may be reversed when asking ‘1058 = ? 46s’. First 1058 is written with an overload as 10BB 5B 8 = 8BB 25B 8. Since 4B*2B = 8BB, the 2B contributes 2B*6 to the 25 Bs. The rest 13B 8 may be rewritten as 12B 18, which recounted in 3s gives 4B6. So, the answer is 1058 = 23 46s.

Skill building. The action is repeated on a BBBoard with other numbers.

End test. 7 8s = ? tens.

MC12. Recounting in another physical unit creates per-numbers, 3$/5kg

The total here exists as a rectangular bundle-number on a BBBoard. Here the ‘T=?’ question is answered by changing the unit in the per-number rectangle.

Recounting a physical total T as 3$ and 5 kg gives a ‘per-number’ 3$/5kg called the price and marked as a 3x5 rectangle on a BBBoard.

The question “20kg = ?$” then is answered by recounting in the per-number:

20kg = (20/5)*5kg = (20/5)*3$ = 12 $.

On a BBBoard the counting sequences now are 5, 10, 15, 20kg, and 3, 6, 9, 12$ since the per-number here has changed from 3/5 to 12/20.

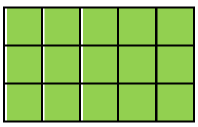


Figure 13. The per-number 3$/5kg show as 3 5s, or 6 10s, or 9 15s, or 12 20s, etc.

Or we may introduce a new unit to make the digits like: 3$ = (3/5)*5$ = n*5$ = 5n$ with the new n = 3/5. So, 20 kg = 20$n$$ = 20*3/5$ = 12$.

Alternatively, the units may be recounted:

$ = ($/kg)*kg = (3/5)*20 = 12

Or we may equate the per-numbers: $/kg = u/20 = 3/5. Moving to opposite side with opposite sign we then get 5*u = 3*20, or u = 3*20/5 = 12.

Skill building. The action is repeated with other numbers.

End test. With 5$/2kg, 12kg = ?$, and ?kg = 12$.

MC13. With the same unit, per-numbers become fractions, 3$/5$ = 3/5

The total here exists as a rectangular bundle-number on a BBBoard. Here the ‘T=?’ question is answered by changing the unit in the per-number rectangle.

If a whole contains a part, they have the same unit. In this case the per-number becomes a fraction without units. Still, we may use the units ‘p’ and ‘w’ for the part and the whole.

To get the fraction 3/5 of 20$ thus means to get 3p/5w of a 20$ whole. Recounting in the per-number thus gives 20w = (20/5)*5w = (20/5)*3p = 12p, or 12$ of 20$.

To get the fraction 3/5 of 100 thus means to get 3p/5w of a 100 whole. Recounting in the per-number thus gives 100w = (100/5)*5w = (100/5)*3p = 60p, or 60 of 100, written as 60%.

To ask “20$ is what percentage of 80$” means asking about the fraction 20/80 of 100. Or we may introduce a new unit 80$ = 100% to see that 20$ = (20/80)*80$ = (20/80)*100% = 40%.

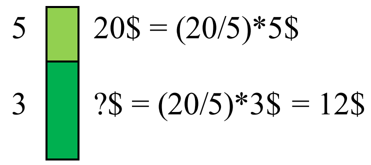


Figure 14. A fraction column with per-numbers to the left and unit-numbers to the right

To add 10% to 200$ we introduce the per-number 200$/100%. After the addition the total is

T = 100% + 10% = 110% = (110/100)*100% = (110/100)*200$ = 220$.

So, adding 10% means multiplying with 110%, and adding 10% 5 times means multiplying with 110%^5 = 161.1% thus giving 50% plus 11.1% extra, also called compound interest.

Skill building. The action is repeated on a BBBoard with other numbers.

End test. With 2p/5w, 10p = ?w, and ?p = 20w, and 2p/5w = ?%.

MC14. Recounting a stack’s sides gives trigonometry, rise = (rise/run)*run = tanA*run

The total here exists as a rectangular bundle-number on a BBBoard. Here the ‘T=?’ question is answered by changing the unit in the per-number rectangle.

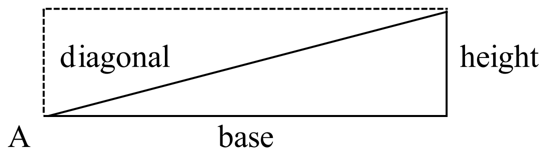


Figure 15. A stack with a base and a height and a diagonal

On a BBBoard, we mark a 3x4stack as a rectangle with height 3 and base 4. If we recount the height and the base in the diagonal, we get the per-numbers sine and cosine:

height = (height/diagonal) * diagonal = sine Angle * diagonal, shortened to

$h = (h/d) * d = \sin A * d = \sin A$ ds,

This gives the formula sin A = height / diagonal, or sin A = h/d, or sin A = 3/5 in our case.

Likewise, cos A = base / diagonal, or sin A = b/d, or cos A = 4/5 in our case.

height = (height/base) * base = tangent Angle * base, shortened to

$h = (h/b) * b = \tan A * b = \tan A$ bs, This gives the formula tangent A = height / base, or tan $A = h/b$, or tan A = 5/10 in our case.

A protractor shows that the angle A is a little above 25 degrees. Testing this we get tan 25 = 0.466. The reverse tan-button 'tan^-1' gives the precise result, tan^-1(0.5) = 26.6 degrees.

Using the words 'run' and 'rise' instead of 'base' and 'height', we get the diagonal's slope-formula: tan A = rise/run. Here the tangent-number describes the steepness of the diagonal.

In a x-y coordinate system a curve may be generated by a formula $y = f(x)$. Here the curve between two close neighbor points is a diagonal in a rectangle, and since the run and the rise are changes in x, Δx, and in y, Δy, the tangent-number here describes the steepness of the curve as the per-number $\Delta y/\Delta x$ called the local slope of the curve.

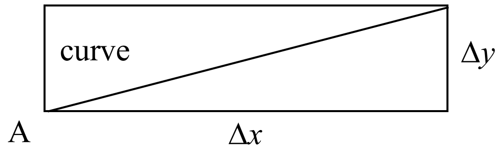


Figure 16. A curving curve is linear locally with small changes in x and y

The word 'tangent' is used since the height will be a tangent in a circle with center in A, and with the base as its radius. This gives a formula for the circumference since a circle contains many right triangles leaving the center. In a circle with radius 1, h = tan A.

A half circle is 180 degrees that split in 100 small parts as 180 = (180/100)*100 = 1.8 100s = 100 1.8s. With A as 1.8 degrees, the circle and the tangent, h, are almost identical. Half the circumference in a circle with radius 1 is called π, and π = 100 * h = 100 * tan 1.8 = 100 * tan (180/100) = 3.1416. This gives a formula for the number π: π = tan (180/n) * n, for n large enough. We also see that in a circle with radius r, the circumference is 2*π*r, and the area is π*r^2, or π/4* d^2 where d is the circle's diameter. So, a d-circle takes up almost 80% of the space inside the surrounding d-square.

Skill building. The action is repeated on a BBBoard with other numbers.

End test. Add a 4-square and a 6-square as a square.

MC15. Adding next-to or on-top, T = 2 3s + 4 5s = ? 8s; T = ? 3s; T = ? 5s

The total here exists as a rectangular bundle-number on a BBBoard. Here the 'T=?' question is answered by recounting.

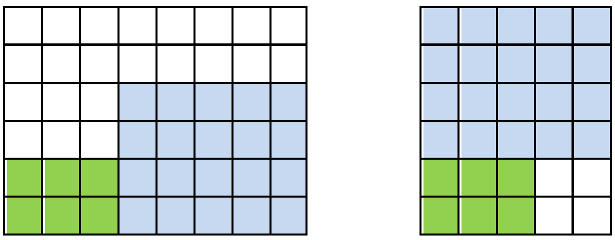


Figure 17. Two totals 2 3s and 4 5s added next-to, and added on-top

Adding 2 3s and 4 5s as 8s means adding areas, which is called integral calculus. Reversing the process by asking '2 3s and how many 5s total 4 8s' is called differential calculus because you find the difference between the known totals before recounting it in 5s, (T2-T1)/5, or ΔT/5.

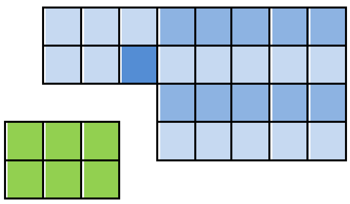


Figure 18. Next-to addition is reversed when asking 2 3s + ? 5s = 4 8s

Adding 2 3s and 4 5s as 3s or 5s, first recounting must make the units like To recount 2 3s in 5s, first we enter '2*3/5' giving '1.more', then we enter '2*3-1*5' giving '1', so 2 3s is 1B1 5s, which give the total 1B1 5s + 4B0 5s = (1B1 + 4B0) 5s = 5B1 5s.

Skill building. The action is repeated on a BBBoard with other numbers.

End test. 3 4s + 6 5s = ? 9s. And 3 4s + 6 5s = ? 4s. And 3 4s + 6 5s = ? 5s. And 3 2s + ? 5s = 4 6s.

MC16. Subtracting and adding single digit numbers, 8+6 = 1B2 + 1B0 = 2B2 6s

The total here exists as lines of pegs on a BBBoard. The 'T=?' question is answered by using rubber bands to mark the bundles.

With a subtraction as '8 – 6 =?', a rubber band marks 8 on a BBBoard, and fingers hide the 6 that is pulled-away, so 8 – 6 = 2.

With an addition as '8+6=?', two rubber bands marks 8 and 6 on two BBBoard parallel lines to show that the sum may exist in two ways, as 2B2 6s, or as 2B-2 8s.

*Figure 19. Adding 6 and 8 as 2B2 6s, or as 2B-2 8s, or as 2*½B + 1 + 3*

Here, using half bundles, 5s, will easy recounting in tens since 6+8 = ½B1 + ½B3 = 1B4 = 14.

Multidigit numbers may be added and subtracted with an over- or an under-load, which makes carrying and borrowing unneeded.

T = 36 + 47 = 3B6 + 4B7 = 7B13 = 8B3 = 83
T = 86 – 37 = 8B6 – 3B7 = 5B-1 = 4B9 = 49
T = 4*67 = 4*6B7 = 24B28 = 26B8 = 268
T = 268 /4 = 26B8 /4 = 24B28 /4 = 6B7 = 67

Skill building. The action is repeated with other one-digit and two-digit numbers.

End test. 9 – 7 = ?, 9+7 = ?, T = 38+46 = ?; T = 82 – 54 = ?

MC17. Adding per-numbers and fractions by integral calculus

The total here exists as a rectangular bundle-number on a BBBoard. The 'T=?' question is answered by using rubber bands to mark the bundles.

Asking "2kg at 3$/kg and 4 kg at 5$/kg total what?" the unit numbers 2kg and 4kg add directly whereas the per-numbers 3$/kg and 5$/kg first must be multiplied to unit-numbers before adding, thus added as areas, i.e., as integral calculus. Here the per-numbers are piecewise constant, but they may also be locally constant as in the case with a falling object having an increasing meter/second number.

Before adding, fractions must also be multiplied to unit, numbers. So, with apples, 1red of 2 plus 2red of 3 gives 3red of 5, and of course not 7red of 6 as taught by 'mathematism'.

Figure 20. Per-numbers add as areas and fractions add with units, both as integral calculus

Adding like per-numbers is predicted by power where, e.g., 6% 10 times gives 106%^10 or 179%, i.e., the expected 60% plus additional 19%, and where 6% 20 times gives 321%, i.e., the expected 120% plus additional 201% showing the benefit of pensions.

	B	~~B~~	
Left	1	2	3
Right	1	1	2
Total	2	3	5

	B	~~B~~	
Left	1/3	2/3	1
Right	1/2	1/2	1
Total	-	-	-

	B	~~B~~	
Left	1/2	2/3	-
Right	1/2	1/3	-
Total	1	1	-

Figure 21. In cross tables, per-numbers must pass through the unit-numbers

Looking at my right hand I see 3 fingers to the left, the Ls, and 2 fingers to the right, the Rs. I bent the two outer fingers. So, 1/3 of the Ls are bent, and ½ of the Rs. Does '1/3 of the Ls are bent' mean that '1/3 of the bent are Ls'? No, ½ is. So, in a cross table we cannot go from the per-numbers in one direction to those in the other direction without going through the unit-number table. This is called the Bayes-principle.

Skill building. The action is repeated with other numbers.

End test. 3kg at 4$/kg and 5 kg at 6$/kg total what?"

MC18. Adding and subtracting Bundle-Bundle squares

The total here exists as bundle-bundle number squares on a BBBoard. The 'T=?' question is answered by using rubber bands or cubes to mark the bundles.

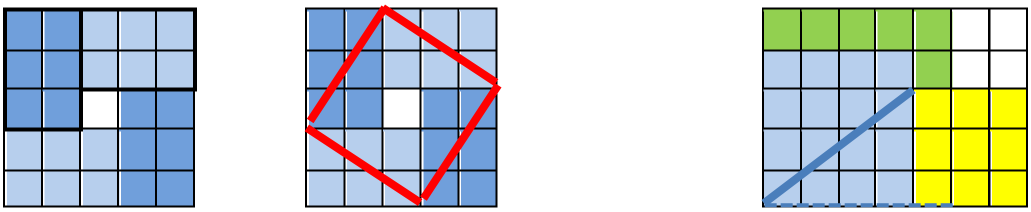


Figure 22. The two squares add as the square formed by the mutual Bottom-Top BT line

On a BBBoard we place four 7 3s so they form a ten-by-ten square that inside contains two squares, 7 7s and 3 3s as well as two stacks. But it also contains one square formed by the diagonals in the stacks as well as four half stacks. So, the two squares add as the square formed by the mutual Bottom-Top BT line thus having the length as the square-root of the sum, i.e., √(7^2 + 3^2) = 7.62. So, in this stack, adding the height and the bundle as squares gives the square of the diagonal. This rule is named by the ancient Greek thinker, Pythagoras.

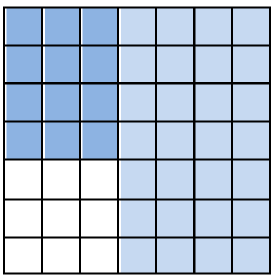


Figure 23. On a BBBoard we see that 7^2 – 3^2 = 7(7-3) + (7-3)*3 = (7+3)*(7 – 3)*

On a BBBoard we pull-away the a 3-by-3 square from a 7-by-7 square. This leaves 7 (7-3)s and (7-3) 3s that may be turned to 3 (7-3)s totaling (7+3) (7-3)s. So 7^2 – 3^2 = (7+3)*(7 – 3).

Adding squares may also be involved when solving a quadratic equation. On a BBBoard we see that T = (x+3)*(x+3) is a square with four parts, two squares x^2 and 3^2, and two stacks 2*3*x, so that T = x^2 + 6*x + 9. The quadratic equation x^2 + 6*x + 8 = 0 then makes the whoe square go away except for 9-8 = 1. So $(x+3)^2 = 1$. This gives two solutions , $x= -2$ and $x = -4$ that hold when tested: $(-2)^2 + 6*(-2)+ 8 = 4-12+8 = 0$, and $(-4)^2 + 6*(-4) + 8 = 16-24+8 = 0$.

The quadratic equation '$x^2 + 6*x + 10 = 0$' has no solutions since here '$(x + 3)^2 = -1$'.

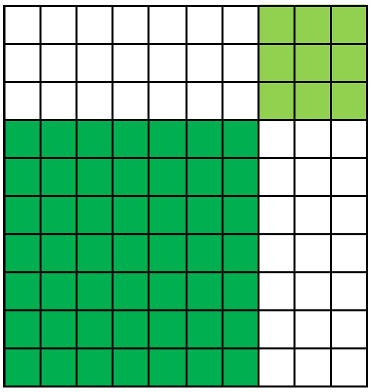


*Figure 24. An (x+3)-square contains an x-square and a 3-square and two 3*x stacks*

Alternatively, we can rewrite the equation $x^2 + 6*x + 8 = 0$, first as
$x^2 + 2*3*x + 3^3 – 3^2 + 8 = 0$, then as
$(x+3)^2 – 9 + 8 = 0$, then as
$(x+3)^2 = 9 – 8 = 1$ again with the solutions $x= -2$ and $x = -4$ that hold when tested:
$(-2)^2 + 6*(-2)+ 8 = 4-12+8 = 0$, and, $(-4)^2 + 6*(-4) + 8 = 16-24+8 = 0$

Skill building. The action is repeated with other numbers.

End test. Add 3 3s and 4 4s as a bubndle-bundle square. Solve the equation $x^2+8*x+12 = 0$

MC19. Adding unspecified letter-numbers

The total here exists as a rectangular bundle-number on a BBBoard. The 'T=?' question is answered by using rubber bands to mark the bundles.

In the letter-number $T = 3ab$ the multiplication sign is invisible, and the letters stands for unspecified numbers. Since any factor may be a unit, T may be seen as 3 abs, or as $(3a)$ bs, or as $(3b)$ as. To avoid being confused by the 's' we will omit it, so $T = 3ab = 3 * ab = 3a * b$ or $3b * a$.

Since totals need a common unit to add, this must be first found:

$T = 3ab + 4ac = 3b * a + 4c * a = (3b+4c) * a$

$T = 2ab^2 + 4bc = ab * 2b + 2c * 2b = (ab+2c) * 2b$

Skill building. The action is repeated with other numbers and letters.

End test. $T = 4ab^2d + 8bcd$

MC20. Change in time

The total here exists as dots on a BBBoard. The 'T=?' question is answered by transferring the results to a squared paper and connect the dots with a curve.

In time, a total grows by being added or multiplied by a number, called addition-growth and multiplying-growth, or linear and exponential growth.

Addition-growth:

Final number = Initial number + growth-number * growth times, or shortly, $T = B + a*n$.

The number a is also called the slope.

Multiplying-growth:

Final number = Initial number * growth-factor ^ growth times, or shortly, $T = B * a^n$, since 200\$+5% = (200*105%) \$, so here a is 1+interest rate = 100% + 5% = 105%.

Combined growth (savings in a bank): Here we have that $A/a = R/r$, where A is end-dollars, a is the period-dollars, R is the end-rate, r is the period-rate, and $1+R = (1+r)^n$, where n is the number of periods.

100% split in n parts will give the Euler number $e = (1+1/n)^n = 2.718$ for n large enough.

Changing the growth-number constantly will give a quadratic growth with a parabola curving upwards or downwards if the number increases or decreases.

