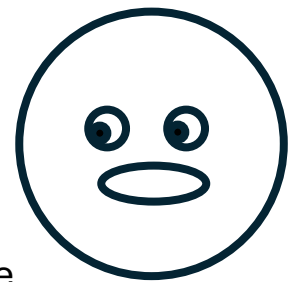


# From **Loser** to **User**

## BundleBundleMath Wonders



Can we Math?  
**No**, we can't!



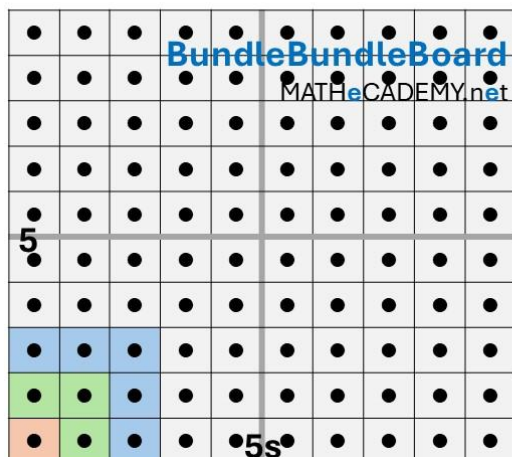
Math is **not** an action word.  
But we can **C**ount & **A**dd in **T**ime & **S**pace.

So, let's learn, not how to **Math**, but how to **CATS**. Then we can **forget** about **place values** and **carrying**. We just **Bundle-count** and **Recount** before we add next-to or on-top with calculus and proportionality. With **one hand** only and a **BundleBundleBoard** we learn **core math**. Remember: **EXISTENCE** before **essence** makes wonders.

### BundleBundleBoard

Front & Back

Allan.Tarp@gmail.com,  
January 2026



**Numeracy before Math** – with a Hand and a BBBoard

OSS, opposite side & sign:  $6 = 3 \times 2$   
 $6 = (6/2) \times 2$

ReCount Formula:  $T = (T/B) \times B$   
 $\$ = (\$/\text{kg}) \times \text{kg} = \text{price} \times \text{kg}$   
 $m = (\text{m/s}) \times s = \text{speed} \times \text{sec}$   
 $8m = (8/2) \times 2m = (8/2) \times 3\$ = 12\$$   
 $\text{up} = (\text{up/out}) \times \text{out} = \text{tanA} \times \text{out}$   
 $\pi = \tan(180/n) \times n, \quad n \text{ big}$   
 $e = (1 + 1/n)^n, \quad n \text{ big}$

STEM  $\rightarrow$  STeN  
 economy  
 Numeracy

6 - 2 = 4  
 6 = 4 + 2  
 6 = (6 - 2) + 2  
 T = (T - B) + B  
 ReUnite Formula

4 B 6  
 1 B 9  
 5 B 15  
 6 B 5  
 6 5

4 B 6  
 1 B 9  
 3 B -3  
 2 B 7  
 2 7

4 B 6 m \$  
 1 B 9 8 ?  
 BB B 2 3  
 4 42 54  $\frac{8}{2} \times 3$  12  
 8 7 4

Integral Calculus adds 1BBB 2s and 1BB 3s next-to as 3B2 5s.  
 $6 \times 7 = 6 \times 7s = 6 \times \frac{1}{2}B 2 = 3B 12 = 4B 2 = 42.$

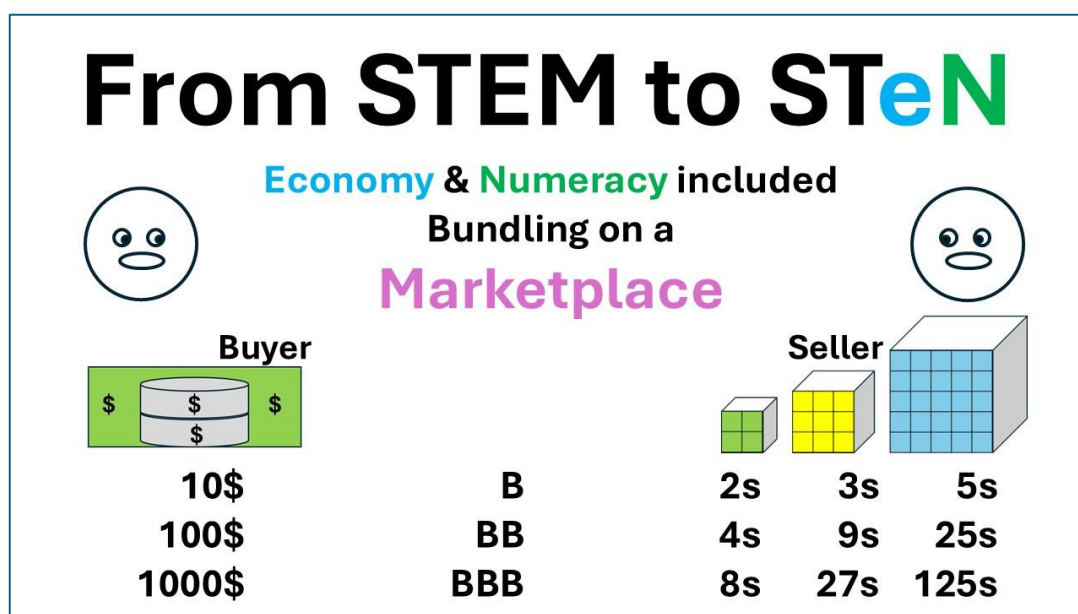
| Unit                                  | Split into | Unlike                         | ALGEBRA SQUARE Like                              |
|---------------------------------------|------------|--------------------------------|--------------------------------------------------|
| Unit-numbers<br>(meter, sec.)         |            | $T = a + b$<br>$T - b = a$     | $T = a \times b$<br>$T/b = a$                    |
| Per-numbers<br>(m/sec,<br>m/100m = %) |            | $T = \int f dx$<br>$dT/dx = f$ | $T = a^b$<br>$b\sqrt{T} = a \quad \log_a(T) = b$ |

MATHeCADEMY.net

January 2026

## Contents

|                                                                                                     |    |
|-----------------------------------------------------------------------------------------------------|----|
| BundleBundleMath Wonder 01. Adding last numbers.....                                                | 1  |
| BundleBundleMath Wonder 02. Numbers as Pictures, Viking Numbers. ....                               | 1  |
| BundleBundleMath Wonder 03. Addition .....                                                          | 2  |
| BundleBundleMath Wonder 04. Subtraction.....                                                        | 2  |
| BundleBundleMath Wonder 05. Multiplication. ....                                                    | 2  |
| BundleBundleMath Wonder 06. Long multiplication.....                                                | 3  |
| BundleBundleMath Wonder 07. Short division.....                                                     | 3  |
| BundleBundleMath Wonder 08. Long division.....                                                      | 4  |
| BundleBundleMath Wonder 09. Equations Solved: to Opposite Side with Opposite Sign.....              | 4  |
| BundleBundleMath Wonder 10. Square Roots. ....                                                      | 5  |
| BundleBundleMath Wonder 11. Add and Subtract Squares. ....                                          | 5  |
| BundleBundleMath Wonder 12. Finding the Square Root of a number.....                                | 6  |
| BundleBundleMath Wonder 13. Squaring Rectangles.....                                                | 7  |
| BundleBundleMath Wonder 14. Squaring Triangles. ....                                                | 7  |
| BundleBundleMath Wonder 15. Squaring Circles. ....                                                  | 8  |
| BundleBundleMath Wonder 16. Squares in STEM and STeN .....                                          | 8  |
| BundleBundleMath Wonder 17. Pouring Water with Quadratic equations. ....                            | 9  |
| BundleBundleMath Wonder 18. Multiplication tables on a BundleBundleBoard.....                       | 10 |
| BundleBundleMath Wonder 19. Adding on-top with proportional ReCounting.....                         | 11 |
| BundleBundleMath Wonder 20. Adding next-to with Integral Calculus. ....                             | 12 |
| BundleBundleMath Wonder 21. ReCounting Goods gives PerNumbers and Fractions .....                   | 12 |
| BundleBundleMath Wonder 22. The ReCount Formula and per-numbers are the core of STEM and STeN ..... | 13 |
| BundleBundleMath Wonder 23. Per-numbers add as Areas (Integral Calculus).....                       | 13 |
| BundleBundleMath Wonder 24. Algebra ReUnites Unlike & Like Unit- & Per-numbers.....                 | 14 |
| Slides to the BundleBundleMath Wonders.....                                                         | 15 |



More about BBM-Numeracy Counting and Adding with units in Time and Space, the CATS method,  
on MrAlTarp YouTube videos,  
on MATHeCADEMY dot net, that also offers free online teacher education  
on [linkedin.com/in/allantarp/](https://www.linkedin.com/in/allantarp/)

*Published on LinkedIn January 2026, Aarhus Denmark*

### **BundleBundleMath Wonder 01. Adding last numbers.**

A grade 3 child sat alone with the desk turned ninety degrees to the wall in the back of the class. Why are you here? I confuse the last numbers 7 and 8 and mostly add them wrong. Try 6 plus 8. OK, I start with 6 and say, 7 or eight, seven or eight, nine, ten, ten-one sorry eleven, thirteen, fourteen, fifteen, no wait I am not sure. Do you use number-sticks? No, we use snap cubes, here is a 6 row, and here an 8 row, so I place them after each other and start counting, but it is so long and after ten I become confused.

Can I show you what we did when I was a child? Yes. As eight we used two diamond signs from the cards, one below the other. Then we let a finger take a trip around the two while we counted the roads. 1, 2, 3, 4, break, 5, 6, 7, 8. In this way we saw that 8 contains two fours both on the cards and on our hands if we bend the thumbs. OK. And, when counting our fingers, sometimes we said 'Hand and 1' instead of six.

Let me try. 1, 2, 3, 4, 5, 1 Hand 1, 1 Hand 2, 1 Hand 3, 1 Hand 4, 1 Hand 5, or two Hands. Right, and two hands we called one Bundle if we count by bundling in tens: ten, twenty, thirty etc.

But then we don't need the numbers between five and ten? No.

And, to add 6 to 8 we just add 1 Hand 1 to 1 Hand 3, which gives two Hands and 4, or ten and four, fourteen, or 1 Bundle 4, right? Right.