Changing the curvature constantly will give cubic growth with a double parabola with curvature and counter-curvature.

Decreasing the growth-factor constantly give logistic saturation growth with a hill-curve in infections. Confusing exponential and saturation growth can cause unnecessary damage.

MC21. Bundle-numbers in a coordinate system

The total here exists as dots on a BBBoard. The 'T=?' question is answered by rubber bands as lines on the BBBoard. The bundle-number '*y x*s' with a height y and a width x may be called a 'changing bundle-number'. Here $y = 2*x$ gives a rising and $y = 9-x$ a falling bundle-number.

Marking the top right corners we get two lines. To inside predict the outside intersection point we equate the two heights, $2*x = 9 – x$. Moving to opposite side with opposite sign we get $3*x = 9$, and $x = 9/3 = 3$, which makes $y = 2*3 = 6$.

So, the prediction is that the two bundle-numbers become like as 3 6s, which is validated on the BBBoard where the first dot now is 0 instead of 1.

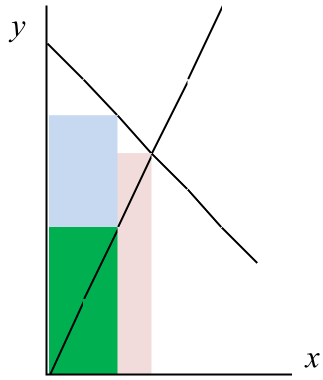


Figure 25. In an x-y coordinate system bundle-number stacks may rise and fall

In a rising bundle-number its total T will increase since here the height increases with increasing width. In the falling bundle-number this is not the case since the height decreases with increasing width. Here the total is $T = y*x = (9-x)*x = 9x – x^2$.

Setting up a table with x = 0, 1, 2, …, 9 we see that first T increases and then T decreases; and that T tops as 20 for x = 4 and x = 5; and that T = 20.25 for x = 4.5.

x	0	1	2	3	4	5	6	7	8	9
y	0	8	14	18	20	20	18	14	8	0

In general, even if a bundle-number is rising, its rise may fall, so its marked corners will lay on a bending line called a parabola where $y = b*x + a*x^2$.

Passing the points (x,y) = (1,6) and (2,10) we find that $10 = b*2+ a*4$, and $6 = b*1 + a*1$, or $12 = b*2 +a*2$. We now equate the two equations for $b*2$: $10 – a*4 = 12 – a*2$. Moving to opposite side with opposite sign we get $10–12 = a*4–a*2$, or $-2 = a*2$, or $-1 = a$. With $6 = b+a$, this gives $b = 7$.

So on the parabola, the points (x,y) are connected by the formula $y = 7*x – x^2$. It thus passes through the points (0,0), (1,6), (2,10), (3,12), (4,12), (5,10), (6,6), and (7,0).

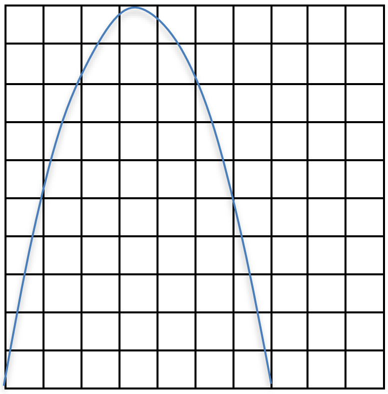


*Figure 26. Passing through (0,0) and (1,6) and (2,10), the parabola formula is y = 7*x – x^2*

Can the intersection points be predicted between the parabola and the two line above?

Can it be predicted that a falling bundle-number $b-a*x$ will have its maximum at the width $b/(2*a)$?

Skill building. The action is repeated with other rising and falling bundle-numbers to find when they are like and when the falling bundle-numbers tops.

End test. $y = 9-2*x$ and $y = x$.

MC22. Games Theory and damage control

The total here exists as towers of cubes and dots on a BBBoard. The 'T=?' question is answered by rubber bands as lines on the BBBoard.

In a Game Theory 2x2 zero-sum game two players A and B each have 2 strategies resulting in four different payments from B to A. It is called a zero-sum game since one player's gain is the other player's loss. In some game, if B chooses strategy B1 then the payment to A is 8$ or 2$ if A chooses strategy A1 or A2. And if B chooses strategy B2 then the payment to A is 4$ or 6$ and A chooses strategy A1 or A2. We may show this game by building four towers with cubes.

First, we assume that B chooses strategy B1. If A now p times chooses A2 and $n-p$ times A1 then A's outcome after n rounds will total $T = 2*p + 8*(n-p) = 8*n – 6*p = (8*n – 6*p)/n * n = (8 –6*p/n) * n$.

Which is $T1 = 8 – 6*p/n$ per round, shown on a BBBoard as a line connecting 8 to the left where p is 0, to 2 to the right where p is n. Next, we assume that B chooses strategy B2. If A now p times chooses A2 and $n-p$ times A1 then A's outcome after n rounds will total $T = 6*p + 4*(n-p) = 4*n + 2*p = (4*n + 2*p)/n*n = (4 + 2*p/n)*n$. Which is $T2 = 4 + 2*p/n$ per round, shown on a BBBoard as a line connecting 4 to the left where p is 0, to 6 to the right where p is n.

With p/n as u we find the intersection point by equating the two totals: $T1 = T2$, or $8 – 6*u = 4 + 2*u$, or $8*u = 4 = (4/8)*8$, or $u = 4/8 = ½$ giving $T1 = T2 = 5$$.

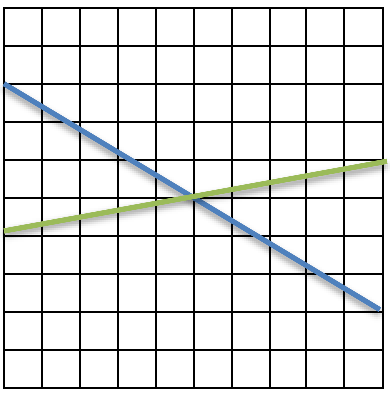


Figure 27. If B choses strategy B1, A will receive between 8 and 2$, else between 4 and 6$

So, if A mixes the strategies 1-to-1by flipping a coin, the average result will be 5$ per round.

Seen from B's side we also get the two lines S1 = 4 + 4*u, and S2 = 6 – 4*u that intersect where 4 + 4*u = 6 – 4*u, or 8u = 2, or u = 2/8, or u = 1/4 giving S1 = S2 = 5$.

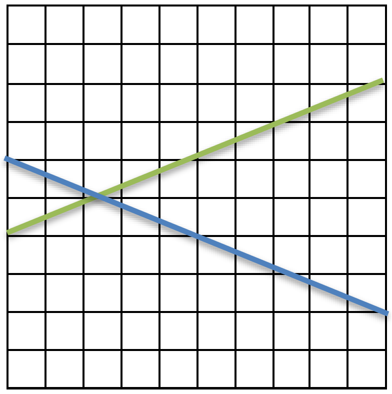


Figure 28. If A choses strategy A1, A will receive between 4 and 8$, else between 6 and 4$

So, if B mixes the strategies 1-to-3 by flipping two coins then the average result again will be 5$ per round. 5$ then is called the value of the game, i.e., the amount B must receive per round to make the game fair with no winner or loser in the long run.

From A's side the 5$ is called the 'maxi-min' value since deviating from it will decrease the value.

From B's side the 5$ is called the 'mini-max' value since deviating from it will increase the value.

In a similar game 4$ is changed to 8$. Here the strategy A1 dominates A2 that will always be lower to A. Likewise, the strategy B1 dominates B2 that will always be higher to B. So, here the value of the game is 6. This point is called a saddle point since the payment goes up one way and down the other.

Skill building. The game is repeated with other payments found, e.g., by rolling some dice.

End test. Replace the four payments 8,2,4,6 with 9,3,5,8.

MC23. Simple board games

The goal is to experience how different games may take place on a BBBoard.

• A racetrack. A 4x4 lawn is placed in the middle of a BBBoard. The start- and end-line goes from (5,0) to (5,3). A trip may change zero or one unit in the horizontal and vertical direction. You may touch but not cross the interior or exterior boundaries. If you do so you restart with 0 speed as close to the crossing point as possible. You may cross your opponent's track, but not end in the same point. The race may be repeated with different lawn shapes.

• Survival. You begin in (5,5). You roll a dice and move the number to the right if the number is even, else to the left. You roll again and now move the number up if the number is even, else down. You may touch but not cross the boundary. How many steps can you survive?

• Vertical race. A rubber band splits a BBBoard vertically in the middle. Two players each have three bricks placed at level one. They roll a dice and pick a brick to move the number upwards, and down again if there is a surplus. The winner is the first to have all three bricks at level ten.

The Algebra Square

There are two kinds of numbers in the world, unit-numbers, and per-numbers, which may be unlike or like and which may be reunited. The aim of mathematics is therefore not to 'math', because you can't do that, but to act: 'Re-Unite Un-like & Like Unit-Numbers & Per-Numbers'.

The four operations that unite unlike and like unit-and per-numbers are: addition, multiplication, integration, and power as shown in the Algebra-square above that also includes the ways to split a total: subtraction, division, differentiation, as well as the factor-finding root and the factor-counting logarithm.

Operations **unite**/ *split Totals in*	Unlike	Like
Unit-numbers m, s, kg, $	$\boldsymbol{T = a + n}$ $T – n = a$	$\boldsymbol{T = a * n}$ $T/n = a$
Per-numbers m/s, $/100$ = %	$\boldsymbol{T = \int f\, dx}$ $dT/dx = f$	$\boldsymbol{T = a\text{^}n}$ $n\sqrt{T} = a \quad loga(T) = n$

Figure 29. The Algebra square shows how to unite and split our four number-types, and how to solve equations by moving 'to opposite side with opposite sign'.

Fact and fiction and fake, the three genres of number-models

Once we know how to count and recount totals, and how to unite and split the four number-types, unlike and like unit-numbers and per-numbers, we can actively use this number-language to produce inside tales about outside totals existing in space and time. This is called modeling.

As in the word-language, number-language tales also come in three genres: fact, fiction, and fake models that are also called since-then, if-then and what-then models, or room, rate, and risk models.

Fact stories are 'since-then' stories that quantify and predict predictable quantities by using factual numbers and formulas. Typically, they model the past and the present. They need to be checked for correctness and units.

Fiction stories are 'if-then' stories that quantify and predict unpredictable quantities by using assumed numbers and formulas. Typically, they model the future. They need to be supplied with scenarios building on alternative assumptions.

Fake stories are 'what-then' stories that quantify and predict unpredictable qualities by using fake numbers and formulas. Typically, they add without units or hide alternatives. Here, number stories need to be replaced by word stories.

Modeling and de-modeling

The goal is to experience how formulas calculating y from x form curves that express change in time, and how totals in space may be split in parts that each then becoming a percentage of the total.

Modeling means solving an outside problem inside with four steps. First an outside problem is translated to an inside problem. Then the inside problem leads to an inside solution that then is translated to an outside solution, that finally is evaluated to see if another cycle is needed.

A typical example is mixture problems. the outside problems here may ask "2kg at 3$ per kg and 4kg at 5$ per kg total what?" The inside problem places the second information under the first ready to add. The inside solution my then simply add all numbers, which leads to the outside solution "2kg at 3$ per kg and 4kg at 5$ per kg total 6 kg at 8$/kg".

This model is not accepted, so another cycle is needed. This time the per-numbers are multiplied to unit numbers before adding, which leads to the outside solution "2kg at 3$ per kg and 4kg at 5$ per kg total 6 kg at 26$/6kg". This model is accepted.

De-modeling is the opposite process: It means solving an inside problem outside with four steps. First an inside problem is translated to an outside problem, then the outside problem leads to an outside solution that then is translated to an inside solution, that finally is evaluated to see if another cycle is needed.

A typical example is uniting fractions.

Adding fractions as 1/2 + 2/3 only has meaning when taken of the same unit,

$u = (u/6)*6 = k*6$, where $k = u/6$, and $6 = 2*3$

$T = (½ + 2/3)*u = (½ + 2/3)*6*k = (3+4)*k = 7*k = 7*u/6 = 7/6*u$,

So, in this case, $1/2 + 2/3 = 7/6$.

• We now model the orbit of a ball sent away with an angle. A constant up-number will give a line that goes up or down or horizontal.

But here gravity makes the up-number decrease so the line curves down as a bent line called a parabola.

We choose the initial angle A determined by $\tan A = 6$.

From (0,0) we assume that the ball takes a '1 out, 5 up' step followed by a '1 out, 3 up' and a '1 out, 1 up', etc., to reach the points (1,5), (2,8), (3, 9), (4,8), (5, 5), (6,0).

Since $y = 0$ for $x = 0$ and for $x = 6$, the formula may contain the two factors (x-0) and (6-x), so a guess could be $y = a*x*(6-x)$.

In the point (1,5) this formula becomes an equation,

$5 = a*1*(6-1)$, or $5 = a*5$, solved by $a = 1$.

So, the parabola formula may be $y = 1*x*(6-x)$, or

$y = -x^2 + 6*x$.

This formula holds when tested on the other points:

$8 = -2^2 + 6*2$, or $8 = -4 + 12$, or $8 = 8$, etc.

We find that with 4 as the first up-number, the orbit formula will be $y = -x^2 + 5*x$, etc.

The height after 5 steps is found by the equation $y = -5^2+6*5 = -25+30 = 5$.

The height 8 is reached after x steps found by the equation

$8 = -x^2+6*x$, or $x^2-6*x+8 = 0$, solved by $x = 2$ and $x = 4$.

The height 10 is never reached since there are no solutions to the equation:

$10 = -x^2+6*x$, or $x^2-6*x+10 = 0$.

Instead, that top-point is found in the middle at $x = 6/2 = 3$, giving $y = -3^2+6*3$, or $y = 9$.

To see if it breaks through a roof with the formula $y = 12-x$, we equate the two ys and get the equation $12 – x = -x^2 + 6*x$, or $x^2 - 7x + 12 = 0$, that is solved for $x = 3$ and $x = 4$.

• We now model the beginning monthly income of a business trying to establish itself at a market. We use the formula $y= x^2-6x+9$ where the steps form a parabola curving up when passing the points (0,9), (1,4), (2, 1), (3,0), (4,1), (5,4), and (6,9).

Later the monthly income changes its curvature from up to down until it reaches a maximum level.

So, from $x = 3$ we use a different model that contains the up-numbers 0, 1, 2, 3, 2, 1, 0, 0. This gives a 'logistical' s-shaped curve describing growth with saturation.

When prompted, AI may give a formula for this curve as $y = 9/(1+25*2^(-1.9*x))$

We see that the up-numbers form a hill. When prompted, AI may give a formula for this curve as $y = 3/(2^(0,44*(x-3)^2))$

• A cats and mice cohabitation on an island is an example of a predator-prey model where cats eat mice.

We expect a cycle in time since many cats and many mice leads to many cats and few mice, which leads to few cats and few mice, which leads to few cats and many mice, which leads to many cats and many mice once again. In a model we assume that a mice-population at 7 and 2 will make the cat-population change with 7-5 and 2-5 respectively.

Also, a cat-population at 7 and 2 will make the mice-population change with 5-7 and 5-2 respectively. We see that initial populations at level 5 will give a stable model. Here we assume that the initial populations for cats and mice are 8 and 1 respectively. The following period the two populations will then be 8 + (1-5) = 4, and 1 + (5-4) = 2.

Continuing, we see that the cat population will change as 8, 4, 1, 2, 6, 9, 8; and the mice population will change as 1, 2, 6, 9, 8, 4, 1. This allows the points (8,1), (4,2), etc., to be marked on a BBBoard, showing a cycle continuing again and again. Different initial numbers will give different cycles.

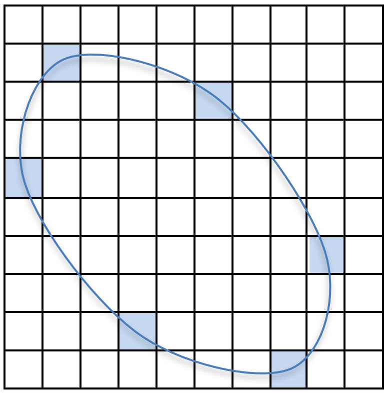


Figure 30. Cats eat mice, so the mice-number decreases, so the cat-number decreases, so the mice-number increases, so the cat number increases, so the mice-number decreases, etc.

• We now model cyclic movements up and down as observed in nature with day and night, with summer and winter, and with tide in an ocean. A cyclic movement may be created by the up-numbers +2, +1, +0, -1, -2, -2, -1, +0, +1 +2.

Beginning at the point (0,5), AI may be prompted to give a formula for this curve as

$y = 5+3*\sin(0.63*x)$.

• Saving money may take place at home or in a bank. At home the terminal capital c after n months will be $c = b + a*n$, where b is the initial capital, and a is the change-number per month. In a bank, the terminal capital after n months will be $c = b*(1+r)$^n where r is the change-percent per month. Combining the two in a bank, the terminal capital C may be found by the formula $C/a = R/r$ where R is the total interest rate including the compound interest, $1+R = (1+r)$^n. This capital may be used as an installment plan to pay out a debt D that has grown to $E = D*(1+R)$ in the same period.

• If an interest rate at 100% is split in 12 portions the total interest is found from the equation $1+R$ = (1+1/12)^12 = 2.613 so that R = 1.613 = 161.3% = 100% plus 61.3% as additional compound interest. This leads to the Euler number $e = (1+1/n)$^n = 2.7183 for n large, which shows that the additional compound interest cannot surpass 71.8% when splitting up 100%.

• Biological populations typically grow exponentially with a constant periodical rate giving a constant doubling time. This may be shown on a BBBoard where the vertical numbers are in tens. Beginning with ¼, a doubling sequence will be ¼, ½, 1, 2, 4, 8, 16, 32, 64, 128.

Alternatively, a capital may decrease by always taking away half of what is left. Beginning with 8 this gives a halving sequence 8, 4, 2, 1, ½, ¼ .

This exponential decay may be recognized as a mirror of the exponential growth.

• On a BBBoard showing 6 7s, a triangle is formed by the three lines connecting the points (0,0) and (7,10) and (10,7). Typically, we want to find the 7 important triangle numbers, its area, its three angels and its three sides.

We see that these 7 numbers may be found indirectly by looking at the three half rectangles that is pulled away from the triangle's wrapping rectangle.

In the lower pull-away half-rectangle the angle is predicted by the formula tan $A = 6/10$, which on a calculator gives $A = 31.0$ degrees. And the area is ½*6*10 = 30. And that the diagonal d is found by squaring: d^2 = 10^2 + 6^2 = 136, giving $d = \sqrt{136} = 11.7$.