OK, so 1Hand1 plus 1Hand2 is 2Hand3; and 1Hand3 plus 1Hand4 is 2Hand7, so now I can also add the last numbers? Yes.

And when we use snap cubes, we can place the rows on top instead of after each other, so that we can see the two hands as a ten so only the small numbers are counted? Yes.

And we don't even have to begin with five sticks, we just use a black cube for a hand; and when we write it, we can write 6 as 1H1? Yes, we can.

And the black cube could also mean ten or a bundle, so that 1B1 and 1B2 is 2B3, or 23; and so that 2B1 + 3B2 is 5B3 or 53? Yes.

In the break the child stayed at the desk and counted without using the names between five and ten. The class was curious and gathered around to see the child adding 6 to 8 by placing two cubes on-top of four cubes, which made the class protest "That is not 8 and 6 that is 4 and 2". The child refused: "The black cube is a hand, so the four cubes are 1Hand3, and the two cubes are 1Hand1 that add as four and two hands that is ten, so fourteen, also called 1 Bundle 4 since we count by bundling in tens."

In the next lesson they asked the teacher if also they could use hands when counting and adding, so the child was invited to take the front seat to instruct the teacher, staying there for the rest of the school year.

● Existence before essence makes wonders. As does numeracy before math.

### **BundleBundleMath Wonder 02. Numbers as Pictures, Viking Numbers.**

A grade 3 child sat alone with the desk turned ninety degrees to the wall in the back of the class. Why are you here? I confuse the numbers and their names, and they are so difficult to draw. Try to draw the number 5. Here it is, or is it the other way around? There are so many different letters, and now there are numbers also.

Can I show you what we did when I was a child? Yes. On squared paper you have many rows with squares on top of each other. Yes. Put your pencil in upper left corner of a square and take it down the side while you say 'one'. Like this? Yes, now while counting 'one, two' you take the pencil down and to the right so that there are two strokes in the number 2. Like this? Yes, now let me try. OK.

So next time I want three strokes in number three? Right. So, I go down, right and down? Yes. And with four I add an up-stroke to the right? Yes. With five, let us see, I begin in the upper right corner and go one left, one down, one right, then one down and finally one left, that gives five strokes? Yes. But how about ten, why is there no sign for ten when there is a word for ten, and why is ten one and zero and not zero and one?

Well, let us try to count our five fingers in threes after we have bundled them. Then we have 1 Bundle and 2. Yes, and in fours? Then we have 1 Bundle and 1. And in fives? Then we have one Bundle and

zero. Right. Oh, you mean that if we count in fives, then we don't have five fingers, then we have one bundle and zero unbundled. Right. So, when we count in tens, we don't have ten fingers but one bundle and zero unbundled. Right.

So, if we continue after ten, we have 1 bundle one, 1 bundle two, one bundle three. But why then say eleven and twelve, and why say thirteen instead of ten three?

The names come from the Vikings that counted 'nine, ten, one left, two left, three and ten'.

OK, but then sixteen and sixty is very close?

Yes, so instead they counted in twenties called scores, so sixty became 3 scores, and seventy became half-four scores. But half of 4 is 2, and two twenties are only forty, not seventy. Right, but the Vikings saw 70 as half the way from 3 to 4 scores.

In the break the child stayed at the desk to practice Viking counting: Bundle, one, half two, two, half three, three, half four, four, half five, five or hundred. The class was curious and gathered around the child. In the next lesson they asked the teacher if they could also be allowed to try Viking-counting, so the child was invited to take the front seat to instruct the teacher, staying there for the rest of the school year.

● Existence before essence makes wonders. As does numeracy before math.

### **BundleBundleMath Wonder 03. Addition**

A grade 3 child sat alone with the desk turned ninety degrees to the wall in the back of the class.

Why are you here? I cannot add. Try to do this ' $87 + 95$ '. Well,  $8+9$  is 17 and  $7+5$  is 12, so the answer is 1712. Are you sure? Yes. What does a calculator say? Let us see, 182, well that is the same if we are allowed to add 7 and 1, are we?

Can I show you what we did when I was a child? Yes. We included the Bundle-units and wrote ' $8B7 + 9B5$ ' that added gives the same as your answer, but with units, 17B12, or 18B2 since 12 also contains a bundle,  $12 = 1B2$ , and without the unit, 18B2 is 182.

Can I try it? Yes, try ' $64+89$ '. Well, ' $64 + 89$ ' is really ' $6B4 + 8B9$ ', and  $6+8$  is 14 and  $4+9$  is 13, so the answer is 14B13 or 15B3 or 153 without the unit, and the calculator says the same.

In the break the child stayed at the desk to solve one problem after another. The class was curious and gathered around the child. In the next lesson they asked the teacher if they could also be allowed to use the Bundle method, so the child was invited to take the front seat to instruct the teacher, staying there for the rest of the school year.

● Existence before essence makes wonders. As does numeracy before math.

### **BundleBundleMath Wonder 04. Subtraction.**

A grade 3 child sat alone with the desk turned ninety degrees to the wall in the back of the class.

Why are you here? I cannot subtract. Try to do this ' $86 - 37$ '. Well,  $8-3$  is 5, and  $6-7$  is -1 since you need 1 to take away 7 from 6, so the answer is  $5-1 = 4$ . Are you sure? Yes. What does a calculator say? Let us see, 49, well, where does the 9 come from?

Can I show you what we did when I was a child? Yes. We included the Bundle-units and wrote ' $8B6 - 3B7$ ' that subtracted gives the same as your answer, but with units, 5B-1, or 4B9 since 1 Bundle contains ten unbundled,  $1B = 10$ , and  $10 - 1 = 9$ , and without the unit, 4B9 is 49.

Can I try it? Yes, try ' $84 - 56$ '. Well, ' $84 - 56$ ' is really ' $8B4 - 5B6$ ', and  $8 - 5$  is 3, and  $4 - 6$  is -2, so the answer is 3B-2 or 2B8 or 28 without the unit, and the calculator says the same.

In the break the child stayed at the desk to solve one problem after another. The class was curious and gathered around the child. In the next lesson they asked the teacher if they could also be allowed to use the Bundle method, so the child was invited to take the front seat to instruct the teacher, staying there for the rest of the school year.

● Existence before essence makes wonders. As does numeracy before math.

### **BundleBundleMath Wonder 05. Multiplication.**

A grade 3 child sat alone with the desk turned ninety degrees to the wall in the back of the class. Why are you here? I cannot multiply. Try to do this ' $7 \times 34$ '. Well, 7 times 3 is 21, and 7 times 4 is 28, so the

answer is 2128. Are you sure? Yes. What does a calculator say? Let us see, 238, well that is the same if we are allowed to add 1 and 2, are we?

Can I show you what we did when I was a child? Yes. We included the Bundle-units and wrote '7 x 3B4' that multiplied gives the same as your answer, but with units, 21B28, or 23B8 since 28 also contains 2 bundles,  $28 = 2B8$ , and without the unit, 23B8 is 238.

Can I try it? Yes, try '3 x 89'. Well, '3x 89' is really '3x 8B9', and  $3x8$  is 24 and  $3x9$  is 27, so the answer is 24B27 or 26B7 or 267 without the unit, and the calculator says the same.

In the break the child stayed at the desk to solve one problem after another. The class was curious and gathered around the child. In the next lesson they asked the teacher if they could also be allowed to use the Bundle method, so the child was invited to take the front seat to instruct the teacher, staying there for the rest of the school year.

● Existence before essence makes wonders. As does numeracy before math.

#### **BundleBundleMath Wonder 06. Long multiplication.**

A grade 3 child sat alone with the desk turned ninety degrees to the wall in the back of the class. Why are you here? I cannot do long multiplication. Try to do this '27 x 34'. Well, 2 times 3 is 6, and 7 times 4 is 28, so the answer is 628. Are you sure? Yes. What does a calculator say? Let us see, 918, so we need 300 less 10, i.e., 290, where do they come from? Try to cross multiply. OK, 2 times 4 plus 7 times 3 is 8, plus 21 gives 29, but where does the zero come from?

Can I show you what we did when I was a child? Yes. We included the Bundle-units and wrote '2B7 x 3B4'. Here 2B times 3B gives 6BB, and 2B times 4 gives 8B, and 7 times 3B gives 21B, and 7 times 4 gives 28. With units and finishing the bundling, we get 6BB 29B 28, or 6BB 31B 8, or 9BB 1B 8, or 918 without units.

Can I try it? Yes, try '23 x 79'. Well, '23x 79' is really '2B3x 7B9', and  $2Bx7B$  is 14BB, and  $2Bx9$  is 18B, and  $3x7B$  is 21B and  $3x9$  is 27 Adding with units and finishing the bundling, we get 14BB 39B 27, or 14BB 41B 7, or 18BB 1B 7, or 1817 without units, and the calculator says the same.

In the break the child stayed at the desk to solve one problem after another. The class was curious and gathered around the child. In the next lesson they asked the teacher if they could also be allowed to use the Bundle method, so the child was invited to take the front seat to instruct the teacher, staying there for the rest of the school year.

● Existence before essence makes wonders. As does numeracy before math.

#### **BundleBundleMath Wonder 07. Short division.**

A grade 3 child sat alone with the desk turned ninety degrees to the wall in the back of the class. Why are you here? I cannot do division. Try to do this '81 / 3 '. Well, first 8 divided by 3 is more than 2 where 2 times 3 is 6, then 81 minus 6 is 75. OK. Then, 7 divided by 3 is 2 where 2 times 3 is 6, and 75 minus 6 is 69. OK. Then, 6 divided by 3 is 2 where 2 times 3 is 6, and 69 minus 6 is 63. OK. And then you carry on until you get the result, 222 and some more. Are you sure? Yes. What does a calculator say? Let us see, 27, but why 7, and where are the rest?

Can I show you what we did when I was a child? Yes. We included the Bundle-units and wrote 81 as 8B1 that we changed to 6B21 because 6 can be divided with 3. And 6B21 divided by 3 is 2B7 or 27 without the unit.

Can I try it? Yes, try '68/4'. Well, 68 is really 6B8 or 4B28, and 4B28 divided by 4 is 1B7 or 17 without the unit, and the calculator says the same. Likewise, 69 is really 6B9 or 4B29 or  $4B28 + 1$ , so 69 divided by 4 is 1B7 plus 1 that still needs to be divided by 4, or  $17 \frac{1}{4}$  without the unit.

In the break the child stayed at the desk to solve one problem after another. The class was curious and gathered around the child. In the next lesson they asked the teacher if they could also be allowed to use the Bundle method, so the child was invited to take the front seat to instruct the teacher, staying there for the rest of the school year.

● Existence before essence makes wonders. As does numeracy before math.

### **BundleBundleMath Wonder 08. Long division.**

A grade 3 child sat alone with the desk turned ninety degrees to the wall in the back of the class. Why are you here? I cannot do long multiplication. Try to do this '97/25'. Well, 9 divided by 2 is 4 with 1 as the rest, and 7 divided by 5 is 1 with 2 as the rest, so the results is 4+1 with 1+2 as the rest, i.e., 5.3. Are you sure? Yes. What does a calculator say? Let us see, 3.88, well that is really something else.

Can I show you what we did when I was a child? Yes. We saw long division as water going into two connected vessels with the same level, shown by our two fists. First, we used the reunite -formula to split 25 as  $25 = (25 - 5) + 5 = 20 + 5$ . This gives the width 20 to the left fist and the width 5 to the right fist. Then we used Bundle-units to rewrite '97/25' as '8B17 / 2B5' so that the 80 liters goes to the left fist and the 17 liters to the right fist. To find the levels in the two vessels we used the recount-formula to recount the liters in the widths:

Left:  $80 = (80/20) * 20 = 4 * 20 = 4 \text{ 20s}$

Right:  $17 = (17/5) * 5 = 3.4 * 5 = 3.4 \text{ 5s}$

We look at the right fist. To get from level 3.4 to level 4 it needs  $0.6 \text{ 5s} = 0.6 * 5 = 3$  liters from the left fist, that then will reduce its height with 3 recounted in 20s as  $3 = (3/20) * 20 = 0.15 \text{ 20s}$ , so it ends at the level  $4 - 0.15 = 3.85$ . This is close to what the calculator says:  $97/25 = 3.88$ . So, we could stop here.

To get closer can repeat and get 3.89 next time.

Can I try it? Yes, try '74/23'.

Well, I include the Bundle-units and write '74/23' as '6B14 / 2B3' so that 60 liters go to the left fist having 20 as its width and 14 liters go to the right fist having 3 as its width. The levels we find by recounting in 20s and 3s:

Left:  $60 = (60/20) * 20 = 3 * 20 = 3 \text{ 20s}$

Right:  $14 = (14/3) * 3 = 4\frac{2}{3} * 3 = 4\frac{2}{3} \text{ 3s} = 4.67 \text{ 3s}$

We look at the right fist. To get from level  $4\frac{2}{3}$  to level 3 =  $9/3$  it needs to give  $\frac{5}{3} \text{ 5s} = \frac{5}{3} * 5 = \frac{25}{3}$  liters to the left fist that will increase its level with  $\frac{25}{3}$  recounted in 20s as  $\frac{25}{3} = (\frac{25}{3/20}) * 20 = \frac{25}{60} \text{ 20s}$ , so it ends at the level  $3 + \frac{25}{60} = \frac{180}{60} + \frac{25}{60} = \frac{205}{60} = 3.42$ . This is rather close to what the calculator says:  $74/23 = 3.22$ . So, we can stop here or repeat.

In the break the child stayed at the desk to solve one problem after another by pouring water from one fist to the other and back again. The class was curious and gathered around the child. In the next lesson they asked the teacher if they could also be allowed to use the Bundle method and the reunite and recount formulas so the child was invited to take the front seat to instruct the teacher, staying there for the rest of the school year.

● Existence before essence makes wonders. As does numeracy before math.

### **BundleBundleMath Wonder 09. Equations Solved: to Opposite Side with Opposite Sign**

A grade 3 child sat alone with the desk turned ninety degrees to the wall in the back of the class. Why are you here? I cannot do solve equations, we only have three tries to fill in the empty square. Try to do this ' $\# + 12 = 37$ '. Well, here I never knew where to start my counting, but then someone told me that  $12 + \# = 37$  is the same equation, so let me start counting, 13, 14, ..., 21, 22, oh out of fingers, so my three guesses will be 14, 15 and 16. What does a calculator say? Let us see, 26, 27, 28, well 3 instead of 2, so I was close.

Can I show you what we did when I was a child? Yes.

We used our left hand to set up to formulas, a 'reunite formula' and a 'recount formula'. Show me your left hand and fix a short pencil to your thumb with rubber bands to have 6 fingers on the hand.

'6-2' means "From 6, pull away 2 to split the total". The rest is  $4 = 6 - 2$ . Now, pull back the two fingers. Then you have  $4 + 2 = 6$ , or  $(6 - 2) + 2 = 6$ . This is the 'reunite formula' that works for all Totals and Bundles,  $T = (T - B) + B$ . It says: "I still have the total if I pull back what I pulled away". Also, look at what happened:  $4 = 6 - 2$  and  $4 + 2 = 6$ . The number 2 can move to opposite side with opposite sign. This is how we solve equations with addition and subtraction.

'6/2' means "From 6, push away 2s to count the total". The number of times is  $3 = 6/2$ . Now, 3 times push back the 2s. Then you have  $3 * 2 = 6$ , or  $(6/2) * 2 = 6$ . This is the 'recount formula' that works for all

Totals and Bundles,  $T = (T/B) \times B$ . It says: "Pushing away bundles from a total tells you the number of bundles in the total." Also, look at what happens here: ' $3 = 6/2$  and  $3 \times 2 = 6$ . The number 2 moves to opposite side with opposite sign. This is how we solve equations with multiplication and division.

Now let me try with ' $\# + 12 = 37$ '?

Well, I just move 12 to opposite side with opposite sign and find that

$\# = 37 - 12 = 25$ . That was easy.

Now let me try with ' $\# \times 3 = 54$ '.

Again, I just move 3 to opposite side with opposite sign and find that

$\# = 54 / 3 = 18$ . That was also easy. Thank you.

In the break the child stayed at the desk to solve one problem after another by pulling and pushing fingers away and back on a hand where a short pencil was fixed to the thumb. The class was curious and gathered around the child. In the next lesson they asked the teacher if they could also be allowed to use the reunite and recount formulas and moving to opposite side with opposite sign to solve equations, so the child was invited to take the front seat to instruct the teacher, staying there for the rest of the school year.

● Existence before essence makes wonders. As does numeracy before math.

### **BundleBundleMath Wonder 10. Square Roots.**

A grade 3 child sat alone with the desk turned ninety degrees to the wall in the back of the class. Why are you here? I can only find the squares up to 3, then I give up.

Can I show you what we did when I was a child? Yes. We looked at a ten-by-ten square that we called a Bundle-Bundle-Board, a BBBoard. What is a Bundle? Show me a V-sign. V. Here we have 2 1s, until we close it. II. Now we have bundled the 2 1s into a bundle of 1 2s. OK. On a BBBoard we see that 2 2s is a Bundle of Bundles, a BundleBundle, and we see all the BundleBundles are squares. Yes. Also, we see that we get to the next square if we add two bundles and a corner. Yes. And that we get to the square before if we pull away two bundles and add the corner that was pulled away twice. Oh, I see, the 3-square is 9, and if I add 3 twice and 1 then I get to 16 as the 4-square, and if I add 4 twice and 1 then I get to 25 as the 5-square. That was easy, but there are still many squares left.

Nor really, try to go back from 100. OK, the ten-square is 100 and if I pull away 10 twice and add 1 then I get 81, right? Yes, now try to pull away 20 twice from 100. Then I get to 60, but now I must add a 2-square, so with 8 as ten minus 2 I get 10-2-2 tens plus a 2-square, 64. Yes. And with 7 as ten minus 3 I get 10-3-3 tens plus a 3-square, 49. Yes, now we write them in a row: 1, 4, 9, 16, 25, 36 = 2B 16, 4B 9, 6B 4, 8B 1.

Can you see a pattern? Yes, after five the squares are 2, 4, 6, 8 bundles, and they end with the squares of their less-numbers 4, 3, 2, 1, that is great.

And what do you see on a BBBoard? I see that a square has its less-square up in the right corner, beautiful. Do you see more? Yes, I see that a (6+4) square contains a 6-square and a 4 square and 4 6s twice. Right, so can we write  $(a+b)^2 = a^2 + b^2 + 2 \cdot a \cdot b$ ? Yes, the square of a sum is more than the sum of the squares. Right. And I also see that the square of 14 = 1B 4 is 1BB + 2x4B + 4^2 or 1BB 8B 16 or 1BB 9B 6 or 196, so now I can find all the squares up to 20, and also up to 99, and I can even find them on my two longest fingers without a BBBoard, great.

In the break the child stayed at the desk to solve one problem after another by dotting on two fingers. The class was curious and gathered around the child. In the next lesson they asked the teacher if they could also be allowed to use the Bundle method to find the squares, so the child was invited to take the front seat to instruct the teacher, staying there for the rest of the school year.

● Existence before essence makes wonders. As does numeracy before math.

### **BundleBundleMath Wonder 11. Add and Subtract Squares.**

A grade 3 child sat alone with the desk turned ninety degrees to the wall in the back of the class. Why are you here? I want to add and subtract squares, but we must first learn this in secondary school, so I was told to find it out myself.

Show me what you have found. I use a BundleBundleBoard with two squares, a 6-square and a 4-square and two stacks 4 6s. Yes. Then I push the 4-square down so I can add them next to each other connected by a Bottom-Top line. Yes. Then I replace the two squares with two more stacks so they fill the whole BBBoard except for a small 2-square in the middle. Yes. Now the Bottom-Top line is a diagonal in the stack, and I see that the four diagonals form a square surrounded by four half-stacks, which is the same as two full stacks, that also fill the BBBoard with the two squares. Yes. So the two squares add as the diagonal's square, that is nice isn't it. Yes, it is, it is called the Pythagorean rule in Greece, and the Gougu rule in China, nice, how do you subtract two squares?