• On a BBBoard, twice rolling 2 dices may suggest we go to the two points (3,6) and (4,3) that then constitute one side in a square. We now may find the area of the square and the intersection point of the two diagonals. We notice that in the slopes of the sides the out- and up-number change places, and one changes the sign also.

• Optimizing income under constraints (also called 'Linear Programming'). At a fair, a class sells caps and shirts. They may buy at most 6 boxes with caps and 4 boxes with shirts that each cost 1unit. Their budget is 8units, and their income is 1unit per shirt-box and 2 units per cap-box. How can they optimize the income?

On a BBBoard a horizontal and a vertical rubber band shows the limit on the shirts and on the caps. A line connecting (0,8) and (8,0) shows the budget-line not to be passed. A line connecting (0,10) and (5,0) shows the 10unit income-line that is moved to the right until (6,2) where the first constraint will be violated. So the class should buy 6 boxes with caps and 2 boxes with shirts, which will give them an income at 2*6 + 1*2 or 14 units.

Three footnotes

The total here exists as rectangular bundle-number on a BBBoard.

The goal is to experience the content of three calculation laws.

The commutative law: The order does not matter, $a*b = b*a$

The distributive law: When adding, like units may be bracket out, $a*c + b*c = (a + b)*c$

The associative law: Bracket may be moved at will, $a*(b*c) = (a*b)*c$

On a BBBoard two rubber bands mark 6 3s. Turning the board a quarter round we have 3 6s thus illustrating that 6*3 = 3*6.

A third rubber band split the 6 3s in 4 3s and 2 3s to illustrate that 4*3 + 2*3 = (4+2)*3.

With cubes 2 3s 4 times gives a Total of (2*3)*4. Turning it over, twice we have 3 4s, thus illustrating that (2*3)*4 = 2*(3*4).

Teacher education

The MATHeCADEMY.net is designed to provide material for pre- and in-service teacher education using PYRAMIDeDUCATION allowing professional development to take place on the internet in self-controlling groups with eight participants validating internal predicates by asking the outside subject itself instead of an instructor. This allows mastery of Many with ManyMath to be tested and developed worldwide in small scale design studies ready to be enlarged.

The MATHeCADEMY.net offers a free one-year in-service distance education course in the CATS approach to mathematics, Count & Add in Time & Space. C1, A1, T1 and S1 is for the primary school, and C2, A2, T2 and S2 is for the secondary school. Furthermore, there is a study unit in quantitative literature. The course is organized as PYRAMIDeDUCATION where 8 teachers form 2 teams of 4 choosing 3 pairs and 2 instructors by turn. An external coach helps the instructors instructing the rest of their team.

Each pair works together to solve count&add problems and routine problems; and to carry out an educational task to be reported in an essay rich on observations of examples of cognition, both re-cognition and new cognition, i.e., both assimilation and accommodation.

The coach assists the instructors in correcting the count&add assignments. In a pair, each teacher corrects the other's routine-assignment. Each pair is the opponent on the essay of another pair. Each teacher pays for the education by coaching a new group of 8 teachers.

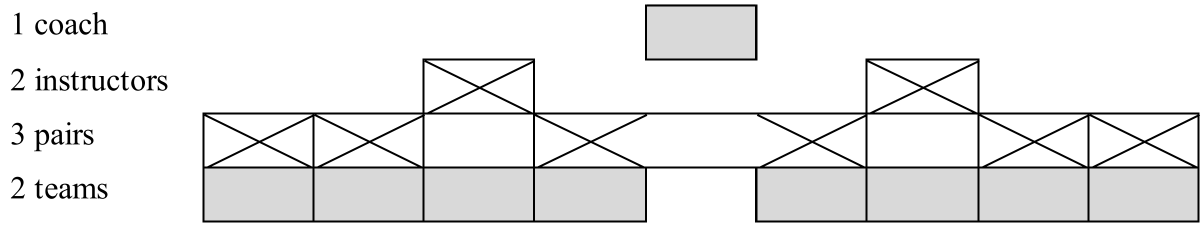


Figure 31. PYRAMIDeDUCATION with 2 teams as 3 pairs and 2 instructors, plus a coach

The material for primary and secondary school has a short question-and-answer format. The question could be: How to count Many? How to recount 8 in 3s? How to count in standard bundles?

The corresponding answers would be: By bundling and stacking the total T predicted by T = (T/B)*B. So, T = 8= (8/3)*3 = 2*3 + 2 = 2*3 + 2/3*3 = 2 2/3*3 = 2B2 3s.

Bundling bundles gives a multiple stack, a stock or polynomial: T = 456 = 4BundleBundle + 5Bundle + 6 = 4tenten5ten6 = 4*B^2 + 5*B + 6*1.

Additional material may be found as MrAlTarp YouTube videos.

How different is the difference?

Digits now are no longer symbols as letters, but icons with as many sticks as they represent. 3 now is called '1B0 3s' or '0B3 tens'. Ten, eleven end twelve now are also called 'one-bundle-zero', 'one-bundle-one', and 'one-bundle two'. And hundred and thousand are also called 'bundle-bundle' and 'bundle-bundle-bundle'.

Multidigit numbers no longer occur without units since with units, 23 now is 2B3 thus making the place value system unneeded. Calculations with overloads and underloads give bundle-numbers with units a flexibility that makes carrying and borrowing unneeded, e.g., 46+37 = 4B6+3B7 = 7B13 = 8B3 = 83. And 86 – 37 = 8B6 – 3B7 = 5B-1 = 4B9 = 49.

Addition now depends on the units so 2+3 is not 5 by necessity. 2weeks + 3weeks = 5weeks, but 2weeks + 3days = 17days. So, without a unit, 3 does not exist, only with a bundle-unit as, e.g., 0B3 tens, or 1B0 3s, or 1B1 2s, or 1B-1 4s, or 1B-2 5s, etc. So, to add, 2 and 3 must have the same unit, e.g., '2+3' = (1B0 + 1B1) 2s = 2B1 2s, or '2+3' = (1B-1 + 1B0) 3s = 2B-1 3s = 1B2 3s. Likewise with subtraction '9-6' = (1B3 -1B0) 6s = 0B3 6s = '3', or '9-6' = (1B0 -1B-3) 9s = 0B--3 9s = 0B3 9s, showing that minus times minus must be plus.

Also, addition now is not well-defined since 2 3s and 4 5s may be added both on-top after a recounting has made the units like, or next-to by areas as integral calculus.

Multiplication now carries units automatically, and 6*8 is not 48 by necessity. Instead, 6*8 exists as 6 8s that may or may not be recounted in another unit, e.g., in 9s or tens: 6 8s is 5B3 9s, or 4B8 tens.

Division now is different, since 8/2 has different meanings in time and space by meaning '8 split in 2 in time', but '8 split in 2s in space' when recounting 8 in 2s.

Solving equations now is different. The equation 'u*2 = 8' asks "How many 2s in 8?" which of course is found by recounting 8 in 2s as 8 = (8/2)*2, so that the solution is u = 8/2 that is found by 'moving to opposite side with opposite sign', which follows the formal definition: 8/2 is the number u that multiplied with 2 gives 8, if u*2 = 8 then u = 8/2. So, the balancing method now is unneeded.

Thus, no longer equations are seen as two equivalent numbers-names that remain equivalent if the same operation is performed on both. And no longer are they transformed by using the communicative, associative, and distributive law; or the two abstract concepts, 1 over 2 as the inverse element to 2, and 0 and 1 as the neutral elements.

And we no longer use the neutralizing 'do the same to both sides' weight-method to solve the equation 2*x = 8 saying: 2*x = 8; (2*x)*½ = 8*½; (x*2)*½ = 4; x*(2*½) = 4; x*1 = 4; x = 4

The multiple calculation 2+3*4 no longer is 14 by definition or by the 'PEMDAS' rule. With units, 2+3*4 exits as 2 1s + 3 4s which is (0B2 + 3B0) 4s or 3B2 4s, or 1B4 tens.

The letter-calculation '2*a + 3*a = (2+3)*a' no longer is an example of a distributive law, but an example of having like units.

Proportionality no longer 'go over one', instead a per-number links the two units by recounting: with 4$ per 5kg, or 4$/5kg, 16$ = (16/4)*4$ = (16/4)*5kg = 20 kg.

Fractions are not numbers by themselves, instead they are per-number with like units, 3meter/4meter = ¾, 3 meter/100meter = 3/100 = 3%. So finally, per-numbers are accepted along with fractions.

Without units, digits, per-numbers, and fractions are not numbers, but operators needing a number to become a number. So, fractions also need units to add: 1 red of two apples plus 2 red of 3 apples total (1+2) red of (3+4) apples, i.e., ½ + 2/3 = (1+2)/(2+3) = 3/5 in this case, and not 7 red of 6 apples as mathematism teaches.

Trigonometry no longer must wait to after plane and coordinate geometry, since it occurs when mutually recounting the sides in a stack split by its diagonal.

Differential calculus no longer precedes integral calculus since the latter answers the core questions: how to add stacks in grade one, and how to add piecewise and locally constant per-numbers in mixture problems in middle school and high school.

Solving a quadratic equation no longer must wait to secondary school since Bundle-Bundles are squares that lead directly to the question 'how to square a rectangle' that provides a double split square containing the three parts of a quadratic equation.

The simplicity of the Algebra Square will no longer be hidden. And no longer will models be seen as mere approximations but as tales with three genres, fact, and fiction, and fake.

Overview of the differences between Essence- math and Existence-math

	Essence-math, mathematism	**Existence-math, Many-math**
Digits	Symbols	Icons
345	Place value system	T = 3BB 4B 5, BB = B^2, BBB = B^3
Operations	Functions, order: + – x / ^	Icons, opposite order: ^ / x – +
3 + 4	3 + 4 = 7	Meaningless without units
3 * 4	3 * 4 = 12	3*4 = 3 4s, may be recounted to 1.2 tens
9 = ? 2s	Meaningless, only ten-counting	9 = 3B3 = 5B-2 = 4B1 = 4½ 2s
8 = ? 2s	Meaningless, only ten-counting	8 = (8/2)*2, T = (T/B)*B, proportionality
2*u = 8	2*u = 8 so, (2*u)*½ = 8*½ so, (u*2)*½ = 4 so, u*(2*½) = 4 so, u*1 = 4 so, u = 4	2*u = 8 = (8/2)*2 so, u = 8/2
6*7 = ?	eh 44? eh 52? eh 42? OK	6*7 = (B-4)*(B-3) = (10-4-3)*B + 4*3 = 3B12 = 4B2 = 42 6*7 = 6 * ½B 2 = 3B 12 = 4B2 = 42

4kg = 5$, 6kg = ?	1kg = 5/4$, 6kg = 6*5/4$	6kg = (6/4)*4kg = (6/4)*5$
1/2 + 2/3 = ?	1/2 + 2/3 = 3/6 + 4/6 = 7/6	1/2*2 + 2/3*3 = 3/5*5
2*3 + 4*5	2*3+4*5 = 6+4*5 = 10*5 ?	2*3+4*5 = 3B2 8s (next-to) or 5B1 5s, or 8B2 3s (on-top)
6 + 9 = ?	6 + 9 = 15 5 – 6	2B3 6s or 2B-3 9s or 2 ½B(1+4) = 1B5 tens
Tangent = ?	tan = sine/cosine	raise = (raise/run)*run, tan = raise/run

Reactions to a BBM BundleBundle Math Curriculum

At the 9th ICMI-East Asia Regional Conference on Mathematics Education (EARCOME 9) in 2025 in Korea, the Special Sharing Groups accepted my paper "Can a Decolonized Mathematics Secure Numeracy for All?" with the following announcement:

> This proposal tackles an urgently needed conversation in mathematics education by challenging deeply ingrained assumptions about number systems and arithmetic instruction and proposing a truly decolonized approach that foregrounds learners' intuitive "bundle-number" language. Its strength lies in weaving together a compelling theoretical critique—drawing on Habermas's colonization concept and rich philosophical underpinnings—with concrete instructional innovations like the Algebra Square that reframe operations as intuitive spatial and bundling processes. By aligning this reconceptualization directly with SDG 4's numeracy targets and illustrating how multiplication-centered reasoning better reflects real-world number use, the paper promises to make a bold and impactful contribution to both research and practice. We look forward to seeing how this work can reshape numeracy instruction and foster truly inclusive mathematical literacy for all.

In Topic Study Groups, reviewers asked for empirical studies where existence- and essence-math had been compared. My answer was that testing both at the same time is ethically problematic:

> Grade one class A has an essence-math curriculum with unit-free 1D line-numbers and the traditional order: addition, subtraction, multiplication, division, power. And only few will hear about calculus. And class B has an existence-math curriculum with 2D Bundle-numbers with units and the opposite order. And, since counting and recounting precedes adding, they will not meet addition until they meet the core of math directly, calculus and linearity. What will happen if a class B student changes to class A, or the other way around? In short, how ethical is it to test essence-math against existence-math in an ordinary school? Or in special education where the students will return far ahead to normal education? Ethical testing is for teacher education and home education.

Grand Theory looks at Mathematics Education

Apparently, two different definitions of 'numerate' exist where existence and essence have different order. The English Oxford Dictionary uses the predicate 'mathematics' when defining 'numerate' as being "competent in the basic principles of mathematics, esp. arithmetic". In contrast, the American Merriam-Webster dictionary uses the verb 'number' when defining 'numerate' as "having the ability to understand and work with numbers."

To understand this difference and to enlighten and discuss the core of education formulated as 'teach learners something', we now consult the three grand theories, philosophy and sociology and psychology. Philosophy may be able to illuminate the different nature of predicates and verbs and to discuss the 'something'. Sociology may be able to illuminate the different inter-human power effects coming from using predicates instead of verbs and in the textbook-teacher-learner interaction. And psychology may be able to illuminate the different learning results coming from listening to predicates or practicing verbs.

Within philosophy, a discussion between existentialism (Sartre 2007) and essentialism about the precedence of existence or essence has taken place since philosophy began in ancient Greece (Russell 1945). Here the 'knowing' sophists argued that to practice democracy we must tell nature from choice to avoid being patronized by choice masked as nature. In other words, we must be able to tell outside existence from its many chosen inside essences and especially the ones that have been institutionalized as the tradition and thus colonizes its outside existence (Habermas, 1981). Against this, the 'better knowing' philo-sophists argued that choice is nonexistent since everything physical is only imperfect examples of metaphysical essence as illustrated in Plato's Cave allegory, and that essence is only accessible to philosophers educated at the Plato Academy.

This disagreement between sophists and philosophers about nature versus choice, and existence versus essence, runs through history. Medieval times saw a controversy between the realists and the nominalists as to whether a name is naming something or a mere sound. In the late Renaissance, a controversy occurred between Hobbes arguing that their destructive nature forces humans to accept patronization, and Locke arguing, like the sophists, that enlightenment enables humans to practice democracy without any physical or metaphysical patronization. In the counter-enlightenment, Hegel reinstalled a patronizing Spirit expressing itself through art and through the history of different people. This created the foundation of Europe's line-organized office-preparing Bildung schools; and for Marxism and socialism, and for the critical thinking of the Frankfurter School, reviving the ancient sophist-philosopher debate by fiercely debating across the Rhine with the French Enlightenment republic's post-structuralism inspired by Heidegger (1962) who argued that "P is Q" is a statement judging an outside existence, P, with an inside constructed essence-predicate that may be a preconceived prejudice, e.g., gossip (Gerede), and which therefore should be met with skepticism and be deconstructed. With 'essence' coming from Latin 'esse' meaning 'being', Heidegger gave four answers to the question "What is 'is'?" pointing up, down, over, or nowhere: is an example of, is for example, is like, or is period. In mathematics, a function thus may be an example of a subset in a set-product where first-component identity implies second-component identity, or a name for calculation containing both specified and unspecified numbers, or a number-language sentence including a subject and a verb and a predicate as in a word-language sentence, or simply what it is, a stand-by calculation awaiting a number, e.g., '3 * x' in contrast to '3 * 5'.

Psychology (Skemp, 1971) has a controversy within constructivism where Vygotsky (1986) sees education as adapting to the institutionalized essence-regime by building ladders down from it to the learners' proximal learning zones. Piaget (1969) replaces this top-down view with a bottom-up view inspired by American Grounded Theory (Glaser et al, 1967) allowing categories to emerge from concrete experiences. Siding with Piaget, Ausubel says that "The most important single factor influencing learning is what the learner already knows. Ascertain this and teach him accordingly" (Ausubel, 1968, p. vi). Also siding with Piaget, Luhmann (1995) sees the brain as a closed self-referring psychic system inside a world of complexity from which it learns by itself reducing the complexity presented to it by another closed self-referring social system called education with its own reductions that it cannot export. Education thus means disturbing the students' systems with outside complexity or hiding complexity if instead wanting uneducated students not able to reduce this, e.g., hiding that numbers occur differently in time and space. This conflict has sociological consequences. Presented top-down from above as examples of inside abstractions, concepts become hard to learn, which forces many learners to stop learning what is meaningless to them and to accept patronization by those who accept such meaninglessness. In contrast, bottom-up concepts grounded from below in the outside world are natural to learn for children through meeting and acting on the concrete examples that exemplify and root the concepts; and for teenagers since knowing the subject in a sentence creates automatic gossip-like learning.

Within Sociology, reviving the ancient Greek sophist skepticism towards patronization masking choice as nature, the Enlightenment created two democracies, one in North America still having its first republic, and one in France now having its fifth republic. In Germany, Weber (1930) was the first to theorize the increasing social goal-oriented rationalization that disenchants the world and create an 'iron cage' if carried to wide. Later, wanting to establish a third German Enlightenment democracy based upon communicative action and the convincing force of the better argument, Habermas (1981) accepts the role of science in rationalizing society, but warns against its instrumentalism spreading from the system to the life world to colonize it.

North American showed skepticism towards rationalist philosophy by developing American Pragmatism (Menand, 1997) leading to Symbolic Interactionism (Blumer, 1998) and Grounded Theory (Glaser et al, 1967).

Here, Mills (1959) sees imagination as the core of sociology. Bauman (1990) agrees by saying that sociological thinking "renders flexible again the world hitherto oppressive in its apparent fixity; it shows it as a world which could be different from what it is now" (p. 16).

Living together involves deciding upon which tasks to do individually and which to do collectively by creating rational institutions, "in which the end is clearly spelled out, and the actors concentrate their thoughts and efforts on selecting such means to the end as promise to be most effective and economical (p. 79)." However, Bauman warns against a possible 'goal displacement' where "The survival of the organization, however useless it may have become in the light of its original end, becomes the purpose in its own right (p. 84)."