Well, to get from one square to another we know that we just add bundles on-top and next-to and a square in the corner. So, I just turn the upper bundles to get a stack the is 6+4 long and 6-4 wide. That was easy.

In the break the child stayed at the desk to form squares of four books, four cards, and four like rectangles drawn on a paper and cut out. Then measuring the length of the sides and the diagonal. Then finding the squares by multiplying on two fingers. And then testing if the diagonal's square was the same at the sum of the sides' squares. Then testing the same on books, the table, a window and the door. The class was curious and gathered around the child. In the next lesson they asked the teacher if they could also be allowed to add and subtract squares, so the child was invited to take the front seat to instruct the teacher, staying there for the rest of the school year.

● Existence before essence makes wonders. As does numeracy before math.

### **BundleBundleMath Wonder 12. Finding the Square Root of a number.**

A grade 3 child sat alone with the desk turned ninety degrees to the wall in the back of the class. Why are you here? I try to find square roots. We now know that two squares add as the square of their Bottom-Top line. The teacher says that the side of a square is called its square root, but we must wait until late in secondary school to learn about it. So, I was told to find it out myself.

Show me what you have found.

On this BundleBundleBoard I add a 5-square and a 3 square. I have measured their Bottom-Top line to around 5.6, but I also like to find it as the square root of the sum of the two squares,  $25 + 9 = 34$ , that is between 5 squared, 25, and 6 squared, 36.

If I add two bundles and a corner, I get from the 5-square to the 6-square, 36. But I only need to get to 34. So, instead of a full bundle, I only need two parts of the bundles that I call  $2*5*p$ , to add  $34-25 = 9$ . First, I forget about the corner.

With  $2*5*p = 9$ , I get  $10*p = 9$ . Moving to opposite side with opposite sign, I get  $p = 9/10 = 0.9$ .

My first guess therefore is that the square root of 34 is 5.9. The calculator says 5.83, so I am close.

To get closer I repeat my calculations to fill the corner that now is a 0.9 square. Here, the equation is  $2*5*p = 0.9*0.9$ , or  $10 * p = 0.81$ , or  $p = 0.81/10 = 0.081$ .

So, my second guess is that the square root of 34 is  $5.9 - 0.081 = 5.819$ , which is below since I made the corner too big.

Each time, it is the same calculation, so I can use letters instead of numbers. So, next time I will find the square root of a number, N, I just find B as the last number with a lower square. Then instead of a full bundle I just add a part, p, found from the formula  $2*p = (N-B^2)/B$ . In this case,  $2*p = (34-25)/5 = 1.8$ , or  $p = 1.8/2 = 0.9$ . So, now we know that the square root of 34 is a little less than  $5+0.9 = 5.9$ .

My uncle sais that I can use the formula  $2*p = N/B - B$  instead. I don't know why, but it works OK,  $2*p = 34/5 - 5 = 6.8 - 5 = 1.8$ .

In the break the child stayed at the desk to find square roots of numbers between 1 and 100 by sliding a white paper over the BundleBundleBoard and putting numbers into the letter-formula. The class was curious and gathered around the child to suggest numbers to find the square root of. In the next lesson they asked the teacher if they could be allowed to find square roots also, so the child was invited to take the front seat to instruct the teacher, staying there for the rest of the school year. The next day the class was asked to find as many decimals in the square root of 2 as they could.

● Existence before essence makes wonders. As does numeracy before math.

### **BundleBundleMath Wonder 13. Squaring Rectangles.**

A grade 3 child sat alone with the desk turned ninety degrees to the wall in the back of the class. Why are you here? I would like to be able to square rectangles. But the teacher says we must wait until late secondary school to learn about it. So, I was told to find it out myself.

Show me what you have found.

On this BundleBundleBoard I have 6 4s with a 4-square below and 2 4s on top. To square it I take half of the top to the side. Now I have a 5-square, but with an empty corner. So, I remove a thin part called  $4 \cdot p$  from the side and the top to fill the corner. This gives me the equation  $2 \cdot 4 \cdot p = 1$ , or  $p = 1/4/2 = 0.125$ . So, my first guess is that the side is  $5 - 0.125 = 4.875$ . The calculator says that the square root of  $6 \cdot 4 = 24$  is 4.899, so I am below since the corner is filled too much.

Therefore, I need to fill the sides by reducing the corner from a 1-square to a 0.875-square. This gives the equation  $2 \cdot 4 \cdot p = 1 - 0.875^2 = 0.234$  where  $p = 0.234/4/2 = 0.029$ . So, my second guess will be  $4.875 + 0.029 = 4.904$ , which is very close to the calculator.

Then I tried to square my desk which is 80 cm and 40. So, on my BBBoard I have a stack of 8 4s if I count in tens. To square it I take half of the top to the side. Now I have a 6-square, but with an empty corner. So, I remove a thin part called  $6 \cdot p$  from the side and the top to fill the corner. This gives me the equation  $2 \cdot 4 \cdot p = 2 \cdot 2$ , or  $p = 4/4/2 = 0.5$ . So, my first guess is the side is  $6 - 0.5 = 5.5$ . The calculator says that the square root of  $8 \cdot 4 = 32$  is 5.657, so I am below since the corner is filled too much and therefore needs to be reduced from a 4-square to a 1.5-square. This gives the equation  $2 \cdot 4 \cdot p = 4 - 1.5^2 = 1.75$  where  $p = 1.75/4/2 = 0.219$ . So, my second guess will be  $5.5 + 0.219 = 5.719$ , which is a little above the calculator.

Of course, the 8 4s may also be seen as two 4 squares added together by their Bottom-Top up line's square i.e. as  $4^2 + 4^2 = 16 + 16 = 32$ , which is between 5-squared and 6-squared. So, I can use my formula for finding a square root,  $2 \cdot p = N/B - B$ , that here gives  $2 \cdot p = 32/5 - 5 = 6.4 - 5 = 1.4$ , with  $p = 1.4/2 = 0.7$ . So, here the first guess is  $5 + 0.7 = 5.7$  that is too much since I need to fill the corner also. Here  $2 \cdot 4 \cdot p = 4 - 1.7^2 = 1.11$  with  $p = 1.11/4/2 = 0.139$ . So, my second guess is  $5.7 - 0.139 = 5.561$ , which is below since the corner now is too big.

In the break the child stayed at the desk to find square rectangles found on books, windows, doors and floors. The class was curious and gathered around the child to suggest rectangles to be squared. In the next lesson they asked the teacher if they could be allowed to square rectangles, so the child was invited to take the front seat to instruct the teacher, staying there for the rest of the school year.

● Existence before essence makes wonders. As does numeracy before math.

### **BundleBundleMath Wonder 14. Squaring Triangles.**

A grade 3 child sat alone with the desk turned ninety degrees to the wall in the back of the class. Why are you here? I would like to square triangles. But the teacher says we must wait until late secondary school to learn about it. So, I was told to find it out myself.

Show me what you have found.

On this BundleBundleBoard I have 6 4s with 6 as the height and 4 as the base, and a diagonal that creates a triangle. I want to find the biggest square under the diagonal.

From the bottom right corner, I draw bigger squares with the same diagonal line until they meet the other diagonal and create a 2.4 square.

Funny enough, 2.4 is  $(6 \cdot 4)/(6+4)$ . So, I wondered: To find the biggest square under the diagonal, can you just recount the stack in half of its outside length? So, I tried again with letters instead of numbers. I have  $h$  bs with  $h$  as the height and  $b$  as the base, and a diagonal that creates a triangle. I want to find the biggest square under the diagonal. From the bottom right corner, I draw bigger squares with the same diagonal line until they meet the other diagonal and create a  $u$ -square.

The other diagonal now is a diagonal in two stacks, a small  $u$  ( $b-u$ )s stack and the big  $h$  bs stack. So the heights have the same number if they are recounted in the bases, in bs and in ( $b-u$ )s. So,  $u/(b-u) = h/b$ .

I move to opposite side with opposite sign and get  $b \cdot u = h \cdot (b-u) = h \cdot b - h \cdot u$ .

I move to opposite side with opposite sign and get  $b \cdot u + h \cdot u = (b+h) \cdot u = h \cdot b$

I move to opposite side with opposite sign and get  $u = h*b/(h+b)$

So, it works for all diagonal triangles: In a stack, you just divide the product with the sum to find the biggest square under the diagonal.

Let me try with this book. Here the numbers are 235 and 155, so this should give a  $(235*155)/(235+155)$ -square or a 93-square. To test, I take a ruler and my BBBoard. Yes, it works.

In the break the child stayed at the desk to test on one more book. Then the table was measured and calculated to find the length of the inside square before cutting it out of paper and placed in the corner. Finally she placed the diagonal with a string. The class was curious, gathered around the child and was amazed to see that the square could be predicted by a simple calculation. In the next lesson they asked the teacher if they could also be allowed to use the Bundle method to square triangles, so the child was invited to take the front seat to instruct the teacher, staying there for the rest of the school year.

● Existence before essence makes wonders. As does numeracy before math.

### **BundleBundleMath Wonder 15. Squaring Circles.**

A grade 3 child sat alone with the desk turned ninety degrees to the wall in the back of the class. Why are you here? I would like to square circles. But the teacher says we must wait until late secondary school to learn about it. So, I was told to find it out myself.

Show me what you have found.

On my BundleBundleBoard I have a circle with 10 as the radius. I can see that the left edge and the circle go together in the beginning before the circle curves away from the line, that therefore is called a tangent to the circle in the starting point.

A calculator says that ' $\tan(2) = 0.035$ ', so if the radius is 10 then the first part of the circle arc is very close to, 10 times 0.035 which is 0.35.

A full circle is 360 degrees, so a half circle is 180 degrees that recounted in 2s gives  $180 = (180/2)*2$  degrees =  $90*2$  degrees. And 90 times 0,35 is 3.14, a number that is called pi. So, with radius r, the length of the full circle arc is 2 times pi times radius.

If we split the circle into small 2-degree strips, we can rearrange them as a rectangle where the base is pi times r and the height is r, which shows how much a circle fills, pi times r times r. This rectangle can then be squared into a 1.77-square.

That is very nice, but do you know what a tangent does?