An example of a goal displacement may be institutionalizing a knowledge-regime (Foucault 1995) saying 'The goal of mathematics education is to master mathematics". By this self-reference, such a goal is meaningless. To master math cannot be the goal of math education, but it can be a means to reach the real end goal, to master Many. Which of course should be reached by different means if a goal displacement has changed institutionalized mathematics from a means to a goal itself colonizing any further road to mastery of Many and numeracy.

The debate on patronization beginning in ancient Greece between the philosophers and the sophists is still with us today between socialist top-down critical theorists and skeptical bottom-up existentialist theorists that are inspired by the French post-structuralist thinker Foucault saying

> It seems to me that the real political task in a society such as ours is to criticize the workings of institutions, which appear to be both neutral and independent; to criticize and attack them in such a manner that the political violence which has always exercised itself obscurely through them will be unmasked, so that one can fight against them. (Chomsky & Foucault, 2006, p. 41; also on YouTube)

So, seeing mathematics education as an institution (Freudenthal, 1973), a Weberian viewpoint would ask if the set-concept is a rationalization of Many gone too far thus leaving Many disenchanted and leaving the learners in an iron cage. And a Baumanian viewpoint would suggest that, by monopolizing the road to mastery of Many, university mathematics has created a goal displacement. So, as an institution, mathematics is a means, hence the word 'mathematics' should go from goal descriptions. If not, there is a risk that mathematics education will practice 'the banality of evil' (Arendt, 1963). So, sociology would recommend replacing the truth regime saying 'the goal of teaching math is to learn mastery of math' by the real end goal, mastery of Many, e.g., by uncovering and developing the existing mastery of Many children create through adaptation to Many before school. In short, it may be time to show skepticism towards institutionalized mathematics and use difference research (Tarp, 2018) to look for differences that may make a difference.

We therefore ask the Cinderella question: If mastery of Many is the end goal, essence-math may be a means for some but not for all as indicated by the more than fifty years since the first International Congress on Mathematical Education was held in 1969, but are there different means also, e.g., an existence-math, that will make the prince dance? And if so, what is its content and what kind of mathematics does the prince learn during the dance?

Essence-math sees Many as an example of 1D linear cardinality always able to absorb one more by being built on the assumption that '1+1 = 2'. In contrast, a different existence-math occurs when asking a 3-year-old child "How many years next time?". The answer is four with four fingers shown. But seeing the four fingers together two by two, the child reacts: "That is not four, that is two 2s".

This observation shows that numbers occur both as bundle-numbers in space, 2s, and counting-numbers in time, two, that need to be together since asking, "What is 1+1?" leads to asking back "One what + one what?". And asking, "What is 1s + 1s?" leads to asking back "How many 1s + how many 1s?". And asking, "What is 1 + 1s" leads to asking back "What??"

We therefore designed a different existence-math curriculum built on numbers with bundle-units as 2 3s and 4 5s that children create before school when adapting to nature's multiplicity in space and repetition in time.

Conclusion. In Numeracy education, a Luhmann understanding is essential

The United Nations Sustainable Development Goal 4.6 says "By 2030, ensure that all youth achieve Numeracy". So, numeracy now, but how, and what is it?

We will reach the definition "Numeracy is mathematics without units, where addition folds and multiplication holds". But first we will look at the two official contradictory definitions of numeracy.

The English Oxford Dictionary defines numerate as being "competent in the basic principles of mathematics, esp. arithmetic". In contrast, the American Merriam-Webster dictionary defines numerate as "having the ability to understand and work with numbers."

The word 'competent' is a predicate, a non-action word, I cannot 'competent' something, I can only be judged as competent by someone who is competent. In contrast, 'work' is an action word, a verb, since with my body and mind I can work on something and test the result myself to see if it works.

Also, there is a difference between the words 'mathematics' and 'numbers.' Again, mathematics is a non-action word, I cannot 'mathematics' a thing. In contrast, number' is both a verb and a noun since I can number different degrees of Many to produce numbers for later calculations.

Why this difference? We ask the three Grand Theories.

In Philosophy, Existentialism holds that 'existence precedes essence' to quote Sartre, i.e., that, to prevent inside essence from colonizing it, outside existence must come first.

In Sociology, Weber warns that if over-rationalized, society may turn into an iron cage disenchanting the world. Likewise, Habermas warns against a colonization of the life-world by the system-world. And Bauman warns that institutionalizing one essence as the only means may lead to a goal displacement where the means becomes the goal instead by working, not for, but against reaching the original goal, existence. And via Heidegger's warning against reducing a subject's being-potential by imposed predicates, Foucault warns that, to stay in power to create docile bodies, an essence needs to install a regime which the original existence has no need for by its mere existence.

In Psychology, Vygotsky sees learning as inside adapting to the inside institutionalized essence, where Piaget sees learning as inside adapting to outside existence. Luhmann instead talks about individual psychic systems that reflect and collective social systems than communicate, where all systems are closed and self-creating and self-referring and construct essence as inside reductions of outside complexity. So, as a self-referring social system, mathematics cannot transfer its essence to self-referring psychic systems that construct their own essence when disturbed by outside existence, e.g., when meeting Many leads to Counting & Adding in Time & Space.

So, to answer the question "math before or after numeracy?" we must first see how numerate children are before school. We therefore ask a 3year-old "How many years next time?" The answers is 'four', with four fingers shown. But, held together tow by two, the child says "No, that is not four, that is two 2s."

We, the educated, see only the essence, four. But the un-educated child sees the existence, two 1s bundled as one 2s in space, and two of them when counted in time.

Children thus understand a number as a two-dimensional combination of a vertical counting-number and a horizontal bundle-unit visible and tangible on a ten-by-ten BundleBundleBoard allowing children to represent outside totals with their own numbers with Bundle-units.

And Totals may be split or counted. Splitting a total by pulling a Bundle away and back will restore the Total as shown by a re-uniting formula, T = (T – B) + B. Counting a total by pushing Bundles away and back will restore the Total as shown by a re-counting formula, T = (T/B) x B. Show on fingers, these two core formulas are available to all children learning numeracy. In mathematics education without units, they never occur.

A BBBoard allows children to work with 'Full-matics' using both counting numbers in time and bundle-units in space instead of having their own number-language colonized by 'Half-Matics' using counting-numbers only. And becoming 'Mathema-tism' by claiming that 2+1 = 3 despite a closed and an open V-sign shows that two 1s and one 2s total one 4s, and not three 3s as claimed by the ruling essence-regime, or essence-paradigm. So, the moment mathematics accepts units, its '2+1=3' foundation will collapse. So, maybe the time has come for a Kuhnian paradigm shift in mathematics education where introducing units will create a change from a 'flat-earth math education' to a 'round earth math education'?

Yes, units matter. Therefore, if all youth are to be numerate by 2030 as wished by the United Nations Sustainable Development Goal, then mathematics without units must go to allow numeracy help the children develop their already existing reductions of outside complexity into a number-language used to count and add in time and space in order to create number-tales about the world and to see to which genre they belong, fact or fiction or fake, the same three genres that is used in the word-language.

So, in Numeracy education, a Luhmann understanding is essential. In Math education, it is not.

References

Arendt, H. (1963). *Eichmann in Jerusalem, a report on the banality of evil.* London. Penguin Books.

Ausubel, D. (1968). *Educational psychology. A cognitive view.* New York. Holt, Rinehart & Winston.

Bauman, Z. (1990). *Thinking sociologically.* Oxford, UK. Blackwell.

Blumer, H. (1998). *Symbolic Interactionism.* Berkeley. University of California Press.

Chomksky, N. & Foucault, M. (2006). *The Chomsky-Foucault debate on human nature.* New York. The New Press.

Foucault, M. (1995). *Discipline & punish.* New York. Vintage Books.

Freudenthal, H. (1973). *Mathematics as an educational task.* Dordrecht-Holland. D. Reidel Publishing Company.

Glaser B. G. & Strauss A. L. (1967). *The discovery of grounded theory.* New York. Aldine de Gruyter.

Habermas, J. (1981). *Theory of communicative action.* Boston, Mass. Beacon Press.

Kuhn T.S. (1962). *The structure of scientific revolutions.* Chicago: Univ. of Chicago Press.

Luhmann, N. (1995). *Social Systems.* Stanford Ca. Stanford University Press.

Menand, L. (1997). *Pragmatism.* New York. Vintage Books.

Mills, C. W. (1959). *The sociological imagination.* UK. Oxford University Press.

Piaget, J. (1971). *Science of education of the psychology of the child.* New York. Viking Compass.

Russell, B. (1945). *A history of western philosophy.* New York. A Touchstone Book.

Sartre, J.P. (2007). *Existentialism is a humanism.* CT. Yale University Press.

Sartre, J.P. (2007). *Existentialism is a humanism.* New Haven, CT: Yale University Press.

Skemp, R. R. (1971). *The psychology of learning mathematics.* Penguin Books.

Tarp, A. (2001). Fact, fiction, fiddle - three types of models, in J. F. Matos & W. Blum & K. Houston & S. P. Carreira (Eds.), Modelling and Mathematics Education: ICTMA 9: Applications in Science and Technology. *Proceedings of the 9th International Conference on the Teaching of Mathematical Modelling and Applications* (62-71). Chichester UK: Horwood Publishing.

Tarp, A. (2016). *Difference-Research powering PISA performance: count and recount before you add.* Retrieved at http://mathecademy.net/master-many-recount-before-adding/.
Tarp, A. (2017). *Math competenc(i)es - catholic or protestant?* Retrieved at http.//mathecademy.net/wp-content/uploads/2020/01/MADIF-papers-2000-2020.pdf.
Tarp, A. (2018). Mastering Many by counting, re-counting and double-counting before adding on-top and next-to. *Journal of Math. Education, 11*(1), 103-117. https.//doi.org/10.26711/007577152790023.
Tarp, A. (2018). Good, bad & evil mathematics - tales of totals, numbers & fractions. In Hsieh, F. J. (Ed.), (2018). *Proceedings of the ICMI-East Asia Regional Conference on Mathematics Education, Vol2*, Taipei, Taiwan: EARCOME8, 163-173.
Tarp, A. (2019). *A decolonized curriculum*. Mathecademy.net/a-decolonized-curriculum/
Tarp, A. (2019). *Master many to later master math, an opportunity for an existence-based mathematics using flexible bundle-numbers with units, the canceled curriculum chapter in the ICMI study 24 school mathematics curriculum reforms. challenges, changes and opportunities.* Retrieved at http.//mathecademy.net/appendix-to-curriculum-study-icmi-24/.
Tarp, A. (2020). De-modeling numbers & operations. From inside-inside to outside-inside understanding. *Ho Chi Minh City University of Education Journal of Science 17*(3), 453–466. ISSN. 1859-3100.
Tarp, A. (2021). Teaching mathematics as communication, trigonometry comes before geometry, and probably makes every other boy an excited engineer. *Complexity, Informatics and Cybernetics. IMCIC 2021.* Retrieved at https://www.iiisci.org/journal/PDV/sci/pdfs/ZA653AQ21.pdf.
Tarp, A. (2022). *WOKE-math never forces fixed forms upon flexible totals*. Retrieved at http://mathecademy.net/wokemath/.
Tarp, A. (2023). *BundleBundleMath on a BBBoard*. Retrieved at http.//mathecademy.net/bundlebundlemath-on-a-bbboard/.
Tarp, A. (2024). Decolonizing math by demodeling essence as existence, teach that 1+1 = 1, while 1s+1s = 2s. Third Symposium Proceedings. *New Ways of Teaching and Learning*. Morska, J. & Rogerson, A. (editors). Bologna, Italy. https.//doi.org/10.37626/GA9783959872881.0.57.
Tarp, A. (2025). 1+1 is 1 not 2 say the children in their declaration of independence. *Journal of Mathematics Education, 2025 Special Issue*, forthcoming. Preprint on http.//mathecademy.net/childrens-declaration-of-independence/.
Tarp, A. (2025). Textbooks in a decolonized future where two 1s and one 2s total one 4s, and not three 3s. In B. Pepin, I. Kohanová, & M. B. Langfeldt (Eds.), *Proceedings of the Fifth International Conference on Mathematics Textbook Research and Development* (pp. 397-404). Norwegian University of Science and Technology
Tarp, A. (2025). Math is fun with bundle-numbers on a bundle-bundle-board. In Kwon, O., Kaur, B., Pang, J., Noh, J., Lee, S., Han, S., Yeo, S., & Lim, M. (Eds.). (2025). *Proceedings of the 9th ICMI-East Asia Regional Conference on Mathematics Education (Vol. 1)*. Seoul National University, Siheung Campus, Korea: EARCOME9, 363-392.
Tarp, A. YouTube video (2012). *Deconstructing pre-calculus mathematics.* https://youtu.be/3C39Pzos9DQ.
Tarp, A. YouTube video (2013). *Deconstructing calculus.* https://youtu.be/yNrLk2nYfaY.
Tarp, A. YouTube video (2013). *Deconstructing pre-school mathematics.* https://youtu.be/qgCwVZnALXA.
Tarp, A. YouTube video (2021). *Flexible bundle numbers develop the child's innate mastery of Many*. https://youtu.be/z_FM3Mm5RmE.
Tarp, A. YouTube video (2023). *AI and difference research in mathematics education*. https://youtu.be/4EPqjz8evd4.
Vygotsky, L. (1986). *Thought and language.* Cambridge MA. MIT press.
Weber, M. (1930). *The protestant ethic and the spirit of capitalism.* London UK. Unwin Hyman.
Widdowson, H. G. (1978). *Teaching language as communication.* Oxford, UK: Oxford Univ. Press.

BundleBundleMath Wonders 01-24

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BundleBundleMath Wonder 01. Adding last numbers.

A grade 3 child sat alone with the desk turned ninety degrees to the wall in the back of the class.

Why are you here? I confuse the last numbers 7 and 8 and mostly add them wrong. Try 6 plus 8. OK, I start with 6 and say, 7 or eight, seven or eight, nine, ten, ten-one sorry eleven, thirteen, fourteen, fifteen, no wait I am not sure. Do you use number-sticks? No, we use snap cubes, here is a 6 row, and here an 8 row, so I place them after each other and start counting, but it is so long and after ten I become confused.

Can I show you what we did when I was a child? Yes.

As eight we used two diamond signs from the cards, one below the other. Then we let a finger take a trip around the two while we counted the roads. 1, 2, 3, 4, break, 5, 6, 7, 8. In this way we saw that 8 contains two fours both on the cards and on our hands if we bend the thumbs. OK. And, when counting our fingers, sometimes we said 'Hand and 1' instead of six.

Let me try. 1, 2, 3, 4, 5, 1 Hand 1, 1 Hand 2, 1 Hand 3, 1 Hand 4, 1 Hand 5, or two Hands. Right, and two hands we called one Bundle since we count by bundling in tens: ten, twenty, thirty etc.

But then we don't need the numbers between five and ten? No.

And, to add 6 to 8 we just add 1 Hand 1 to 1 Hand 3, which gives two Hands and 4, or ten and four, fourteen, or 1 Bundle 4, right? Right.

OK, so 1Hand1 plus 1Hand2 is 2Hand3; and 1Hand3 plus 1Hand4 is 2Hand7, so now I can also add the last numbers? Yes.

And when we use snap cubes, we can place the rows on top instead of after each other, so that we can see the two hands as a ten so only the small numbers are counted? Yes.

And we don't even have to begin with five sticks, we just use a black cube for a hand; and when we write it, we can write 6 as 1H1? Yes, we can.

And the black cube could also mean ten or a bundle, so that 1B1 and 1B2 is 2B3, or 23; and so that 2B1 + 3B2 is 5B3 or 53? Yes.

In the break the child stayed at the desk and counted without using the names between five and ten. The class was curious and gathered around to see the child adding 6 to 8 by placing two cubes on-top of four cubes. The class protested "That is not 8 and 6 that is 4 and 2". The child refused: "The black cube is a hand, so the four cubes are 1Hand3, and the two cubes are 1Hand1 that add as four and two hands that is ten, so fourteen, also called 1 Bundle 4 since we count by bundling in tens."

In the next lesson they asked the teacher if also they could use hands when counting and adding, so the child was invited to take the front seat to instruct the teacher to stay there for the rest of the school year.

● Existence before essence makes wonders. As does numeracy before math.

More about BBM-Numeracy Counting and Adding with units in Time and Space, the CATS method, on MrAlTarp YouTube videos, and on MATHeCADEMY dot net, that also offers free online teacher education. #math #numeracy #BundleBundleMath #specialeducation

BundleBundleMath Wonder 02. Numbers as Pictures, Viking Numbers.

A grade 3 child sat alone with the desk turned ninety degrees to the wall in the back of the class.

Why are you here? I confuse the numbers and their names, and they are so difficult to draw. Try to draw the number 5. Here it is, or is it the other way around? There are so many different letters, and now there are numbers also.

Can I show you what we did when I was a child? Yes.

On squared paper you have many rows with squares on top of each other. Yes. Put your pencil in upper left corner of a square and take it down the side while you say ‘one’. Like this? Yes, now while counting ‘one, two’ you take the pencil down and to the right so that there are two strokes in the number 2. Like this? Yes, now let me try. OK.

So next time I want three strokes in number three? Right. So, I go down, right and down? Yes. And with four I add an up-stroke to the right? Yes. With five, let us see, I begin in the upper right corner and go one left, one down, one right, then one down and finally one left, that gives five strokes? Yes. But how about ten, why is there no sign for ten when there is a word for ten, and why is ten one and zero and not zero and one?

Well, let us try to count our five fingers in threes after we have bundled them. Then we have 1 Bundle and 2. Yes, and in fours? Then we have 1 Bundle and 1. And in fives? Then we have one Bundle and zero. Right. Oh, you mean that if we count in fives, then we don’t have five fingers, then we have one bundle and zero unbundled. Right. So, when we count in tens, we don’t have ten fingers but one bundle and zero unbundled. Right.

So, if we continue after ten, we have 1 bundle one, 1 bundle two, one bundle three. But why then say eleven and twelve, and why say thirteen instead of ten three?

The names come from the Vikings that counted ‘nine, ten, one left, two left, three and ten’.

OK, but then sixteen and sixty is very close?

Yes, so instead they counted in twenties called scores, so sixty became 3 scores, and seventy became half-four scores. But half of 4 is 2, and two twenties are only forty, not seventy. Right, but the Vikings saw 70 as half the way from 3 to 4 scores.

In the break the child stayed at the desk to practice Viking counting: Bundle, one, half two, two, half three, three, half four, four, half five, five or hundred. The class was curious and gathered around the child. In the next lesson they asked the teacher if they could also be allowed to try Viking-counting, so the child was invited to take the front seat to instruct the teacher to stay there for the rest of the school year.

● Existence before essence makes wonders. As does numeracy before math.

BundleBundleMath Wonder 03. Addition

A grade 3 child sat alone with the desk turned ninety degrees to the wall in the back of the class.

Why are you here? I cannot add. Try to do this ’87 + 95’. Well, 8+9 is 17 and 7+5 is 12, so the answer is 1712. Are you sure? Yes. What does a calculator say? Let us see, 182, well that is the same if we are allowed to add 7 and 1, are we?

Can I show you what we did when I was a child? Yes. We included the Bundle-units and wrote ‘8B7 + 9B5’ that added gives the same as your answer, but with units, 17B12, or 18B2 since 12 also contains a bundle, 12 = 1B2, and without the unit, 18B2 is 182.

Can I try it? Yes, try ‘64+89’. Well, ’64 + 89’ is really ‘6B4 + 8B9’, and 6+8 is 14 and 4+9 is 13, so the answer is 14B13 or 15B3 or 153 without the unit, and the calculator says the same.

In the break the child stayed at the desk to solve one problem after another. The class was curious and gathered around the child. In the next lesson they asked the teacher if they could also be allowed to use the Bundle method, so the child was invited to take the front seat to instruct the teacher to stay there for the rest of the school year.

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BundleBundleMath Wonder 04. Subtraction.

A grade 3 child sat alone with the desk turned ninety degrees to the wall in the back of the class.

Why are you here? I cannot subtract. Try to do this ’86 – 37’. Well, 8-3 is 5, and 6-7 is -1 since you need 1 to take away 7 from 6, so the answer is 5-1 = 4. Are you sure? Yes. What does a calculator say? Let us see, 49, well, where does the 9 come from?

Can I show you what we did when I was a child? Yes. We included the Bundle-units and wrote ‘8B6 – 3B7’ that subtracted gives the same as your answer, but with units, 5B-1, or 4B9 since 1 Bundle contains ten unbundled, 1B = 10, and 10 – 1 = 9, and without the unit, 4B9 is 49.

Can I try it? Yes, try ‘84 – 56’. Well, ’84 – 56’ is really ‘8B4 – 5B6’, and 8 – 5 is 3, and 4 – 6 is -2, so the answer is 3B-2 or 2B8 or 28 without the unit, and the calculator says the same.

In the break the child stayed at the desk to solve one problem after another. The class was curious and gathered around the child. In the next lesson they asked the teacher if they could also be allowed to use the Bundle method, so the child was invited to take the front seat to instruct the teacher to stay there for the rest of the school year.

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BundleBundleMath Wonder 05. Multiplication.

A grade 3 child sat alone with the desk turned ninety degrees to the wall in the back of the class.

Why are you here? I cannot multiply. Try to do this ’7 x 34’. Well, 7 times 3 is 21, and 7 times 4 is 28, so the answer is 2128. Are you sure? Yes. What does a calculator say? Let us see, 238, well that is the same if we are allowed to add 1 and 2, are we?

Can I show you what we did when I was a child? Yes. We included the Bundle-units and wrote ‘7 x 3B4’ that multiplied gives the same as your answer, but with units, 21B28, or 23B8 since 28 also contains 2 bundles, 28 = 2B8, and without the unit, 23B8 is 238.

Can I try it? Yes, try ‘3 x 89’. Well, ’3x 89’ is really ‘3x 8B9’, and 3x8 is 24 and 3x9 is 27, so the answer is 24B27 or 26B7 or 267 without the unit, and the calculator says the same.

In the break the child stayed at the desk to solve one problem after another. The class was curious and gathered around the child. In the next lesson they asked the teacher if they could also be allowed to use the Bundle method, so the child was invited to take the front seat to instruct the teacher to stay there for the rest of the school year.

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BundleBundleMath Wonder 06. Long multiplication.

A grade 3 child sat alone with the desk turned ninety degrees to the wall in the back of the class.

Why are you here? I cannot do long multiplication. Try to do this ’27 x 34’. Well, 2 times 3 is 6, and 7 times 4 is 28, so the answer is 628. Are you sure? Yes. What does a calculator say? Let us see, 918, so we need 300 less 10, i.e., 290, where do they come from? Try to cross multiply. OK, 2 times 4 plus 7 times 3 is 8, plus 21 gives 29, but where does the zero come from?

Can I show you what we did when I was a child? Yes. We included the Bundle-units and wrote ‘2B7 x 3B4’. Here 2B times 3B gives 6BB, and 2B times 4 gives 8B, and 7 times 3B gives 21B, and 7 times 4 gives 28. With units and finishing the bundling, we get 6BB 29B 28, or 6BB 31B 8, or 9BB 1B 8, or 918 without units.

Can I try it? Yes, try ‘23 x 79’. Well, ’23x 79’ is really ‘2B3x 7B9’, and 2Bx17B is 14BB, and 2Bx9 is 18B, and 3x7B is 21B and 3x9 is 27 Adding with units and finishing the bundling, we get 14BB 39B 27, or 14BB 41B 7, or 18BB 1B 7, or 1817 without units, and the calculator says the same.

In the break the child stayed at the desk to solve one problem after another. The class was curious and gathered around the child. In the next lesson they asked the teacher if they could also be allowed to use the Bundle method, so the child was invited to take the front seat to instruct the teacher to stay there for the rest of the school year.

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BundleBundleMath Wonder 07. Short division.

A grade 3 child sat alone with the desk turned ninety degrees to the wall in the back of the class.

Why are you here? I cannot do division. Try to do this '81 /3 '. Well, first 8 divided by 3 is more than 2 where 2 times 3 is 6, then 81 minus 6 is 75. OK. Then, 7 divided by 3 is 2 where 2 times 3 is 6, and 75 minus 6 is 69. OK. Then, 6 divided by 3 is 2 where 2 times 3 is 6, and 69 minus 6 is 63. OK. And then you carry on until you get the result, 222 and some more. Are you sure? Yes. What does a calculator say? Let us see, 27, but why 7, and where are the rest?

Can I show you what we did when I was a child? Yes. We included the Bundle-units and wrote 81 as 8B1 that we changed to 6B21 because 6 can be divided with 3. And 6B21 divided by 3 is 2B7 or 27 without the unit.

Can I try it? Yes, try '68/4'. Well, 68 is really 6B8 or 4B28, and 4B28 divided by 4 is 1B7 or 17 without the unit, and the calculator says the same. Likewise, 69 is really 6B9 or 4B29 or 4B28 + 1, so 69 divided by 4 is 1B7 plus 1 that still needs to be divided by 4, or 17 ¼ without the unit.

In the break the child stayed at the desk to solve one problem after another. The class was curious and gathered around the child. In the next lesson they asked the teacher if they could also be allowed to use the Bundle method, so the child was invited to take the front seat to instruct the teacher to stay there for the rest of the school year.

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BundleBundleMath Wonder 08. Long division.

A grade 3 child sat alone with the desk turned ninety degrees to the wall in the back of the class.

Why are you here? I cannot do long multiplication. Try to do this '97/25'. Well, 9 divided by 2 is 4 with 1 as the rest, and 7 divided by 5 is 1 with 2 as the rest, so the results is 4+1 with 1+2 as the rest, i.e., 5.3. Are you sure? Yes. What does a calculator say? Let us see, 3.88, well that is really something else.

Can I show you what we did when I was a child? Yes. We saw long division as water going into two connected vessels with the same level, shown by our two fists. First, we used the reunite-formula to split 25 as 25 = (25 – 5) + 5 = 20 + 5. This gives the width 20 to the left fist and the width 5 to the right fist. Then we used Bundle-units to rewrite '97/25' as '8B17 / 2B5' so that the 80 liters goes to the left fist and the 17 liters to the right fist. To find the levels in the two vessels we used the recount-formula to recount the liters in the widths:

Left: 80 = (80/20) * 20 = 4 * 20 = 4 20s

Right: 17 = (17/5) * 5 = 3.4 * 5 = 3.4 5s

We look at the right fist. To get from level 3.4 to level 4 it needs 0.6 5s = 0.6*5 = 3 liters from the left fist, that then will reduce its height with 3 recounted in 20s as 3 = (3/20)*20 = 0.15 20s, so it ends at the level 4 – 0.15 = 3.85. This is close to what the calculator says: 97/25 = 3.88. So, we could stop here.

To get closer can repeat and get 3.89 next time.

Can I try it? Yes, try '74/23'.

Well, I include the Bundle-units and write ‘74/23’ as ‘6B14 / 2B3’ so that 60 liters go to the left fist having 20 as its width and 14 liters go to the right fist having 3 as its width. The levels we find by recounting in 20s and 3s:

Left: 60 = (60/20)*20 = 3*20 = 3 20s

Right: 14 = (14/3)*3 = 14/3 * 3 = 14/3 3s = 4.67 3s

We look at the right fist. To get from level 14/3 to level 3 = 9/3 it needs to give 5/3 5s = 5/3*5 = 25/3 liters to the left fist that will increase its level with 25/3 recounted in 20s as

25/3 = (25/3/20)*20 = 25/60 20s,

so it ends at the level 3 + 25/60 = 180/60 + 25/60 = 205/60 = 3.42.

This is rather close to what the calculator says: 74/23 = 3.22. So, we can stop here or repeat.

In the break the child stayed at the desk to solve one problem after another by pouring water from one fist to the other and back again The class was curious and gathered around the child. In the next lesson they asked the teacher if they could also be allowed to use the Bundle method and the reunite and recount formulas so the child was invited to take the front seat to instruct the teacher to stay there for the rest of the school year.

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BundleBundleMath Wonder 09. Equations Solved: to Opposite Side with Opposite Sign

A grade 3 child sat alone with the desk turned ninety degrees to the wall in the back of the class.

Why are you here? I cannot do solve equations, we only have three tries to fill in the empty square. Try to do this ‘# + 12 = 37’ . Well, here I never knew where to start my counting, but then someone told me that 12 + # = 37 is the same equation, so let me start counting, 13, 14, …, 21, 22, oh out of fingers, so my three guesses will be 14, 15 and 16. What does a calculator say? Let us see, 26, 27, 28, well 3 instead of 2, so I was close.

Can I show you what we did when I was a child? Yes.

We used our left hand to set up to formulas, a ‘reunite formula’ and a ‘recount formula’. Show me your left hand and fix a short pencil to your thumb with rubber bands to have 6 fingers on the hand.

‘6-2’ means “From 6, pull away 2 to split the total”. The rest is 4 = 6 – 2. Now, pull back the two fingers. Then you have 4 + 2 = 6, or (6 – 2) + 2 = 6. This is the ‘reunite formula’ that works for all Totals and Bundles, T = (T – B) + B. It says: “I still have the total if I pull back what I pulled away”. Also, look at what happened: 4 = 6-2 and 4+2 = 6. The number 2 can move to opposite side with opposite sign. This is how we solve equations with addition and subtraction.

‘6/2’ means “From 6, push away 2s to count the total”. The number of times is 3 = 6/2. Now, 3 times push back the 2s. Then you have 3x2 = 6, or (6/2)x2 = 6. This is the ‘recount formula’ that works for all Totals and Bundles, T = (T/B)xB. It says: “Pushing away bundles form a total tells you the number of bundles in the total.” Also, look at what happens here: ‘3 = 6/2 and 3x2 = 6. The number 2 moves to opposite side with opposite sign. This is how we solve equations with multiplication and division.

Now let me try with ‘# + 12 = 37’?

Well, I just move 12 to opposite side with opposite sign and find that

= 37 -12 = 3B 7 – 1B 2 = 2B 5 = 25. That was easy.

Now let me try with ‘# x 3 = 54’.

Again, I just move 3 to opposite side with opposite sign and find that

= 54 /3 = 5B 4 /3 = 3B 24 /3 = 1B 8 = 18. That was also easy. Thank you.

In the break the child stayed at the desk to solve one problem after another by pulling and pushing fingers away and back on a hand where a short pencil was fixed to the thumb. The class was curious and gathered around the child. In the next lesson they asked the teacher if they could also be allowed to use the reunite and recount formulas and moving to opposite side with opposite sign to solve equations, so the child was invited to take the front seat to instruct the teacher to stay there for the rest of the school year.

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BundleBundleMath Wonder 10. Square Roots.

A grade 3 child sat alone with the desk turned ninety degrees to the wall in the back of the class.

Why are you here? I can only find the squares up to 3, then I give up.

Can I show you what we did when I was a child? Yes. We looked at a ten-by-ten square that we called a Bundle-Bundle-Board, a BBBoard. What is a Bundle? Show me a V-sign. V. Here we have 2 1s, until we close it. II. Now we have bundled the 2 1s into a bundle of 1 2s. OK. On a BBBoard we see that 2 2s is a Bundle of Bundles, a BundleBundle, and we see all the BundleBundles are squares. Yes. Also, we see that we get to the next square if we add two bundles and a corner. Yes. And that we get to the square before if we pull away two bundles and add the corner that was pulled away twice. Oh, I see, the 3-square is 9, and if I add 3 twice and 1 then I get to 16 as the 4-square, and if I add 4 twice and 1 then I get to 25 as the 5-square. That was easy, but there are still many squares left.

Nor really, try to go back from 100. OK, the ten-square is 100 and if I pull away 10 twice and add 1 then I get 81, right? Yes, now try to pull away 20 twice from 100. Then I get to 60, but now I must add a 2-square, so with 8 as ten minus 2 I get 10-2-2 tens plus a 2-square, 64. Yes. And with 7 as ten minus 3 I get 10-3-3 tens plus a 3-square, 49. Yes, now we write them in a row:

1, 4, 9, 16, 25, 36 = 2B 16, 4B 9, 6B 4, 8B 1.

Can you see a pattern? Yes, after five the squares are 2, 4, 6, 8 bundles, and they end with the squares of their less-numbers 4, 3, 2, 1, that is great.

And what do you see on a BBBoard? I see that a square has its less-square up in the right corner, beautiful. Do you see more? Yes, I see that a (6+4) square contains a 6-square and a 4 square and 4 6s twice. Right, so can we write (a+b) = a^2 + b^2 + 2*a*b? Yes, the square of a sum is more than the sum of the squares. Right. And I also see that the square of 14 = 1B 4 is 1BB + 2x4B + 4^2 or 1BB 8B 16 or 1BB 9B 6 or 196, so now I can find all the squares up to 20, and also up to 99, and I can even find them on my two longest fingers without a BBBoard, great.

In the break the child stayed at the desk to solve one problem after another by doting on two fingers The class was curious and gathered around the child. In the next lesson they asked the teacher if they could also be allowed to use the Bundle method to find the squares, so the child was invited to take the front seat to instruct the teacher to stay there for the rest of the school year.

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BundleBundleMath Wonder 11. Add and Subtract Squares.

A grade 3 child sat alone with the desk turned ninety degrees to the wall in the back of the class.

Why are you here? I want to add and subtract squares, but we must first learn this in secondary school, so I was told to find it out myself.

Show me what you have found. I use a BundleBundleBoard with two squares, a 6-square and a 4-sqaure and two stacks 4 6s. Yes. Then I push the 4-square down so I can add them next to each other connected by a Bottom-Top line. Yes. Then I replace the two squares with two more stacks so they fill the whole BBBoard except for a small 2-sqaure in the middle. Yes. Now the Bottom-Top line is a diagonal in the stack, and I see that the four diagonals form a square surrounded by four half-stacks, which is the same as two full stacks, that also fill the BBBoard with the two squares. Yes. So the two squares add as the diagonal's square, that is nice isn't it. Yes, it is, its is called the Pythagorean rule in Greece, and the Gougu rule in China, nice, how do you subtract two squares?

Well, to get from one square to another we know that we just add bundles on-top and next-to and a square in the corner. So, I just turn the upper bundles to get a stack the is 6+4 long and 6-4 wide. That was easy.

In the break the child stayed at the desk to form squares of four books, four cards, and four like rectangles drawn on a paper and cut out. Then measuring the length of the sides and the diagonal. Then finding the squares by multiplying on two fingers. And then testing if the diagonal's square was the same at the sum of the sides' squares. Then testing the same on books, the table, a window and the door. The class was curious and gathered around the child. In the next lesson they asked the teacher if they could also be allowed to add and subtract squares, so the child was invited to take the front seat to instruct the teacher to stay there for the rest of the school year.

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BundleBundleMath Wonder 12. Finding the Square Root of a number.

A grade 3 child sat alone with the desk turned ninety degrees to the wall in the back of the class.

Why are you here? I try to find square roots. We now know that two squares add as the square of their Bottom-Top line. The teacher says that the side of a square is called its square root, but we must wait until late in secondary school to learn about it. So, I was told to find it out myself.

Show me what you have found.

On this BundleBundleBoard I add a 5-square and a 3 square. I have measured their Bottom-Top line to around 5.6, but I also like to find it as the square root of the sum of the two squares, 25 + 9 = 34, that is between 5 squared, 25, and 6 squared, 36.

If I add two bundles and a corner, I get from the 5-square to the 6-square, 36. But I only need to get to 34. So, instead of a full bundle, I only need two parts of the bundles that I call 2*5*p, to add 34-25 = 9. First, I forget about the corner.

With 2*5*p = 9, I get 10*p = 9. Moving to opposite side with opposite sign, I get p = 9/10 = 0.9.

My first guess therefore is that the square root of 34 is 5.9. The calculator says 5.83, so I am close.

To get closer I repeat my calculations to fill the corner that now is a 0.9 square. Here, the equation is 2*5* p = 0.9*0.9, or 10 * p = 0.81, or p = 0.81/10 = 0.081.

So, my second guess is that the square root of 34 is 5.9 – 0.081 = 5.819, which is below since I made the corner too big.

Each time, it is the same calculation, so I can use letters instead of numbers. So, next time I will find the square root of a number, N, I just find B as the last number with a lower square. Then instead of a full bundle I just add a part, p, found from the formula 2*p = (N-B^2)/B. In this case, 2*p = (34-25)/5 = 1.8, or p = 1.8/2 = 0.9. So, now we know that the square root of 34 is a little less than 5+0.9 = 5.9.

My uncle sais that I can use the formula 2*p = N/B – B instead. I don't know why, but it works OK, 2*p = 34/5 – 5 = 6.8 – 5 = 1.8.