Yes, the tangent uses the diagonal angle to tell you what the height is if you know the base. We can always recount the height in the base as

$\text{height} = (\text{height}/\text{base}) * \text{base} = \text{tangent}(\text{angle}) * \text{base}$

Therefore,  $\text{tangent}(\text{angle}) = \text{height}/\text{base}$ .

So, the tangent is a per-number telling you how much height there is per base-unit. Once we have the actual base-number, we just multiply with the tangent of the angle to find the height-number.

In the break the child stayed at the desk to draw circles on a paper. Then the circle was changed into a rectangle that had the radius and pi times the radius as the sides. Finally the rectangle was changed into a square drawn inside the circle. The class was curious and gathered around the child to see how a circle was squared. In the next lesson they asked the teacher if they could also be allowed to square circles. "That is not possible", said the teacher, but the class showed what the child had done, so the child was invited to take the front seat to instruct the teacher, staying there for the rest of the school year.

● Existence before essence makes wonders. As does numeracy before math.

### **BundleBundleMath Wonder 16. Squares in STEM and STeN**

A grade 3 child sat alone with the desk turned ninety degrees to the wall in the back of the class. Why are you here? I was asked to do three square tasks for a STEM-day in next week.

Tell me what you are doing.

We are working with a ten fives stack on a BundleBundleBoard, and on paper, and on the floor. We see it as a window that needs a diagonal to not collapse.

Task one is to measure and calculate the length of the diagonal and its angles with the sides.

Task two is to square the 10 5s stack in case that in some rooms the light needs to come through quadratic windows.

Task three is to find both the square and the rectangle with the biggest area under the diagonal to insert colored window parts here.

Task four is to raise the diagonal by pushing a 1-square under it. We pause each time we have an extra ten degrees. So, with  $\tan(10) = 0.176 = 1/\text{place}$ , the first place we pause is at the  $1/0.176 = 5.7$  mark.

Task five is to find out if lines with length 6 and 7 and 8 all can pass round a corner from a 4- and a 2-corridor. Here they pass the two corridor squares on the way.

In the break the child stayed at the desk to finish the work. The class was curious and tried to gather around to see the tasks. In the next lesson they asked the teacher if they could also be allowed to propose some tasks. The teacher said "Yes, next time we have a STEM day we will all make proposals for tasks to do. And next month we will also have a STeN day where we solve tasks on a marketplace where economy and numeracy is included and where the sellers and buyers meet and use math differently". So, this time the child stayed at the desk to finish the STEM tasks.

● Existence before essence makes wonders. As does numeracy before math.

### **BundleBundleMath Wonder 17. Pouring Water with Quadratic equations.**

A grade 3 child sat alone with the desk turned ninety degrees to the wall in the back of the class. Why are you here? I would like to use and solve equations with squares. But the teacher says we must wait until late secondary school to learn about it. So, I was told to find it out myself.

Show me what you have found.

On my BundleBundleBoard I have an 8 4s cup and an 8 3s cup that both are quadratic at the bottom. Up the sides there are marks for Full and +2.

● First, I pour  $u$  units of water into a 4-square glass to the +2 mark:

$$u = 4 \cdot 4 + 2 \cdot 4 = 16 + 8 = 24$$

● Then, I pour 24 units into the 3-square glass to the  $u$  mark and get an equation that I solve by moving to opposite side with opposite sign:

$$3 \cdot 3 + u \cdot 3 = 24$$

$$9 + 3 \cdot u = 24$$

$$3 \cdot u = 24 - 9 = 15$$

$$u = 15/3 = 5$$

So, here the water rises until the +5 mark

● Next, I pour 35 units into a glass with an unknown bottom square,  $u$ , up to the +2 mark. This gives me a quadratic equation,  $u^2 + 2 \cdot u = 35$ , that I solve on my BBBoard arranged with four sections showing that

$$(u+1)^2 = u^2 + 2 \cdot u + 1.$$

So, with  $u^2 + 2 \cdot u = 35$ , I can now write

$$(u+1)^2 = u^2 + 2 \cdot u + 1 = 35 + 1 = 36 = 6^2$$

$$u+1 = 6$$

$$u = 6 - 1 = 5$$

So, here the glass needs to be a 5-square glass.

● Finally, I pour 35 units into a glass with an unknown bottom square,  $u$ , up to the +2 mark. But now the glass has a thick bottom going up to the 2 mark.

This gives me a quadratic equation,  $u^2 - 2 \cdot u = 35$ , that I solve on my BBBoard arranged with four sections showing that

$$(u - 1)^2 = u^2 - 2 \cdot u + 1$$

coming from pulling away the two  $2*u$  stacks from the top and the side and then adding the  $1*1$  that was pulled away twice.

So, with  $u^2 - 2*u = 35$ , I can now write

$$(u - 1)^2 = u^2 - 2*u + 1 = 35 + 1 = 36 = 6^2$$

$$u - 1 = 6$$

$$u = 6 + 1 = 7$$

So, here the glass needs to be a 7-square glass.

To get a formula I use letters instead of numbers.

So, I pour  $c$  units into a glass with an unknown bottom square,  $u$ , up to the  $+2*p$  mark. This gives me a quadratic equation,

$$u^2 + 2*p*u = c$$

that I solve on my BBBoard arranged with four sections showing that

$$(u+p)^2 = u^2 + 2*p*u + p^2.$$

So, with  $u^2 + 2*p*u = c$ , I can now write

$$(u+p)^2 = u^2 + 2*p*u + p^2 = c + p^2$$

$$u+p = \sqrt{p^2+c}$$

$$u = \sqrt{p^2+c} - p, \text{ which is a formula for the squared glass.}$$

In the first case we had  $c = 24$  and  $p = 1$ , so here

$$c = \sqrt{p^2+c} - p = \sqrt{1^2 + 24} - 1 = \sqrt{25} - 1 = 5 - 1 = 4.$$

In the break the child stayed at the desk to finish the work. The class was curious and gathered around the child. In the next lesson they asked the teacher if they could also be allowed to pour water in different glasses, so the child was invited to take the front seat to instruct the teacher, staying there for the rest of the school year.

● Existence before essence makes wonders. As does numeracy before math.

### **BundleBundleMath Wonder 18. Multiplication tables on a BundleBundleBoard**

A grade 3 child sat alone with the desk turned ninety degrees to the wall in the back of the class. Why are you here? I cannot do the multiplication tables. I can use my fingers to multiply numbers up to 5, but not more than that. Try to do this '6 x 7'. You want me to add 6 to itself 7 times? Well. I cannot do that. Are you sure? Yes. What does a calculator say? Let us see, 42, so twenty-four, but that is six time four?

Can I show you what we did when I was a child? Yes.

We used this ten-by-ten BundleBundleBoard with two bands after 6 & 7s so that we could both feel and see and recounting 6 7s in tens, which is what multiplying is about, to recount from small bundles to tens. Well, I have never heard that.

● With the first method we count the half-Bundles,  $\frac{1}{2}Bs$

$$6 * 7 = (5 + 1 + 2) * \frac{1}{2}B + 2 = 8 * \frac{1}{2}B + 2 = 4B 2 = 42$$

● With the second method we recount 7 as  $\frac{1}{2}B 2$

$$6 * 7 = 6 * \frac{1}{2}B 2 = 3B 12 = 4B 2 = 42 \text{ where we called } 3B 12 \text{ an overload since } 12 \text{ is really } 1B 2.$$

● With the third method we pull-away the outside Bundles

$$6 * 7 = (B - 4) * (B - 3) = BB - 3B - 4B - - 3 * 4 = (10 - 3 - 4)B + 12 = 3B 1B 2 = 4B 2 = 42.$$

Here we can see that minus times minus is plus since we must add the upper right corner that we pulled away twice.

● With the fourth method we use our fingers where on the left hand, 6 is 1up and 4down, and where on the right finger, 7 is 2up and 3down. Then we add the up-fingers and multiply the down fingers and get:

$$6 * 7 = (1+2)B (4*3) = 3B 12 = 4B 2 = 42$$

● With the fifth method we use our fingers where on the left hand, 6 is 1up, and where on the right finger, 7 is 2up. Then we add the up-fingers to five and multiply the up-fingers and this time we use half-bundles,  $\frac{1}{2}B$ :

$$6 * 7 = (1+2+5)*\frac{1}{2}B (1*2) = 8*\frac{1}{2}B (1*2) = 4B 2 = 42$$

We can see why on the BundleBundleBoard: We already have the 5 half-bundles and then get one more above and two more to the right, and the two in the corner.

● With the sixth method uses the upper right part of that BundleBundleBoard that are split in four parts. Below to the left we add the sides  $1+2 = 3$  and up to the right we multiply the sides:

Below:  $1+2 = 3$ , above:  $4*3 = 12$ , so again,  $6*7 = 3B 12 = 4B 2 = 42$ .

In the break the child stayed at the desk to solve one problem after another. The class was curious and gathered around the child to see this new way to multiply. In the next lesson they asked the teacher if they could also be allowed to use a BundleBundleBoard when learning the multiplication tables, so the child was invited to take the front seat to instruct the teacher, staying there for the rest of the school year.

● Existence before essence makes wonders. As does numeracy before math.

### **BundleBundleMath Wonder 19. Adding on-top with proportional ReCounting.**

A grade 3 child sat alone with the desk turned ninety degrees to the wall in the back of the class. Why are you here? I will tell you why:

The teacher said: "Welcome children, I am your teacher in math which is about the numbers that you can see on this number line and that is built upon the fact that one plus one is two as you can see here. So ..."

Showing a V-sign I interrupted him: "Mister teacher, here is one 1s in space, and here is also one 1s. If we count them in time, we can see how many 1s we have by saying 'one, two'. So, we have two 1s. But only until we add them as a bundle. Then we have one 2s, so 1s plus 1s become 2s, but one plus one is still one when we count it, and not two as you say. The thumb is also one 1s. They cannot be counted since they are not the same. But they can be added to one 3s. So, again one plus one is one. Here is another 3s on the other hand. They are the same, so we can count them as two 3s. And we can add them as one 6s. Or, we can split the two 3s into six 1s and see that two times three is six. So, the counting numbers two and three can be multiplied, but they cannot be added.