In the break the child stayed at the desk to find square roots of numbers between 1 and 100 by sliding a white paper over the BundleBundleBoard and putting numbers into the letter-formula. The class was curious and gathered around the child to suggest numbers to find the square root of. In the next lesson they asked the teacher if they could be allowed to find square roots also, so the child was invited to take the front seat to instruct the teacher to stay there for the rest of the school year. The next day the class was asked to find as many decimals in the square root of 2 as they could.

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BundleBundleMath Wonder 13. Squaring Rectangles.

A grade 3 child sat alone with the desk turned ninety degrees to the wall in the back of the class.

Why are you here? I would like to be able to square rectangles. But the teacher says we must wait until late secondary school to learn about it. So, I was told to find it out myself.

Show me what you have found.

On this BundleBundleBoard I have 6 4s with a 4-square below and 2 4s on top. To square it I take half of the top to the side. Now I have a 5-square, but with an empty corner. So, I remove a thin part called 4*p from the side and the top to fill the corner. This gives me the equation 2*4*p = 1, or p = 1/4/2 = 0.125. So, my first guess is that the side is 5 – 0.125 = 4.875. The calculator says that the square root of 6*4 = 24 is 4.899, so I am below since the corner is filled too much.

Therefore, I need to fill the sides by reducing the corner from a 1-square to a 0.875-square. This gives the equation 2*4*p = 1 – 0.875^2 = 0.234 where p = 0.234/4/2 = 0.029. So, my second guess will be 4.875 + 0.029 = 4.904, which is very close to the calculator.

Then I tried to square my desk which is 80 cm and 40. So, on my BBBoard I have a stack of 8 4s if I count in tens. To square it I take half of the top to the side. Now I have a 6-square, but with an empty corner. So, I remove a thin part called 6*p from the side and the top to fill the corner. This gives me the equation 2*4*p = 2*2, or p = 4/4/2 = 0.5. So, my first guess is the side is 6 – 0.5 = 5.5. The calculator says that the square root of 8*4 = 32 is 5.657, so I am below since the corner is filled too much and therefore needs to be reduced from a 4-square to a 1.5-square. This gives the equation 2*4*p = 4 – 1.5^2 = 1.75 where p = 1.75/4/2 = 0.219. So, my second guess will be 5.5+ 0.219 = 5.719, which is a little above the calculator.

Of course, the 8 4s may also be seen as two 4 squares added together by their Bottom-Top up line's square i.e. as 4^2 + 4^2 = 16 + 16 = 32, which is between 5-squared and 6-sqaured. So, I can use my formula for finding a square root, 2*p = N/B – B, that here gives 2*p = 32/5 – 5 = 6.4 – 5 = 1.4, with p = 1.4/2 = 0.7. So, here the first guess is 5 + 0.7 = 5.7 that is too much since I need to fill the corner also. Here 2*4*p = 4 – 1.7^2 = 1.11 with p = 1.11/4/2 = 0.139. So, my second guess is 5.7 – 0.139 = 5.561, which is below since the corner now is too big.

In the break the child stayed at the desk to find square rectangles found on books, windows, doors and floors. The class was curious and gathered around the child to suggest rectangles to be squared. In the next lesson they asked the teacher if they could be allowed to square rectangles, so the child was invited to take the front seat to instruct the teacher to stay there for the rest of the school year.

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BundleBundleMath Wonder 14. Squaring Triangles.

A grade 3 child sat alone with the desk turned ninety degrees to the wall in the back of the class.

Why are you here? I would like to square triangles. But the teacher says we must wait until late secondary school to learn about it. So, I was told to find it out myself.

Show me what you have found.

On this BundleBundleBoard I have 6 4s with 6 as the height and 4 as the base, and a diagonal that creates a triangle. I want to find the biggest square under the diagonal.

From the bottom right corner, I draw bigger squares with the same diagonal line until they meet the other diagonal and create a 2.4 square.

Funny enough, 2.4 is (6*4)/(6+4). So, I wondered: To find the biggest square under the diagonal, can you just recount the stack in half of its outside length? So, I tried again with letters instead of numbers.

I have h bs with h as the height and b as the base, and a diagonal that creates a triangle. I want to find the biggest square under the diagonal. From the bottom right corner, I draw bigger squares with the same diagonal line until they meet the other diagonal and create a u-square.

The other diagonal now is a diagonal in two stacks, a small u (b-u)s stack and the big h bs stack. So the heights have the same number if they are recounted in the bases, in bs and in (b-u)s. So,

u/(b-u) = h/b.

I move to opposite side with opposite sign and get

b*u = h*(b-u) = h*b – h*u.

I move to opposite side with opposite sign and get

b*u + h*u = (b+h)*u = h*b

I move to opposite side with opposite sign and get

u = h*b/(h+b)

So, it works for all diagonal triangles: In a stack, you just divide the product with the sum to find the biggest square under the diagonal.

Let me try with this book. Here the numbers are 235 and 155, so this should give a (235*155)/(235+155)-square or a 93-square. To test, I take a ruler and my BBBoard. Yes, it works.

In the break the child stayed at the desk to test on one more book. Then the table was measured and calculated to find the length of the inside square before cutting it out of paper and placed in the corner. Finally she placed the diagonal with a string. The class was curious, gathered around the child and was amazed to see that the square could be predicted by a simple calculation. In the next lesson they asked the teacher if they could also be allowed to use the Bundle method to square triangles, so the child was invited to take the front seat to instruct the teacher to stay there for the rest of the school year.

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BundleBundleMath Wonder 15. Squaring Circles.

A grade 3 child sat alone with the desk turned ninety degrees to the wall in the back of the class.

Why are you here? I would like to square circles. But the teacher says we must wait until late secondary school to learn about it. So, I was told to find it out myself.

Show me what you have found.

On my BundleBundleBoard I have a circle with 10 as the radius. I can see that the left edge and the circle go together in the beginning before the circle curves away from the line, that therefore is called a tangent to the circle in the starting point.

A calculator says that 'tan(2) = 0.035', so if the radius is 10 then the first part of the circle arc is very close to, 10 times 0.035 which is 0.35.

A full circle is 360 degrees, so a half circle is 180 degrees that recounted in 2s gives 180 = (180/2)*2 degrees = 90*2 degrees. And 90 times 0,35 is 3.14, a number that is called pi. So, with radius r, the length of the full circle arc is 2 times pi times radius.

If we split the circle into small 2-degree strips, we can rearrange them as a rectangle where the base is pi times r and the height is r, which shows how much a circle fills, pi times r times r. This rectangle can then be squared into a 1.77-sqaure.

That is very nice, but do you know what a tangent does?

Yes, the tangent uses the diagonal angle to tell you what the height is if you know the base. We can always recount the height in the base as height = (height/base) * base = tangent (angle) * base

Therefore, tangent (angle) = height/base.

So, the tangent is a per-number telling you how much height there is per base-unit. Once we have the actual base-number, we just multiply with the tangent of the angle to find the height-number.

In the break the child stayed at the desk to draw circles on a paper. Then the circle was changed into a rectangle that had the radius and pi times the radius as the sides. Finally the rectangle was changed into a square drawn inside the circle. The class was curious and gathered around the child to see how a circle was squared. In the next lesson they asked the teacher if they could also be allowed to square circles. "That is not possible", said the teacher, but the class showed what the child had done, so the child was invited to take the front seat to instruct the teacher to stay there for the rest of the school year.

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BundleBundleMath Wonder 16. Squares in STEM and STeN

A grade 3 child sat alone with the desk turned ninety degrees to the wall in the back of the class.

Why are you here? I was asked to do three square tasks for a STEM-day in next week.

Tell me what you are doing.

We are working with a ten fives stack on a BundleBundleBoard, and on paper, and on the floor. We see it as a window that needs a diagonal to not collapse.

Task one is to measure and calculate the length of the diagonal and its angles with the sides.

Task two is to square the 10 5s stack in case that in some rooms the light needs to come through quadratic windows.

Task three is to find both the square and the rectangle with the biggest area under the diagonal to insert colored window parts here.

Task four is to raise the diagonal by pushing a 1-square under it. We pause each time we have an extra ten degrees. So, with tan(10) = 0.176 = 1/place, the first place we pause is at the 1/0.176 = 5.7 mark.

Task five is to find out if lines with length 6 and 7 and 8 all can pass round a corner from a 4- and a 2-corridor. Here they pass the two corridor squares on the way.

In the break the child stayed at the desk to finish the work. The class was curious and tried to gather around to see the tasks. In the next lesson they asked the teacher if they could also be allowed to propose some tasks. The teacher said “Yes, next time we have a STEM day we will all make proposals for tasks to do. And next month we will also have a STeN day where we solve tasks on a marketplace where the sellers and buyers meet and use math differently”. So, this time the child stayed at the desk to finish the STEM tasks.

● Existence before essence makes wonders. As does numeracy before math.

BundleBundleMath Wonder 17. Pouring Water with Quadratic equations.

A grade 3 child sat alone with the desk turned ninety degrees to the wall in the back of the class.

Why are you here? I would like to use and solve equations with squares. But the teacher says we must wait until late secondary school to learn about it. So, I was told to find it out myself.

Show me what you have found.

On my BundleBundleBoard I have an 8 4s cup and an 8 3s cup that both are quadratic at the bottom. Up the sides there are marks for Full and +2.

● First, I pour u units of water into a 4-square glass to the +2 mark:

u = 4*4 + 2*4 = 16 + 8 = 24

● Then, I pour 24 units into the 3-square glass to the u mark and get an equation that I solve by moving to opposite side with opposite sign:

3*3 + u*3 = 24
9 + 3*u = 24
3*u = 24 – 9 = 15
u = 15/3 = 5

So, here the water rises until the +5 mark

● Next, I pour 35 units into a glass with an unknown bottom square, u, up to the +2 mark. This gives me a quadratic equation, u^2 + 2*u = 35, that I solve on my BBBoard arranged with four sections showing that

(u+1)^2 = u^2 + 2*u + 1.
So, with u^2 + 2*u = 35, I can now write
(u+1)^2 = u^2+2*u+1 = 35+1 = 36 = 6^2
u+1 = 6
u = 6 – 1 = 5

So, here the glass needs to be a 5-square glass.

● Finally, I pour 35 units into a glass with an unknown bottom square, u, up to the +2 mark. But now the glass has a thick bottom going up to the 2 mark.

This gives me a quadratic equation, u^2 – 2*u = 35, that I solve on my BBBoard arranged with four sections showing that

(u – 1)^2 = u^2 – 2*u + 1

coming from pulling away the two 2*u stacks from the top and the side and then adding the 1*1 that was pulled away twice.

So, with u^2 – 2*u = 35, I can now write

(u – 1)^2 = u^2 – 2*u + 1 = 35+1 = 36 = 6^2
u – 1 = 6
u = 6 + 1 = 7

So, here the glass needs to be a 7-square glass.

To get a formula I use letters instead of numbers.

So, I pour c units into a glass with an unknown bottom square, u, up to the +2*p mark. This gives me a quadratic equation,

u^2 + 2*p*u = c

that I solve on my BBBoard arranged with four sections showing that

(u+p)^2 = u^2 + 2*p*u + p^2.

So, with u^2 + 2*p*u = c, I can now write

(u+p)^2 = u^2 + 2*u + p^2 = c + p^2
u+p = √(p^2+c)
u = √(p^2+c) – p , which is a formula for the squared glass.

In the first case we had c = 24 and p = 1, so here

c = √(p^2+c) – p = √(1^2 + 24) – 1 = √25 – 1 = 5 – 1 = 4.

In the break the child stayed at the desk to finish the work. The class was curious and gathered around the child. In the next lesson they asked the teacher if they could also be allowed to pour water in different glasses, so the child was invited to take the front seat to instruct the teacher to stay there for the rest of the school year.

● Existence before essence makes wonders. As does numeracy before math.

BundleBundleMath Wonder 18. Multiplication tables on a BundleBundleBoard

A grade 3 child sat alone with the desk turned ninety degrees to the wall in the back of the class.

Why are you here? I cannot do the multiplication tables. I can use my fingers to multiply numbers up to 5, but not more than that. Try to do this '6 x 7'. You want me to add 6 to itself 7 times? Well. I cannot do that. Are you sure? Yes. What does a calculator say? Let us see, 42, so twenty-four, but that is six time four?

Can I show you what we did when I was a child? Yes.

We used this ten-by-ten BundleBundleBoard with two bands after 6 & 7s so that we could both feel and see and recounting 6 7s in tens, which is what multiplying is about, to recount from small bundles to tens. Well, I have never heard that.

● With the first method we count the half-Bundles, ½Bs

6 * 7 = (5 + 1 + 2) * ½B + 2 = 8 * ½B + 2 = 4B 2 = 42

● With the second method we recount 7 as ½B 2

6 * 7 = 6 * ½B 2 = 3B 12 = 4B 2 = 42 where we called 3B 12 an overload since 12 is really 1B 2.

● With the third method we pull-away the outside Bundles

6 * 7= (B – 4)*(B – 3) = BB – 3B – 4B - - 3 * 4

= (10 – 3 – 4)B + 12 = 3B 1B 2 = 4B 2 = 42.

Here we can see that minus times minus is plus since we must add the upper right corner that we pulled away twice.

● With the fourth method we use our fingers where on the left hand, 6 is 1up and 4down, and where on the right finger, 7 is 2up and 3down. Then we add the up-fingers and multiply the down fingers and get:

6 * 7 = (1+2)B (4*3) = 3B 12 = 4B 2 = 42

● With the fifth method we use our fingers where on the left hand, 6 is 1up, and where on the right finger, 7 is 2up. Then we add the up-fingers to five and multiply the up-fingers and this time we use half-bundles, ½B:

6 * 7 = (1+2+5)*½B (1*2) = 8*½B (1*2) = 4B 2 = 42

We can see why on the BundleBundleBoard: We already have the 5 half-bundles and then get one more above and two more to the right, and the two in the corner.

● With the sixth method uses the upper right part of that BundleBundleBoard that are split in four parts. Below to the left we add the sides 1+2 = 3 and up to the right we multiply the sides:

Below: 1+2 = 3, above: 4*3 = 12, so again, 6*7 = 3B 12 = 4B 2 = 42.

In the break the child stayed at the desk to solve one problem after another. The class was curious and gathered around the child to see this new way to multiply . In the next lesson they asked the teacher if they could also be allowed to use a BundleBundleBoard when learning the multiplication tables, so the child was invited to take the front seat to instruct the teacher to stay there for the rest of the school year.

● Existence before essence makes wonders. As does numeracy before math.

BundleBundleMath Wonder 19. Adding on-top with proportional ReCounting.

A grade 3 child sat alone with the desk turned ninety degrees to the wall in the back of the class.

Why are you here? I will tell you why:

The teacher said: "Welcome children, I am your teacher in math which is about the numbers that you can see on this number line and that is built upon the fact that one plus one is two as you can see here. So …"

Showing a V-sign I interrupted him: "Mister teacher, here is one 1s in space, and here is also one 1s. If we count them in time, we can see how many 1s we have by saying 'one, two'. So, we have two 1s. But only until we add them as a bundle. Then we have one 2s, so 1s plus 1s become 2s, but one plus one is still one when we count it, and not two as you say. The thumb is also one 1s. They cannot be counted since they are not the same. But they can be added to one 3s. So, again one plus one is one. Here is another 3s on the other hand. They are the same, so we can count them as two 3s. And we can add them as one 6s. Or, we can split the two 3s into six 1s and see that two times three is six. So, the counting numbers two and three can be multiplied, but they cannot be added.

Therefore, please forget adding your line-numbers without units. Instead, help us adding the bundle-numbers with units we bring to school, as 2 3s and 4 5s, that we can add next-to as eights, or on-top as 3s or 5s as we can see on a peg board. If we add them next-to, we add plates, which my uncle calls integral calculus. And if we add them on-top the units must be changed to the same unit, which my uncle calls linearity or proportionality. He says it is taught the first year at college, but we need it here to keep and develop the bundle-numbers with units we bring to school, instead of being colonized with your line-numbers without units.

We know that you want to bundle in tens, and in ten-tens, and in ten-ten tens, but we like to bundle also in 2s, in 3s, in 4s, in half-tens, etc. We know that you have not been taught this and that the textbook doesn't teach it. But don't worry, we will teach you what we found out in preschool. Or better, instead of you colonizing our ways let us find out together what math may grow from our bundle-numbers with units. My uncle is a philosopher, and he calls it existentialism if we let existence come before essence.

And, when existence comes before essence, we must count the totals before we can add them. We know you say that 8 divided by 2 is 8 split in 2 parts, but to us 8 divided by 2 is 8 counted in 2s. You cannot split 9 in 2 parts, but we can easily count 9 in 2s as 4 bundles and 1 unbundled that becomes a decimal, 9 = 4B1 2s, or a fraction if we count it in 2s also, 9 = 4 ½ 2s. Or, with negative less-numbers we get 5 bundles less 1, 9 = 5B-1 2s.

Look at the fingers on a hand. You only see the essence, five, but we see all the ways the five fingers may exist when counted in 2s: 0B5, 1B3, 2B1, 3B-1, 1BB0B1."

● Existence before essence makes wonders. As does numeracy before math.

BundleBundleMath Wonder 20. Adding next-to with Integral Calculus.

A grade 3 child sat alone with the desk turned ninety degrees to the wall in the back of the class.

Why are you here?

Well, my uncle told me that the key to core mathematics is one simple question: "5 4s and 3 2s add to what?"

Adding next-to as areas brings you directly to integral calculus. And adding on-top after shifting units brings you directly to proportionality that leads on to per-numbers as 2$/5kg when including physical units.

Which again leads to the root of calculus, adding per-numbers as in mixture problems: Adding 2kg at 3$/kg and 4kg at 5$/kg, the unit-numbers 2 and 4 add directly to 6, but the per-numbers 3 and 5 must be multiplied to unit-numbers before adding, thus, adding as areas as integral calculus, becoming differential calculus when reversing the question: “2kg at 3$/kg and 4kg at how many $/kg add to 6kg at 5$/kg”, or “5 4s and how many 2s add to 5 6s?”.

So, alculus occurs in three versions.

Primary school: adding bundle-numbers by their areas.

Middle school: adding piece-wise constant per-numbers as the area under the per-number graph.

High school: adding locally constant per-numbers as the area under the per-number graph.

In the break the child stayed at the desk to solve one problem after another. The class was curious and gathered around the child. In the next lesson they asked the teacher if they could also be allowed to use the Bundle method, so the child was invited to take the front seat to instruct the teacher to stay there for the rest of the school year.

● Existence before essence makes wonders. As does numeracy before math.