Therefore, please forget adding your line-numbers without units. Instead, help us adding the bundle-numbers with units we bring to school, as 2 3s and 4 5s, that we can add next-to as eights, or on-top as 3s or 5s as we can see on a peg board. If we add them next-to, we add plates, which my uncle calls integral calculus. And if we add them on-top the units must be changed to the same unit, which my uncle calls linearity or proportionality. He says it is taught the first year at college, but we need it here to keep and develop the bundle-numbers with units we bring to school, instead of being colonized with your line-numbers without units.

We know that you want to bundle in tens, and in ten-tens, and in ten-ten tens, but we like to bundle also in 2s, in 3s, in 4s, in half-tens, etc. We know that you have not been taught this and that the textbook doesn't teach it. But don't worry, we will teach you what we found out in preschool. Or better, instead of you colonizing our ways let us find out together what math may grow from our bundle-numbers with units. My uncle is a philosopher, and he calls it existentialism if we let existence come before essence.

And, when existence comes before essence, we must count the totals before we can add them. We know you say that 8 divided by 2 is 8 split in 2 parts, but to us 8 divided by 2 is 8 counted in 2s. You cannot split 9 in 2 parts, but we can easily count 9 in 2s as 4 bundles and 1 unbundled that becomes a decimal,  $9 = 4B 1 2s$ , or a fraction if we count it in 2s also,  $9 = 4 \frac{1}{2} 2s$ . Or, with negative less-numbers we get 5 bundles less 1,  $9 = 5B -1 2s$ .

Look at the fingers on a hand. You only see the essence, five, but we see all the ways the five fingers may exist when counted in 2s: 0B5, 1B3, 2B1, 3B-1, 1BB0B1."

## **BundleMath Wonder 20. Adding next-to with Integral Calculus.**

A grade 3 child sat alone with the desk turned ninety degrees to the wall in the back of the class. Why are you here?

Well, my uncle told me that the key to core mathematics is one simple question: “5 4s and 3 2s add to what?”

Adding next-to as areas brings you directly to integral calculus. And adding on-top after shifting units brings you directly to proportionality that leads on to per-numbers as 2\$/5kg when including physical units.

Which again leads to the root of calculus, adding per-numbers as in mixture problems: Adding 2kg at 3\$/kg and 4kg at 5\$/kg, the unit-numbers 2 and 4 add directly to 6, but the per-numbers 3 and 5 must be multiplied to unit-numbers before adding, thus, adding as areas as integral calculus, becoming differential calculus when reversing the question: “2kg at 3\$/kg and 4kg at how many \$/kg add to 6kg at 5\$/kg”, or “5 4s and how many 2s add to 5 6s?”.

So, calculus occurs in three versions.

Primary school: adding bundle-numbers by their areas. Middle school: adding piece-wise constant per-numbers as the area under the per-number graph. High school: adding locally constant per-numbers as the area under the per-number graph.

In the break the child stayed at the desk to solve one problem after another. The class was curious and gathered around the child. In the next lesson they asked the teacher if they could also be allowed to use the Bundle method, so the child was invited to take the front seat to instruct the teacher, staying there for the rest of the school year.

● Existence before essence makes wonders. As does numeracy before math.

## **BundleMath Wonder 21. ReCounting Goods gives PerNumbers and Fractions**

A grade 3 child sat alone with the desk turned ninety degrees to the wall in the back of the class. Why are you here? I would like to change units with per-numbers since they are all over, dollar per day, meter per second, 5 percent. But the teacher says we must wait until secondary school to learn about what he calls proportional thinking and that there is no per-numbers in mathematics. So, I was told to learn about proportional thinking myself.

Show me what you have found.

The total here exists as a rectangular bundle-number on a BBBoard. Here the ‘T=?’ question is answered by changing the unit in the per-number rectangle.

Recounting a physical total T as 3\$ and 5 kg gives a ‘per-number’ 3\$/5kg called the price and marked as a 3x5 rectangle on a BBBoard.

The question “20kg = ?\$” then is answered by recounting in the per-number:

$$20\text{kg} = (20/5)*5\text{kg} = (20/5)*3\$ = 12 \$.$$

On a BBBoard the counting sequences now are 5, 10, 15, 20kg, and 3, 6, 9, 12\$ since the per-number here has changed from 3/5 to 12/20.

Or we may introduce a new unit to make the digits like:

$$3\$ = (3/5)*5\$ = n*5\$ = 5n\$ \text{ with the new } n = 3/5. \text{ So, } 20 \text{ kg} = 20n\$ = 20*3/5\$ = 12\$.$$

$$\text{Alternatively, the units may be recounted: } \$ = (\$/\text{kg})*\text{kg} = (3/5)*20 = 12$$

$$\text{Or we may equate the per-numbers: } \$/\text{kg} = u/20 = 3/5.$$

$$\text{Moving to opposite side with opposite sign we then get } 5*u = 3*20, \text{ or } u = 3*20/5 = 12.$$

Or we may use our left little finger. Above to the left we write 20. Below to the left and right we write 5 and 3. Then we can see the result directly: 20 divided by 5 and multiplied with 3.

With the same unit, per-numbers become fractions, 3\$/5\$ = 3/5

The total here exists as a rectangular bundle-number on a BBBoard. Here the ‘T=?’ question is answered by changing the unit in the per-number rectangle.

If a whole contains a part, they have the same unit. In this case the per-number becomes a fraction without units. Still, we may use the units 'p' and 'w' for the part and the whole.

To get the fraction  $\frac{3}{5}$  of 20\$ means to get  $3p/5w$  of a 20\$ whole. Recounting in the per-number thus gives  $20w = (20/5)*5w = (20/5)*3p = 12p$ , or 12\$ of 20\$.

To get the fraction  $\frac{3}{5}$  of 100 means to get  $3p/5w$  of a 100 whole. Recounting in the per-number thus gives  $100w = (100/5)*5w = (100/5)*3p = 60p$ , or 60 of 100, written as 60%.

To ask "20\$ is what percentage of 80\$" means asking about the fraction  $20/80$  of 100. Or we may introduce a new unit  $80\$ = 100\%$  to see that  $20\$ = (20/80)*80\$ = (20/80)*100\% = 40\%$ .

In the break the child stayed at the desk to solve one problem after another. The class was curious and gathered around the child. In the next lesson they asked the teacher if they could also be allowed to use the Bundle method, so the child was invited to take the front seat to instruct the teacher, staying there for the rest of the school year.

● Existence before essence makes wonders. As does numeracy before math.

### **BundleBundleMath Wonder 22. The ReCount Formula and per-numbers are the core of STEM and STeN**

A grade 3 child sat alone with the desk turned ninety degrees to the wall in the back of the class. Why are you here? I would like to change units with per-numbers since they are all over, dollar per day, meter per second, 5 percent. But the teacher says we must wait until secondary school to learn about what he calls proportional thinking and that there is no per-numbers in mathematics. So, I was told to learn about proportional thinking myself.

Show me what you have found.

The ReCount Formula and per-numbers are the core of STeN (economics & Numeracy included)

STeN typically contains multiplication formulas about changing units

$\$ = (\$/\text{hour}) * \text{hour} = \text{salary} * \text{hour}$

$\text{kg} = (\text{kg}/\text{cubic-meter}) * \text{cubic-meter} = \text{density} * \text{cubic-meter}$

$\text{force} = (\text{force}/\text{square-meter}) * \text{square-meter} = \text{pressure} * \text{square-meter}$

$\text{meter} = (\text{meter}/\text{sec}) * \text{sec} = \text{speed} * \text{sec}$

$\text{energy} = (\text{energy}/\text{sec}) * \text{sec} = \text{Watt} * \text{sec}$

$\text{energy} = (\text{energy}/\text{kg}) * \text{kg} = \text{heat} * \text{kg}$

$\text{gram} = (\text{gram}/\text{mole}) * \text{mole} = \text{molar mass} * \text{mole}$

$\Delta \text{momentum} = (\Delta \text{momentum}/\text{sec}) * \text{sec} = \text{force} * \text{sec}$

$\Delta \text{energy} = (\Delta \text{energy}/\text{meter}) * \text{meter} = \text{force} * \text{meter} = \text{work}$

$\text{energy}/\text{sec} = (\text{energy}/\text{charge}) * (\text{charge}/\text{sec}) \text{ or Watt} = \text{Volt} * \text{Amp}$

In the break the child stayed at the desk to solve one problem after another. The class was curious and gathered around the child. In the next lesson they asked the teacher if they could also be allowed to use the Bundle method, so the child was invited to take the front seat to instruct the teacher, staying there for the rest of the school year.

● Existence before essence makes wonders. As does numeracy before math.

### **BundleBundleMath Wonder 23. Per-numbers add as Areas (Integral Calculus)**

A grade 3 child sat alone with the desk turned ninety degrees to the wall in the back of the class. Why are you here? I would like to change units with per-numbers since they are all over, dollar per day, meter per second, 5 percent. But the teacher says we must wait until secondary school to learn about what he calls proportional thinking and that there is no per-numbers in mathematics. So, I was told to learn about proportional thinking myself.

Show me what you have found.

The total here exists as a rectangular bundle-number on a BBBoard. The 'T=?' question is answered by using rubber bands to mark the bundles.

Asking "2kg at 3\$/kg and 4 kg at 5\$/kg total what?" the unit numbers 2kg and 4kg add directly whereas the per-numbers 3\$/kg and 5\$/kg first must be multiplied to unit-numbers before adding, thus added as areas, i.e., as integral calculus. Here the per-numbers are piecewise constant, but they may also be locally constant as in the case with a falling object having an increasing meter/second number.

Before adding, fractions must also be multiplied to unit, numbers. So, with apples, 1 red of 2 plus 2 red of 3 gives 3 red of 5, and of course not 7 red of 6.

Adding like per-numbers is predicted by power where, e.g., 6% 10 times gives  $106\%^{10}$  or 179%, i.e., the expected 60% plus additional 19%, and where 6% 20 times gives 321%, i.e., the expected 120% plus additional 201% showing the benefit of pensions.

Looking at my right hand, I see 3 fingers to the left, the Ls, and 2 fingers to the right, the Rs. I bent the two outer fingers. So,  $\frac{1}{3}$  of the Ls are bent, and  $\frac{1}{2}$  of the Rs. Does '1/3 of the Ls are bent' mean that '1/3 of the bent are Ls'? No,  $\frac{1}{2}$  is. So, in a cross table we cannot go from the per-numbers in one direction to those in the other direction without going through the unit-number table. This is called the Bayes-principle.