BundleBundleMath Wonder 21. ReCounting Goods gives PerNumbers and Fractions

A grade 3 child sat alone with the desk turned ninety degrees to the wall in the back of the class.

Why are you here? I would like to change units with per-numbers since they are all over, dollar per day, meter per second, 5 percent. But the teacher says we must wait until secondary school to learn about what he calls proportional thinking and that there is no per-numbers in mathematics. So, I was told to learn about proportional thinking myself.

Show me what you have found.

The total here exists as a rectangular bundle-number on a BBBoard. Here the ‘T=?’ question is answered by changing the unit in the per-number rectangle.

Recounting a physical total T as 3$ and 5 kg gives a ‘per-number’ 3$/5kg called the price and marked as a 3x5 rectangle on a BBBoard.

The question “20kg = ?$” then is answered by recounting in the per-number:

20kg = (20/5)*5kg = (20/5)*3$ = 12 $.

On a BBBoard the counting sequences now are 5, 10, 15, 20kg, and 3, 6, 9, 12$ since the per-number here has changed from 3/5 to 12/20.

Or we may introduce a new unit to make the digits like:

3$ = (3/5)*5$ = n*5$ = 5n$ with the new n = 3/5. So, 20 kg = 20n$ = 20*3/5$ = 12$.

Alternatively, the units may be recounted:

$ = ($/kg)*kg = (3/5)*20 = 12

Or we may equate the per-numbers:

$/kg = u/20 = 3/5.

Moving to opposite side with opposite sign we then get 5*u = 3*20, or u = 3*20/5 = 12.

Or we may use our left little finger. Above to the left we write 20. Below to the left and right we write 5 and 3. Then we can see the result directly: 20 divided by 5 and multiplied with 3.

With the same unit, per-numbers become fractions, 3$/5$ = 3/5

The total here exists as a rectangular bundle-number on a BBBoard. Here the ‘T=?’ question is answered by changing the unit in the per-number rectangle.

If a whole contains a part, they have the same unit. In this case the per-number becomes a fraction without units. Still, we may use the units 'p' and 'w' for the part and the whole.

To get the fraction 3/5 of 20$ means to get 3p/5w of a 20$ whole. Recounting in the per-number thus gives 20w = (20/5)*5w = (20/5)*3p = 12p, or 12$ of 20$.

To get the fraction 3/5 of 100 means to get 3p/5w of a 100 whole. Recounting in the per-number thus gives 100w = (100/5)*5w = (100/5)*3p = 60p, or 60 of 100, written as 60%.

To ask "20$ is what percentage of 80$" means asking about the fraction 20/80 of 100. Or we may introduce a new unit 80$ = 100% to see that 20$ = (20/80)*80$ = (20/80)*100% = 40%.

In the break the child stayed at the desk to solve one problem after another. The class was curious and gathered around the child. In the next lesson they asked the teacher if they could also be allowed to use the Bundle method, so the child was invited to take the front seat to instruct the teacher to stay there for the rest of the school year.

● Existence before essence makes wonders. As does numeracy before math.

BundleBundleMath Wonder 22. The ReCount Formula and per-numbers are the core of STEM and STeN (economics & Numeracy included)

A grade 3 child sat alone with the desk turned ninety degrees to the wall in the back of the class.

Why are you here? I would like to change units with per-numbers since they are all over, dollar per day, meter per second, 5 percent. But the teacher says we must wait until secondary school to learn about what he calls proportional thinking and that there is no per-numbers in mathematics. So, I was told to learn about proportional thinking myself.

Show me what you have found.

The ReCount Formula and per-numbers are the core of STeN (economics & Numeracy included)

STeN typically contains multiplication formulas about changing units

- $ = ($/hour) * hour = salary * hour
- kg = (kg/cubic-meter) x cubic-meter = density x cubic-meter
- force = (force/square-meter) x square-meter = pressure x square-meter
- meter = (meter/sec) x sec = speed x sec
- energy = (energy/sec) x sec = Watt x sec
- energy = (energy/kg) x kg = heat x kg
- gram = (gram/mole) x mole = molar mass x mole
- D momentum = (D momentum/sec) x sec = force x sec
- D energy = (D energy/ meter) x meter = force x meter = work
- energy/sec = (energy/charge) x (charge/sec) or Watt = Volt x Amp

In the break the child stayed at the desk to solve one problem after another. The class was curious and gathered around the child. In the next lesson they asked the teacher if they could also be allowed to use the Bundle method, so the child was invited to take the front seat to instruct the teacher to stay there for the rest of the school year.

● Existence before essence makes wonders. As does numeracy before math.

BundleBundleMath Wonder 23. Per-numbers add as Areas (Integral Calculus)

A grade 3 child sat alone with the desk turned ninety degrees to the wall in the back of the class.

Why are you here? I would like to change units with per-numbers since they are all over, dollar per day, meter per second, 5 percent. But the teacher says we must wait until secondary school to learn about what he calls proportional thinking and that there is no per-numbers in mathematics. So, I was told to learn about proportional thinking myself.

Show me what you have found.

The total here exists as a rectangular bundle-number on a BBBoard. The ‘T=?’ question is answered by using rubber bands to mark the bundles.

Asking “2kg at 3$/kg and 4 kg at 5$/kg total what?” the unit numbers 2kg and 4kg add directly whereas the per-numbers 3$/kg and 5$/kg first must be multiplied to unit-numbers before adding, thus added as areas, i.e., as integral calculus. Here the per-numbers are piecewise constant, but they may also be locally constant as in the case with a falling object having an increasing meter/second number.

Before adding, fractions must also be multiplied to unit, numbers. So, with apples, 1red of 2 plus 2red of 3 gives 3red of 5, and of course not 7red of 6.

Adding like per-numbers is predicted by power where, e.g., 6% 10 times gives 106%^10 or 179%, i.e., the expected 60% plus additional 19%, and where 6% 20 times gives 321%, i.e., the expected 120% plus additional 201% showing the benefit of pensions.

Looking at my right hand, I see 3 fingers to the left, the Ls, and 2 fingers to the right, the Rs. I bent the two outer fingers. So, 1/3 of the Ls are bent, and ½ of the Rs. Does ‘1/3 of the Ls are bent’ mean that ‘1/3 of the bent are Ls’? No, ½ is. So, in a cross table we cannot go from the per-numbers in one direction to those in the other direction without going through the unit-number table. This is called the Bayes-principle.

In the break the child stayed at the desk to solve one problem after another. The class was curious and gathered around the child. In the next lesson they asked the teacher if they could also be allowed to use the Bundle method, so the child was invited to take the front seat to instruct the teacher to stay there for the rest of the school year.

● Existence before essence makes wonders. As does numeracy before math.

BundleBundleMath Wonder 24. The Algebra Square ReUnites Unlike & Like, Unit- & Per-numbers

A grade 3 child sat alone with the desk turned ninety degrees to the wall in the back of the class.

Why are you here? I would like to change units with per-numbers since they are all over, dollar per day, meter per second, 5 percent. But the teacher says we must wait until secondary school to learn about what he calls proportional thinking and that there is no per-numbers in mathematics. So, I was told to learn about proportional thinking myself.

Show me what you have found.

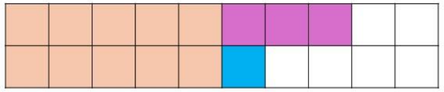
The Arabic word Algebra means to reunite, to unite and split. Numbers are united in four ways: Addition unites unlike unit-numbers. Multiplication unites like unit-numbers. Integration unites unlike per-numbers. Power unites like per-number.

The opposite is to split into. Subtraction splits into unlike unit-numbers. Division splits into like unit-numbers. Differentiation splits into unlike per-numbers. Finally, the factor-finding root and factor-counting logarithm splits into like per-numbers.

In the break the child stayed at the desk to solve one problem after another. The class was curious and gathered around the child. In the next lesson they asked the teacher if they could also be allowed to use the Bundle method, so the child was invited to take the front seat to instruct the teacher to stay there for the rest of the school year.

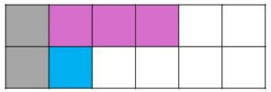
● Existence before essence makes wonders. As does numeracy before math.

BundleBundleMath Wonder 01
Add last Numbers, 8 + 6 = ? = 1H3 + 1H1



8 + 6 = 1H3 + 1H1 = 2H4 = 1B4 = 14

8 – 6 = 1H3 – 1H1 = 3 – 1 = 2



13 + 11 = 1B3 + 1B1 = 2B4 = 24

13 – 11 = 1B3 – 1B1 = 3 – 1 = 2

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BundleBundleMath Wonder 02
Numbers as Pictures & Viking Numbers

Tens:

Bundle	**Score**	**½2**	**2**	**½3**	**3**	**½4**	**4**	**½5**

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BundleBundleMath Wonders 03
Addition

8	7
+ 9	5
17	12

8 **B**	7
+ 9 **B**	5
17 **B**	12
18 **B**	2
18	2

6 **B**	4
8 **B**	9
14 **B**	13
15 **B**	3
15	3

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BundleBundleMath Wonders 04
Subtraction

8	6
– 3	7
5	-1
4	

8 **B**	6
– 3 **B**	7
5 **B**	-1
4 **B**	9
4	9

8 **B**	4
– 5 **B**	6
3 **B**	-2
2 **B**	8
2	8

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BundleBundleMath Wonders 05
Short Multiplication

7 x	
3	4
21	28

7 x	
3 **B**	4
21 **B**	28
23 **B**	8
23	8

3 x	
8 **B**	9
24 **B**	27
26 **B**	7
26	7

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BundleBundleMath Wonders 06
Long Multiplication

2	7
x 3	4
6	28

	2 **B**	7
	x 3 **B**	4
6 **BB**	8+21 **B**	28
6 **BB**	31 **B**	8
9 **BB**	1 **B**	8
9	1	8

	2 **B**	3
	x 7 **B**	9
14 **BB**	18+21 **B**	27
14 **BB**	41 **B**	7
18 **BB**	1 **B**	7
18	1	7

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BundleBundleMath Wonders 07
Short Division

81	/3
8/3	is 2
2x3	is 6
81 – 6	is 75
7/3	2
2x3	is 6
75 – 6	is 69

8**B** 1	/3
6**B** 21	/3
2**B** 7	
27	

6**B** 9	/4
4**B** 29	/4
1**B** 7	+ 1/4
17	1/4

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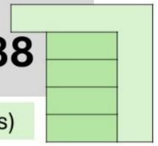
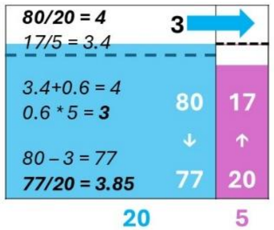
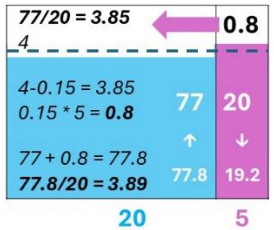
BundleBundleMath Wonders 08
Long Division: $\frac{97}{25} = \frac{9B\,7}{2B\,5} = \frac{8B\,17}{2B\,5} = 3.88$



97/25: 80 + 17 liters Water in two Connected 20 + 5 Vessels (fists)





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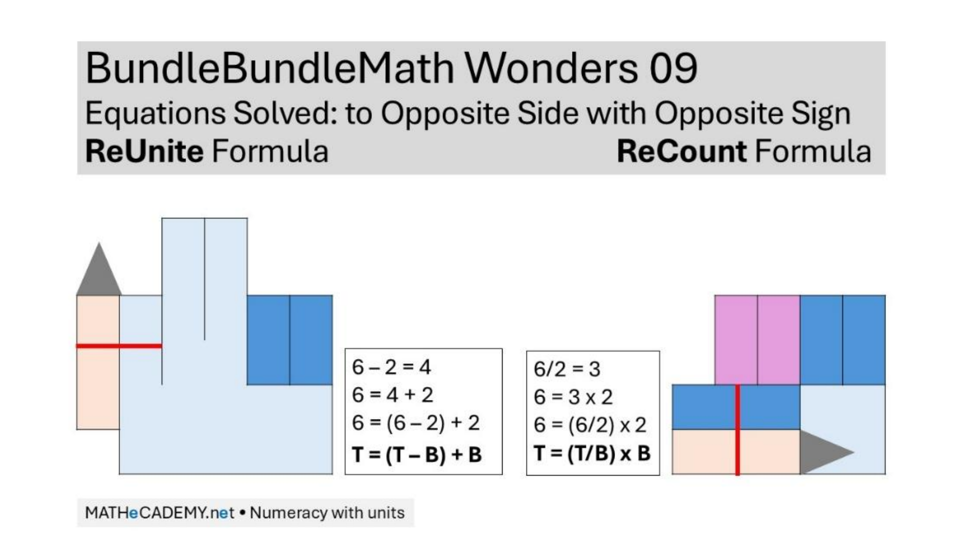
BundleBundleMath Wonders 09
Equations Solved: to Opposite Side with Opposite Sign
ReUnite Formula
ReCount Formula
6 – 2 = 4
6 = 4 + 2
6 = (6 – 2) + 2
T = (T – B) + B
6/2 = 3
6 = 3 x 2
6 = (6/2) x 2
T = (T/B) x B
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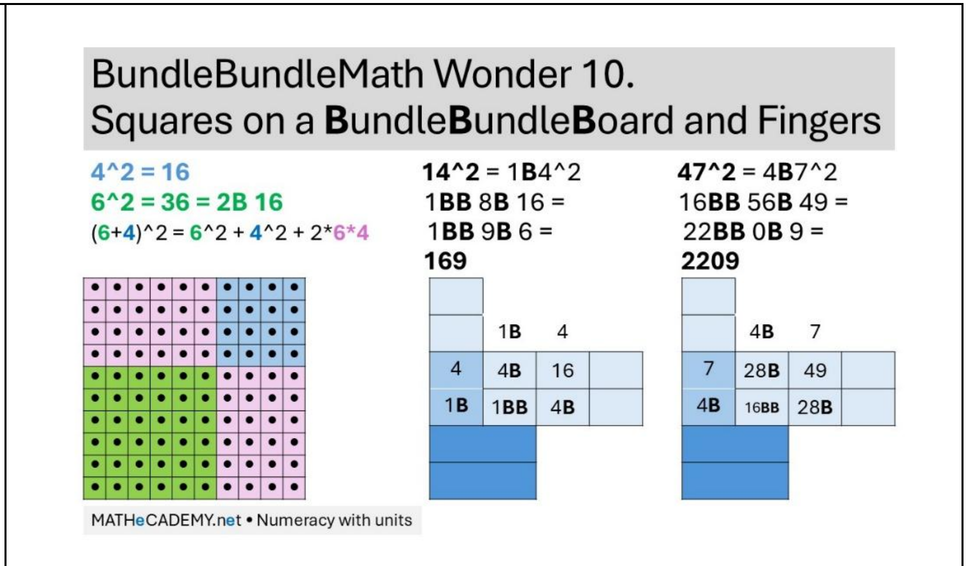
BundleBundleMath Wonder 10.
Squares on a BundleBundleBoard and Fingers
4^2 = 16
6^2 = 36 = 2B 16
(6+4)^2 = 6^2 + 4^2 + 2*6*4
14^2 = 1B4^2
1BB 8B 16 =
1BB 9B 6 =
169
47^2 = 4B7^2
16BB 56B 49 =
22BB 0B 9 =
2209
1B
4
4
4B
16
1B
1BB
4B
4B
7
7
28B
49
4B
16BB
28B
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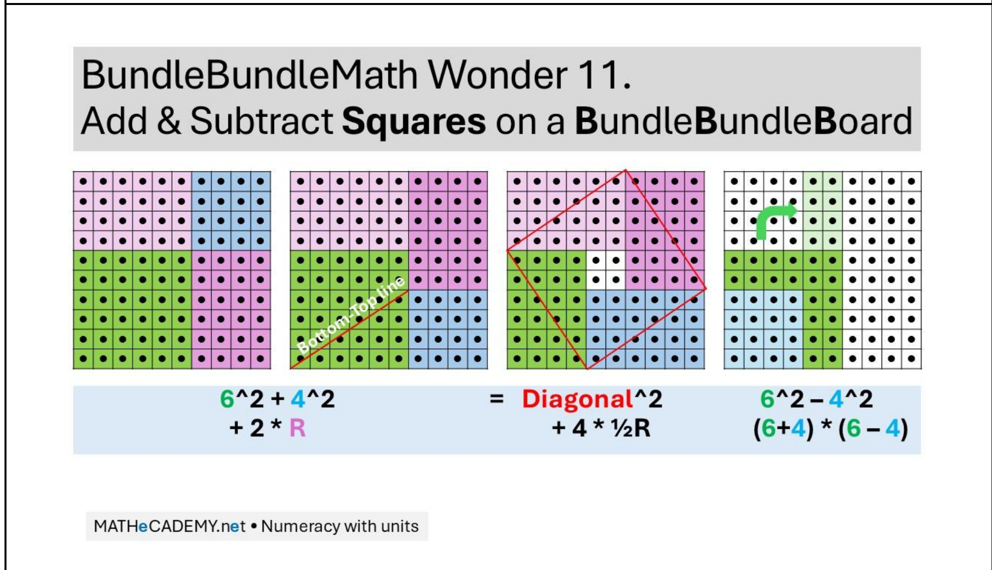
BundleBundleMath Wonder 11.
Add & Subtract Squares on a BundleBundleBoard
Bottom-Top line
6^2 + 4^2
+ 2 * R
= Diagonal^2
+ 4 * ½R
6^2 – 4^2
(6+4) * (6 – 4)
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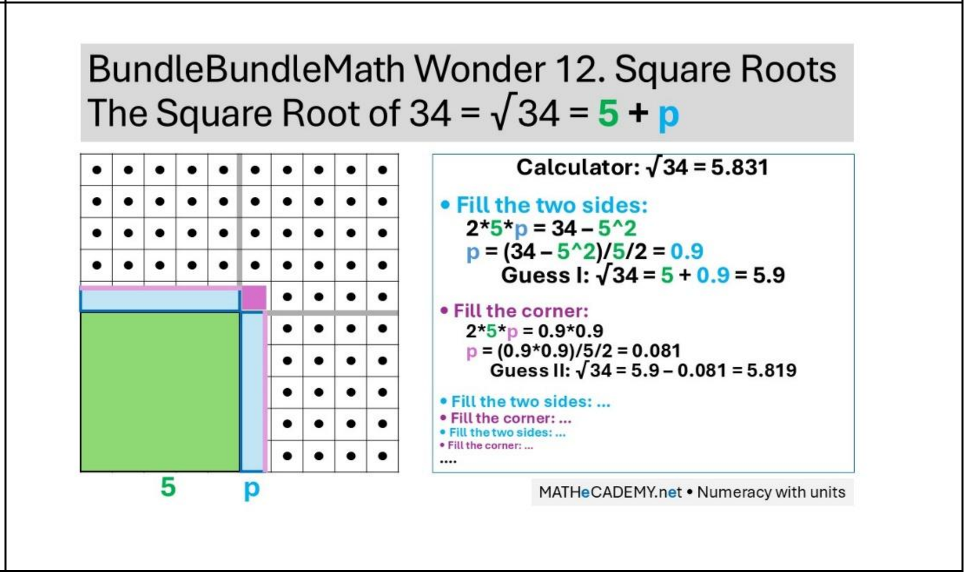
BundleBundleMath Wonder 12. Square Roots
The Square Root of 34 = √34 = 5 + p
5
p
Calculator: √34 = 5.831
• Fill the two sides:
2*5*p = 34 – 5^2
p = (34 – 5^2)/5/2 = 0.9
Guess I: √34 = 5 + 0.9 = 5.9
• Fill the corner:
2*5*p = 0.9*0.9
p = (0.9*0.9)/5/2 = 0.081
Guess II: √34 = 5.9 – 0.081 = 5.819
• Fill the two sides: ...
• Fill the corner: ...
• Fill the two sides: ...
• Fill the corner: ...
....
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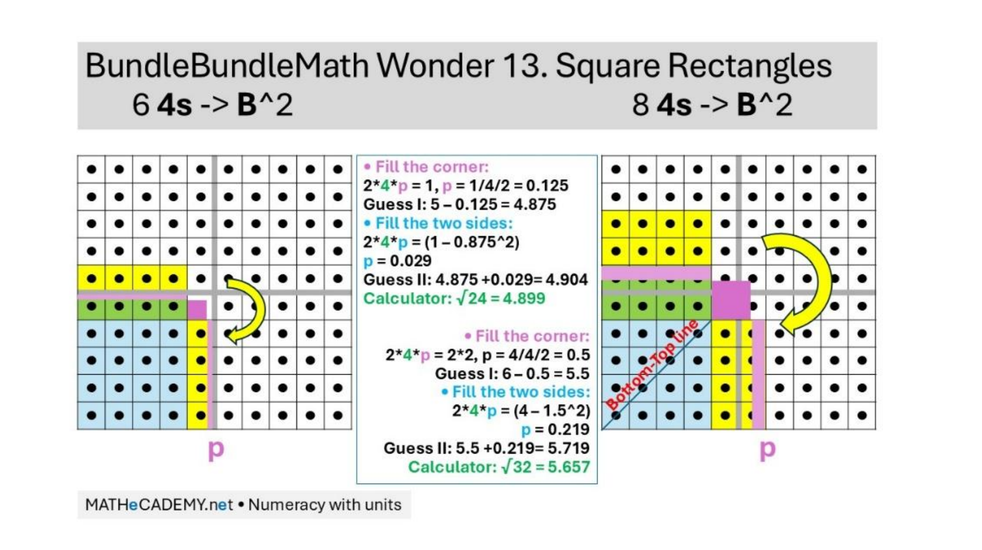
BundleBundleMath Wonder 13. Square Rectangles
6 4s -> B^2
8 4s -> B^2
• Fill the corner:
2*4*p = 1, p = 1/4/2 = 0.125
Guess I: 5 – 0.125 = 4.875
• Fill the two sides:
2*4*p = (1 – 0.875^2)
p = 0.029
Guess II: 4.875 +0.029= 4.904
Calculator: √24 = 4.899
• Fill the corner:
2*4*p = 2*2, p = 4/4/2 = 0.5
Guess I: 6 – 0.5 = 5.5
• Fill the two sides:
2*4*p = (4 – 1.5^2)
p = 0.219
Guess II: 5.5 +0.219= 5.719
Calculator: √32 = 5.657
Bottom-Top line
p
p
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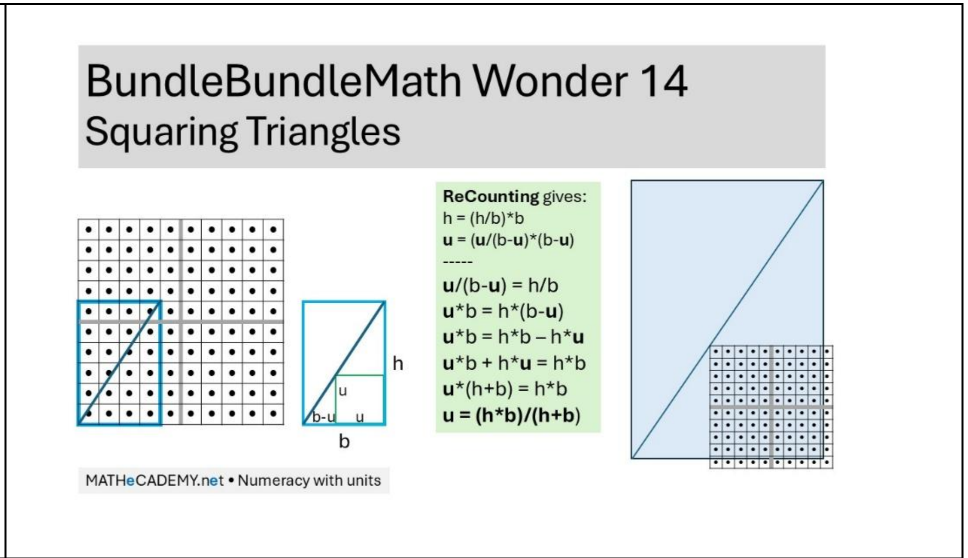
BundleBundleMath Wonder 14
Squaring Triangles
ReCounting gives:
h = (h/b)*b
u = (u/(b-u)*(b-u)
-----
u/(b-u) = h/b
u*b = h*(b-u)
u*b = h*b – h*u
u*b + h*u = h*b
u*(h+b) = h*b
u = (h*b)/(h+b)
h
b
u
b-u
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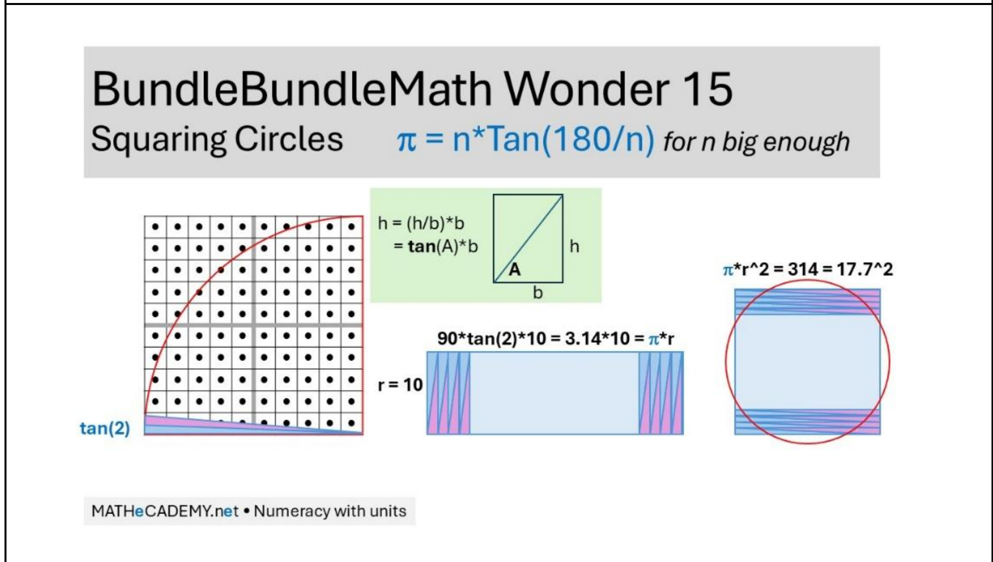
BundleBundleMath Wonder 15
Squaring Circles π = n*Tan(180/n) for n big enough
h = (h/b)*b
= tan(A)*b
A
h
b
tan(2)
r = 10
90*tan(2)*10 = 3.14*10 = π*r
π*r^2 = 314 = 17.7^2
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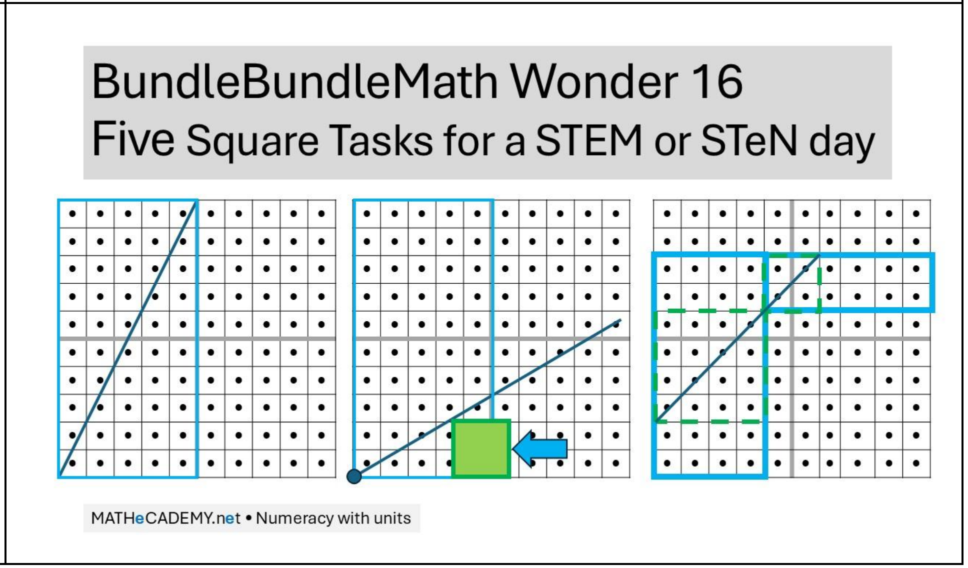
BundleBundleMath Wonder 16
Five Square Tasks for a STEM or STeN day
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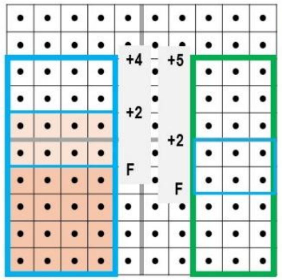
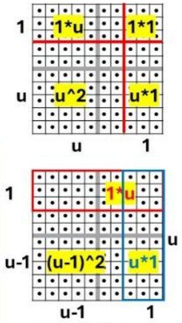
BundleBundleMath Wonder 17
Pouring Water with Quadratic Eqautions



- Pour **u** into a 4-sq. glass to the +2 mark:
  u = 4*4 + 2*4 = 16 + 8 = 24
- Pour 24 into a 3-sq. glass to the **u** mark:
  3*3 + **u***3 = 24
  9 + 3***u** = 24
  3***u** = 24 – 9 = 15
  u = 15/3 = 5, so to the +5 mark
- Pour 35 into a **u**-sq. glass to the +2 mark:
  u***u** + 2***u** = 35
  (**u**+1)^2 = **u**^2+2***u**+1 = 35+1 = 36 = 6^2
  u+1 = 6
  u = 6 – 1 = 5, so into a 5-square glass



- *2*u thick bottom:*
  u***u** - 2***u** = 35
  (**u** – 1)^2 = **u**^2-2***u**+1
  = 35+1 = 36 = 6^2
  u – 1 = 6
  u = 6 + 1 = 7, so into a 7-square glass

BundleBundleMath Wonder 18
Multiplication Table with Early Algebra on a BBBoard

Two bands after 6 & **7s** allow feeling & seeing & recounting 6 **7s** in **tens**.

- <u>Count the half-**B**undles, ½**B**s</u>

6 * 7 = (5 + 1 + 2) * ½**B** + 2 = 8 * ½**B** + 2 = 4**B** 2 = 42

- <u>Recount 7 as ½**B** 2</u>

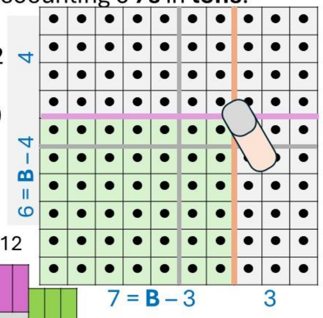
6 * 7 = 6 * ½**B** 2 = 3**B** 12 = 4**B** 2 = 42 *(an overload)*

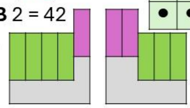
- <u>Pull-away the outside **B**undles</u>

6 * 7= (**B** – 4)*(**B** – 3) = **BB** – 3**B** – 4**B** - - 3 * 4
= (10 – 3 – 4)**B** + 12 = 3**B** 1**B** 2 = 4**B** 2 = 42.

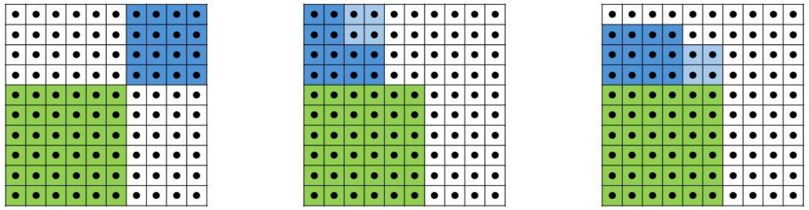
- 6 = 1up, 4down & 7 = 2up, 3down, T = (1+2)**B** (4*3) = 3**B** 12
- 6 = 1up & 7 = 2up, T = (1+2+5)*½**B** (1*2) = 4**B** 2 = 42

The last finger-up method uses half-bundles, ½**B**





BundleBundleMath Wonder 19
OnTop Addition leads to Proportional ReCounting

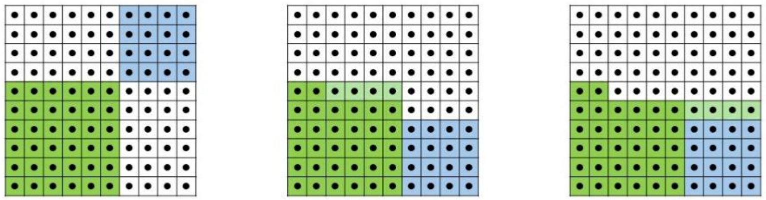


6^2 + 4^2 = **6B 16 6s** = **8B4 = 8 4/6 = 9B-2 6s**

Recounting 4^2 into **6s** as 2**B**4 on a calculator :

4*4/6	2.more
4*4 – 2*6	4

BundleBundleMath Wonder 20
NextTo Addition leads to Integral Calculus



6^2 + 4^2 = **4B 12 10s** = **5B2 = 5 2/10 = 6B-8 10s**

Recounting into **10s** on a calculator :

(6^2+4^2)/10	5.more
6^2+4^2–5*10	2

BundleBundleMath Wonder 21. ReCounting Goods gives PerNumbers and Fractions

A **per-number** 4kg/5$ recounts goods in kg's and dollar's.
ReCounting in the **per-number** changes units (proportionality)

- Question: 20kg = ? $.
- Answer: 20kg = (20/4) * 4kg = (20/4) * 5$ = 25$.

- Question: 20$ = ? kg.
- Answer: 20$ = (20/5) * 5$ = (20/5) * 4kg = 16kg.

Footnote.
With like units, **per-numbers become fractions**: 4$/5$ = 4/5, and 40$/100$ = 40%

Question: 8$ = ?% *with 40$ = 100%,*	**Question: 80% = ?$ *with 40$ = 100%***
Answer: 8$ = (8/40)*40$ = (8/40)*100% = 20%	**Answer: 80% = (80/100)*100% = (80/100)*40$ = 32$**

With PerNumbers, No More **Proportional Reasoning** & **Multiplicative Thinking**

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BundleBundleMath Wonder 22. The ReCount Formula and per-numbers are the core of STEM and STeN
(economics & Numeracy included)

STEM & STeN typically contains multiplication formulas about changing units:

- $ = ($/hour) * hour = salary * hour
- kg = (**kg/cubic-meter**) x cubic-meter = **density** x cubic-meter
- force = (**force/square-meter**) x square-meter = **pressure** x square-meter
- meter = (**meter/sec**) x sec = **speed** x sec
- energy = (**energy/sec**) x sec = **Watt** x sec
- energy = (**energy/kg**) x kg = **heat** x kg
- gram = (**gram/mole**) x mole = **molar mass** x mole
- Δ momentum = (Δ **momentum/sec**) x sec = **force** x sec
- Δ energy = (Δ **energy/ meter**) x meter = **force** x meter = work
- **energy/sec** = (**energy/charge**) x (**charge/sec**) or **Watt** = **Volt** x **Amp**

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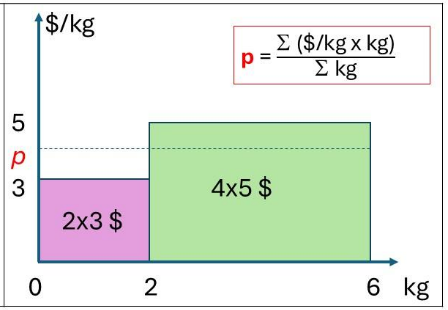
BundleBundleMath Wonder 23. Per-numbers add as Areas (Integral Calculus)

Question: "2kg at **3$/kg** + 4kg at **5$/kg** = 6kg at **? $/kg**?"

2 kg	at	**3 $/kg**
+ 4 kg	at	**5 $/kg**
(2+4) kg	at	***p* $/kg**

- Unit-numbers add directly.
- Per-numbers must be multiplied to unit-numbers, thus adding as **areas** under the per-number curve.
- Here, multiplication before addition



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BundleBundleMath Wonder 24. The **Algebra Square** ReUnites Unlike & Like, Unit- & Per-numbers

The Arabic word Algebra means to reunite, to unite and split. Numbers are united in four ways: **Addition** unites unlike unit-numbers. **Multiplication** unites like unit-numbers. **Integration** unites unlike per-numbers. **Power** unites like per-number.

The opposite is to split into. **Subtraction** splits into unlike unit-numbers. **Division** splits into like unit-numbers. **Differentiation** splits into unlike per-numbers. Finally, the **factor-finding root** and **factor-counting logarithm** splits into like per-numbers.

Unite / Split into	**Unlike**	**Like**
Unit-numbers (meter, second)	**T = a + b** *T – b = a*	**T = a*b** *T/b = a*
Per-numbers (m/sec, m/100m = %)	**T = ∫ f dx** *dT/dx = f*	**T = a^b** $\sqrt[b]{T} = a$ $log_a(T) = b$

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