In the break the child stayed at the desk to solve one problem after another. The class was curious and gathered around the child. In the next lesson they asked the teacher if they could also be allowed to use the Bundle method, so the child was invited to take the front seat to instruct the teacher, staying there for the rest of the school year.

● Existence before essence makes wonders. As does numeracy before math.

### **BundleBundleMath Wonder 24. Algebra ReUnites Unlike & Like Unit- & Per-numbers**

A grade 3 child sat alone with the desk turned ninety degrees to the wall in the back of the class. Why are you here?

I would like to add per-numbers since they are all over, dollar per day, meter per second, 5 percent. But the teacher says we must wait until late secondary school where only few will be able to learn about something called calculus. Also, he says that there are no per-numbers in mathematics. So, I was told to teach myself calculus, which would not be easy since it contains three very, very difficult things called limits, derivatives and primitives, which are too difficult for most students. I think the teacher was joking. Instead, it is very easy if you use units. Only, it comes in the opposite order, first you add per-numbers, then you find smart formulas to do that for you.

Show me what you have found.

We already know that to add, per-numbers must be multiplied with totals to become unit-numbers. And the moment you multiply you create an area, like the bundle numbers with units that we bring to school but are not allowed to use. So, bundle-numbers and per-numbers add or integrate by their areas, that is why it is called integral calculus.

In early secondary school they are piecewise constant, and in late secondary they are locally constant, so here we must add very many tiny area strips.

But that is no problem if we write them as differences, since then all the middle terms disappear and only leave one difference between the start- and the end-number. This is why it is called differential calculus. And we already know that square-numbers grow by two bundles. So, we can always write  $2x$  as a change in  $x^2$ .

I simply don't understand why we always add without units, I mean 1 week and 1 day isn't 2?

Instead, we should use the beautiful Algebra Square that shows there are only four kinds of numbers in the world: Like and unlike unit- and per-numbers. And that addition and subtraction unite and split into unlike unit-numbers. And that multiplication and division unite and split into like unit-numbers. And that integration and differentiation unite and split into unlike per-numbers. And that power and the factor-finding root with factor-counting logarithm unites and splits into like per-numbers. And that equations are solved by moving to opposite side with opposite sign.

In the break the child stayed at the desk to solve one problem after another. The class was curious and gathered around the child. In the next lesson they asked the teacher if they could also be allowed to use the Algebra Square, so the child was invited to take the front seat to instruct the teacher, staying there for the rest of the school year.

● Existence before essence makes wonders. As does numeracy before math.

## Slides to the BundleBundlMath Wonders

### BundleBundlMath Wonder 01

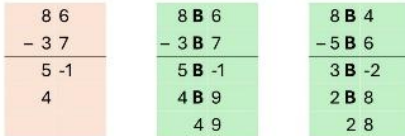
Add last Numbers,  $8 + 6 = ? = 1H3 + 1H1$



MATHCADER.NET • Numeracy with units

### BundleBundlMath Wonders 04

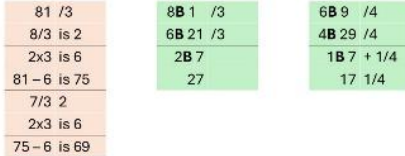
Subtraction



MATHCADER.NET • Numeracy with units

### BundleBundlMath Wonders 07

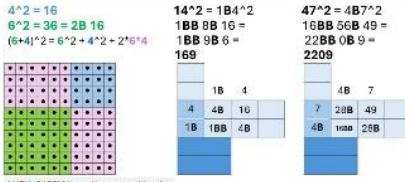
Short Division



MATHCADER.NET • Numeracy with units

### BundleBundlMath Wonder 10.

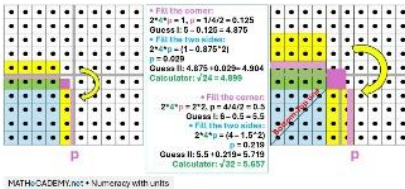
Squares on a BundleBundlBoard and Fingers



MATHCADER.NET • Numeracy with units

### BundleBundlMath Wonder 13. Square Rectangles

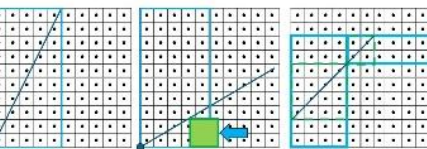
$64s \rightarrow B^2$   $84s \rightarrow B^2$



MATHCADER.NET • Numeracy with units

### BundleBundlMath Wonder 16

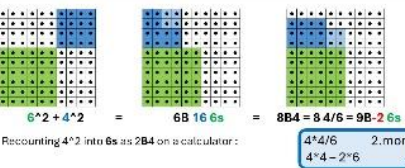
Five Square Tasks for a STEM or StEn day



MATHCADER.NET • Numeracy with units

### BundleBundlMath Wonder 19

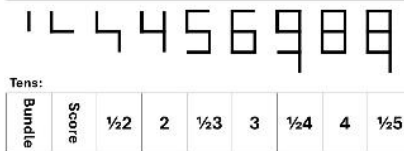
OnTop Addition leads to Proportional ReCounting



MATHCADER.NET • Numeracy with units

### BundleBundlMath Wonder 02

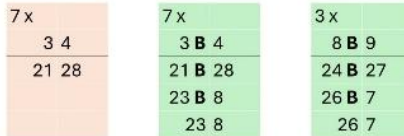
Numbers as Pictures & Viking Numbers



MATHCADER.NET • Numeracy with units

### BundleBundlMath Wonders 05

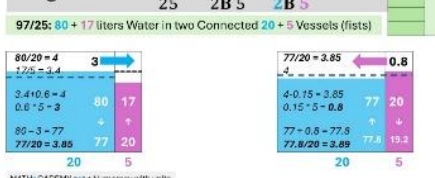
Short Multiplication



MATHCADER.NET • Numeracy with units

### BundleBundlMath Wonders 08

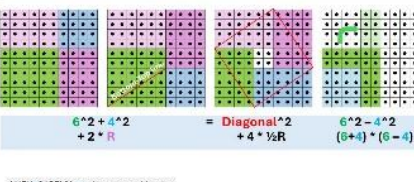
Long Division:  $\frac{97}{25} = \frac{9B7}{2B5} = \frac{8B17}{2B5} = 3.88$



MATHCADER.NET • Numeracy with units

### BundleBundlMath Wonder 11.

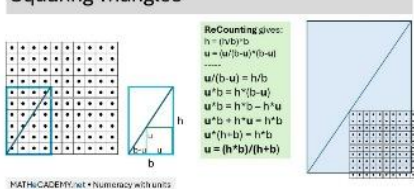
Add & Subtract Squares on a BundleBundlBoard



MATHCADER.NET • Numeracy with units

### BundleBundlMath Wonder 14

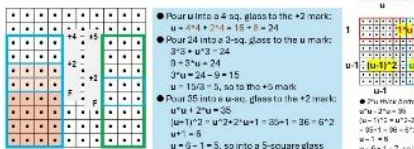
Squaring Triangles



MATHCADER.NET • Numeracy with units

### BundleBundlMath Wonder 17

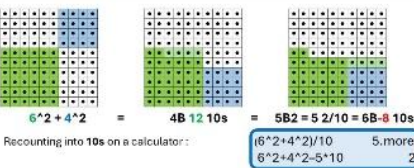
Pouring Water with Quadratic Equations



MATHCADER.NET • Numeracy with units

### BundleBundlMath Wonder 20

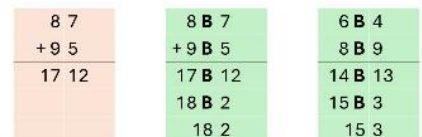
NextTo Addition leads to Integral Calculus



MATHCADER.NET • Numeracy with units

### BundleBundlMath Wonders 03

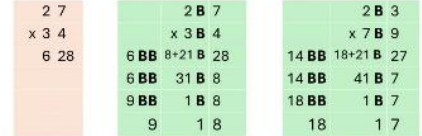
Addition



MATHCADER.NET • Numeracy with units

### BundleBundlMath Wonders 06

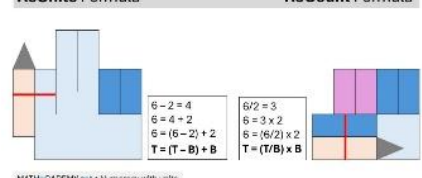
Long Multiplication



MATHCADER.NET • Numeracy with units

### BundleBundlMath Wonders 09

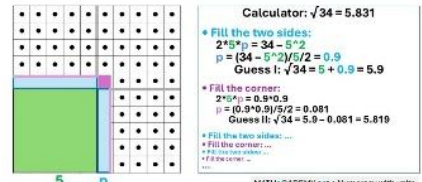
Equations Solved: to Opposite Side with Opposite Sign



MATHCADER.NET • Numeracy with units

### BundleBundlMath Wonder 12. Square Roots

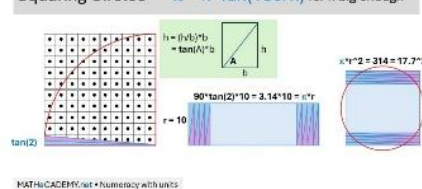
The Square Root of 34 =  $\sqrt{34} = 5 + p$



MATHCADER.NET • Numeracy with units

### BundleBundlMath Wonder 15

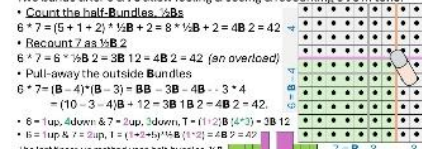
Squaring Circles  $\pi = n^2 \tan(180/n)$  for n big enough



MATHCADER.NET • Numeracy with units

### BundleBundlMath Wonder 18

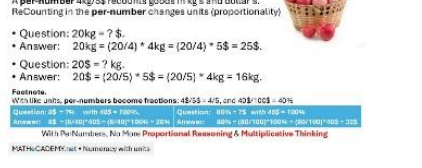
Multiplication Table with Early Algebra on a BBBoard



MATHCADER.NET • Numeracy with units

### BundleBundlMath Wonder 21. ReCounting

Goods gives PerNumbers and Fractions



MATHCADER.NET • Numeracy with units

## BundleBundleMath Wonder 22. The ReCount Formula and per-numbers are the core of STEM and SteN

(economics & Numeracy included)

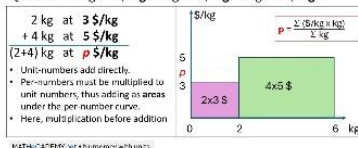
STEM & SteN typically contains multiplication formulas about changing units:

- $5 = (\$/\text{hour}) \times \text{hour} = \text{salary} \times \text{hour}$
- $\text{kg} = (\text{kg}/\text{cubic-meter}) \times \text{cubic-meter} = \text{density} \times \text{cubic-meter}$
- $\text{force} = (\text{force}/\text{square-meter}) \times \text{square-meter} = \text{pressure} \times \text{square-meter}$
- $\text{meter} = (\text{meter}/\text{second}) \times \text{second} = \text{speed} \times \text{second}$
- $\text{energy} = (\text{energy}/\text{second}) \times \text{second} = \text{Watt} \times \text{second}$
- $\text{energy} = (\text{energy}/\text{kg}) \times \text{kg} = \text{heat} \times \text{kg}$
- $\text{gram} = (\text{gram}/\text{mole}) \times \text{mole} = \text{molar mass} \times \text{mole}$
- $\Delta \text{momentum} = (\Delta \text{momentum}/\text{second}) \times \text{second} = \text{force} \times \text{second}$
- $\Delta \text{energy} = (\Delta \text{energy}/\text{meter}) \times \text{meter} = \text{force} \times \text{meter} = \text{work}$
- $\text{energy}/\text{second} = (\text{energy}/\text{charge}) \times (\text{charge}/\text{second}) = \text{Watt} = \text{Volts} \times \text{Amp}$

MATHeCADEMY.net Numeracy with units

## BundleBundleMath Wonder 23. Per-numbers add as Areas (Integral Calculus)

Question: "2kg at 3\$/kg + 4kg at 5\$/kg = 6kg at ?\$/kg?"



## BundleBundleMath Wonder 24. The Algebra Square ReUnites Unlike & Like, Unit- & Per-numbers

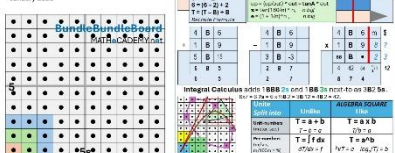
The Arabic word Algebra means to reunite, to unite and split. Numbers like unit numbers, **Addition** unites unlike unit numbers. **Multiplication** unites like unit numbers. **Integration** unites like per-numbers. **Power** unites like per-numbers. The opposite is to split into. **Subtraction** splits into unlike unit numbers. **Division** splits into like unit numbers. **Differentiation** splits into unlike per-numbers. Finally, the factor-finding root and factor-counting logarithm splits into like per-numbers.

| Unit / Split into | Unlike                                  | Like                                                     |
|-------------------|-----------------------------------------|----------------------------------------------------------|
| Unit-numbers      | $T = a + b$<br>(meter, second)          | $T = a \cdot b$<br>$T/b = a$                             |
| Per-numbers       | $T = f \cdot dx$<br>(m/sec, m/100m = %) | $T = a/b$<br>$dT/dx = f$<br>$b/T = a$<br>$\log_2(T) = b$ |

MATHeCADEMY.net Numeracy with units

## BundleBundleBoard Front & Back

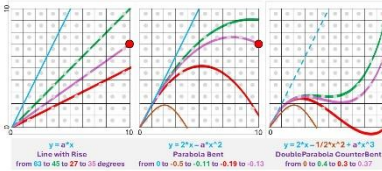
Allan.Tarp@gmail.com, January 2026



## No more PlaceValues or Carrying or Borrowing when Bundling deModels essence into existence

| Overload       | Underload      | Overload | Overload   |
|----------------|----------------|----------|------------|
| 65<br>+ 27     | 65<br>- 27     | 7 x 48   | 336 / 7    |
| 6B 5<br>+ 2B 7 | 6B 5<br>- 2B 7 | 7 x 4B 8 | 33B 6 / 7  |
| 8B 12          | 4B -2          | 28B 56   | 28B 56 / 7 |
| 9B 2           | 3B 8           | 33B 6    | 4B 8       |
| 92             | 38             | 336      | 48         |

## Parabola DartGame: Hit 7 with • Rise • Bent • CounterBent

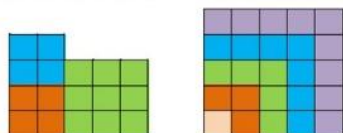


# BundleBundle Math on a BundleBundle Board

$T = 5 = 4 \text{ 111} = 1B 3$  2s  
 $T = 5 = 4 \text{ 44} = 2B 1$  2s  
 $T = 5 = 4 \text{ 44} = 3B -1$  2s  
 $T = 5 = 4 \text{ 44} = 1BB 0B 1$  2s

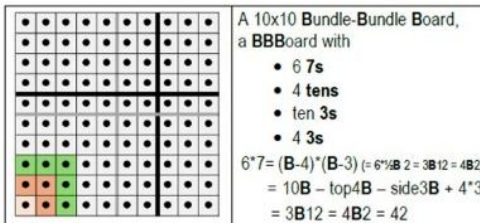
Numeracy = Math with units, where  
 Addition folds while Multiplication holds, using  
 Children's own CountingNumbers with BundleUnits.  
 A paradigm-shift from HalfMath to FullMath

Existence before Essence means Counting before Adding



4 2s, 2BB 2s, 1BBB 2s, 3 3s, 1BB 3s, 1BB 5s = 1BB2B1 4s  
 2 2s, 1BB 2s, 1 3s, 1B 3s, 1BB 4s = 1BB2B1 3s  
 1 2s, 1B 2s, 1BB 3s = 1BB2B1 2s

4 2s plus 3 3s add next-to as 3B2 5s or 3 2/5 5s or 4B3 5s as an example of Integral Calculus adding areas



Allan.Tarp@MATHeCADEMY.net, Denmark, September 2025, beta 05

<http://mathecademy.net/bundlebundlemath-on-a-bbboard/>

SteN solves Problems on a BundleBundleBoard to make all Youth Numerate by 2030

# NUMERACY

BBM • BundleBundleMath with units (Kids: II II "Not 4, but 2 2s")

**PULL** away & back (minus & plus)

$T = (T-B) + B$

$5 - 3 = 2$ . But,  $5 = (5-3) + 3$ , so  $5 - 3 = 2$

$5 - 3 = 2$ . But,  $5 = (5-3) + 3 = 5 - 3 = 2$

**PUSH** away & back (divide & multiply)

$T = 4 \times 8$

$T = (T/8) \times 8$

$4 \times 8 = 32$ . But,  $4 = (4/8) \times 8$ , so  $4 \times 8 = 32$

$4 \times 8 = 32$ . But,  $4 = (4/8) \times 8$ , so  $4 \times 8 = 32$

**Next BundleBundle: 1BB 2B 1**

Before BB-square: 1BB 2B 1

$7 = 0B 7 = 1B -3 = 1/2B 2$  tens

$\Delta B^2 = 2B -1$

Later,  $(x^2)^2 = 2^2 \times x$

**Add & Subtract etc.**

$8 + 6 = (8+3) + (3+3) = 11 + 6 = 17$

$29 + 36 = 2B 8 + 1B 6 = 3B 14 = 4B 4 = 7B 6$

$8 + 48 = 5B 6 = 5B 6 = 5B 6 = 5B 6$

$368 / 8 = 36B 8 / 8 = 36B 4 / 8 = 36B 4 = 46$

$T = 6^*7 = 6^*(7/2) = 3B 12 = 42$  (notice -- is +)

$T = 6^*7 = 6^*(7/2) = 3B 12 = 42$  (notice -- is +)

**ReCount 6 7s into 5B-8 tens and 4B 2 tens**

To square the total  $T = 64$ s, half of the (6-4) 4s move from the top to the side to get a 5 x 5 square, and an unfilled square in the upper corner.

On a BBBBoard four 2 2s arranged as a 5-by-5 square contains 2 squares (3 3s and 2 2s) as well as 2 stacks. But it also contains one square formed by the stacks' diagonals as well as four half stacks.

A triangle has the points A(3,4) and B(6,8) and C(5,5) packed inside at 4 5s stack. We find its angles and sides. Ar is A's right angle.  $\pi = n^* \tan(180/n)$  for n large

On a BBBBoard,  $(u+3)^*(u+3)$  is a square with four parts, two squares ( $u^2 + 3^2$ ), and two stacks,  $2^*3u$ , so that  $T = u^2 + 6^*u + 9$ .

**Add Stacks**

$2 3s + 4 5s = 1B 1 5s + 4 5s = 5B 1 5s$

$2 3s + 4 5s = 2 3s + 6B 2 3s = 8B 2 3s$

On a 2B4 step from the (0,0) dot to the (4,0) dot,  $4^*2 = 8$ . On the way the  $1/2^*2$  line meets the circle,  $1/2^*2 = 1$ . Another line has  $1/2^*2 = 1$ . Where the line meets the circle,  $1/2^*2 = 1$ . This gives  $1/2^*2 = 1$ . So, they meet at point (0,1).

On a circle with radius 10 and center at (0,0), dot,  $1/2^*2 = 10^2 = 100$ . On the way the  $1/2^*2$  line meets the circle,  $1/2^*2 = 100$ . Another line has  $1/2^*2 = 100$ . Where the line meets the circle,  $1/2^*2 = 100$ . This gives  $1/2^*2 = 100$ . So, they meet at point (0,10).

A trip where  $1/2^*2 = 1$  is a bent line called a parabola. It meets the  $1/2^*2$  line in point (1,1). Here  $1/2^*2 = 1$ .  $1/2^*2 = 1$ .  $1/2^*2 = 1$ .  $1/2^*2 = 1$ . There are two solutions,  $1/2^*2 = 1$ . This gives  $1/2^*2 = 1$  and  $1/2^*2 = 1$ . So, they meet at points (1,1) and (1,1).